Editorial

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This special issue is devoted to Vasily Davydov and the ground-breaking work that he introduced together with Daniil Elkonin.[1] Their work, substantiated by many years of rigorous experiments, presents a theoretical extension and an educational concretisation of foremost Lev Semyonovich Vygotsky's cultural historical theory and Alexei Leontiev's activity theory. In the West, the curriculum in mathematics for the youngest students stemming from Davydov's longitudinal series of experiments is most known as the Davydov curriculum or program. Jean Schmittau together with her colleague Ann Morris is perhaps among the first to "import" the Davydov program into the US (Schmittau & Morris, 2004). There are other established implementations of the Davydov curriculum especially in the US, such as the project "Measure up" in Hawaii led by Barbara Dougherty (Dougherty & Slovin, 2004). More recently, Martin Simon (Simon & Placa, 2012) has been leading a five-year research project inquiry into the mechanisms of mathematics conceptual learning, where the Davydov curriculum is used as the main framework to shift the curriculum in mathematics education in the US. To date, the program has become known in other countries as well and a growing number of researchers are exploring its tenets and potential implications in mathematics education. Interestingly, the Davydov program is more frequently referred to within the research field of early algebraisation (see for e.g., Cai & Knuth, 2011) and researchers are looking to Davydov's heritage as a source of inspiration for new developments and new perspectives on what and how to teach elementary mathematics, thus introducing a new paradigm of psychological, mathematical, and pedagogical knowledge in the field of mathematics education.

Within the scope of the 4th International Congress of the International Society for Cultural-historical Activity Research (ISCAR) that took place in 2014 in Australia, a group of researchers organised a symposium titled "Transforming Davydov's Learning Activity Curricula into New Contexts - Examples from Canada, Italy, and Sweden." The symposium focused on the implementation and development of Davydov's ideas in contemporary research and practice in mathematics education. The meeting sparked deeper interest and further research on Davydov's legacy. This special issue continues the conversation initiated during the 2014 ISCAR meeting by bringing forth research projects inspired by Davydov's intellectual heritage.

You may wonder, what makes Davydov's ideas so attractive. Well, the answer is twofold. First, there is a growing pressure on schools to transform the mathematics teaching in a way that students develop agency and interest in mathematics. Thus, there is a societal need to expand mathematical teaching beyond operational methods and routines. Second, today we have rather substantial evidence, both from the US and from Russia that students following Davydov's program outperform other students in problem solving tasks

1 Daniil Elkonin must perhaps be recognised as the first to start the experimental work on how to realise an education that allows students to develop theoretical thinking (higher order thinking) and agency in relation to learning. The research-group that continues Elkonin's and Davydov's work in Moscow is led by Elkonin's son Boris Elkonin. To date, this research program includes not only developments of curricula in mathematics but also in natural science, language, literature, and art.

(see for e.g., Schmittau, 2004; Zuckerman, 2005). There are also results indicating that the Davydov program used during the first three school years has a long-lasting positive impact (Blake et al., 2003; Ching-Shu, 2006; Dougherty & Simon, 2014; Kinnard & Kozulin, 2008; Zuckerman, 2005) helping students to develop a profound understanding of the mathematics they learn. In light of this growing research, we argue that the insights garnered from Davydov's work contribute a great deal to moving the elementary mathematics education to a new higher level.

You may also wonder, what in the Davydov program can meet the type of societal demands we are currently facing. The answer to this question requires some level of scrutiny of Davydov's work. To arrive to a preliminary understanding, we need to examine the program and its underpinning ideas. Davydov's underpinning theoretical framework and his experimental work are intricately complex and in many ways very different from the teaching theories and pedagogical traditions dominating many of the Western countries. Therefore, we think it is important to coalesce recent theoretical and empirical work that highlights some of Davydov's key tenets and principles and brings forth his legacy. We hope that this will spark interest among readers to retrieve Davydov's original works and access what we could not include in this edition for lack of space.

In "One is not born a mathematician: In conversation with Vasily Davydov," Fellus and Biton (this issue) provide an overview of Davydov's tenets and theoretical stance and bring forth some practical implications. The authors invite the reader to listen, through the written mode, to an imaginary conversation with Vasily Davydov where they not only unpack "notions that include, inter alia, language and interaction, learning and teaching, and empirical and theoretical thinking" but also raise issues that are relevant to mathematics education.

