Physical and Virtual Manipulative Framework Conceptions of Undergraduate