Two contradictory aspects come to light. One pertains to the concept of *learning activity;* the other to the cognitive foundations of the teaching of mathematics in elementary school. One of the key theoretical elements we inherited from the Russian scholars is the concept of "learning activity." According to Davydov, a learning activity can only occur if the students themselves identify a problem, develop a motive to find a solution to the problem, and formulate the means to solve the problem. It is important to note that a learning activity is not the same as a teaching activity. In a teaching activity, students may do what they are asked to do but they may not necessarily find value in carrying out these activities, and thus not effectively engage in a learning activity. Students may learn things but they will not develop capabilities to regulate their own learning. Further, being involved in a qualifying theoretical work, such as a learning activity, is not only seen as a means for knowledge formation but also as a means for cognitive development and agency. In many of our Western educational systems, knowledge and development are seen as two separate things where development leads or precedes knowledge acquisition. For Davydov, following Vygotsky's legacy, it is the other way around. The process of knowledge acquisition and construction precedes the cognitive development of a student.

When we think of the foundation of mathematics' teaching and learning in elementary school, the Western tradition preconises concrete manipulations, concrete number knowledge, and numerical operations as the means for abstract and algebraic reasoning development. In the Davydov program, the theoretical algebraic reasoning is the foundation of arithmetic. In many countries and educational contexts, counting and operating with numbers and objects, in line with children's everyday pre-school experiences, is perceived as an ideal way to introduce mathematics. Davydov argues that, from the very beginning, children can - and must - encounter tasks requiring modelling and theoretical reasoning. For example, measurement is a powerful and central concept in the Davydov program. To develop the concept of measurement, students analyse their measuring experience in a general way and only then use the developed abstract knowledge to understand numbers and mathematical operations. This is often described as ascending from the abstract to the concrete (Davydov, 2008). The reader should not confound this approach with the well-known “explicit teaching” approach, where the knowledge is first presented in a theoretical way and then applied to concrete situations.

The issue of abstract versus concrete in mathematical thinking development is further studied by Polotskaia (this issue). The author draws upon a formative experiment (Polotskaia, 2015; Savard & Polotskaia, 2014) conducted in elementary schools in Canada to discuss the transformative power of the Relational Paradigm in mathematics education and suggests the idea of equilibrium between the different types of thinking. In line with this idea, Venenciano (this issue) studied the performance of Grade 2 students who were learning the Measure-Up curriculum and those who were learning in the US-Standard curriculum. Her analysis suggests that the early study of number within the US-Standard curriculum “interfere[s] with students’ thinking about quantitative relationships” in a negative way.

From the very beginning of his experimental work, Davydov explored mathematical and cognitive expressions of young students. “Laboratory observations showed that the "capable" children acted reflectively on their own initiative, while the rest needed some help from an adult. Reflective thinking is the zone of proximal development of children starting school.” (Zuckerman, 2014:2). In line with the Davydov program, students with varying experiences and capabilities can jointly establish a learning activity and thus, be taught at the same high theoretical level (Rubtsov, 2013).

Two studies in this issue examine the possibility of introducing relational, algebraic, and theoretical thinking to novice learners of higher ages. Mellone and Tortora (this issue) conducted a design study in Italy with Grade 5 students. The authors discuss how Davydov’s acute insights can be anticipatory to the urgent issue of early algebra in school. Bobos and Sierpinska (this issue) used the concept of measurement to ground the prospective teachers learning about fractions and “to prepare future elementary teachers to think about fractions of quantities in a theoretical way.”

The idea of a contradiction itself is an important teaching tool in the Davydov program as well as in activity theory. Both propose that contradictions give rise to change and development. While planning a learning situation, a contradiction becomes a source for the students’ curiosity, helps them to identify a problem, and sets them on the path to initiate reflective thinking. Work in Sweden (Eriksson and Jansson, in this issue) proposes a discussion about task design allowing for a learning activity to arise in a Grade 1 elementary class.

The collection of studies, however limited, presented in this special issue addresses different aspects of the Davydov heritage. We hope it will nourish the research interest to his legacy and initiate a larger discussion about theoretical and practical contradictions in mathematics education giving rise to change and development.

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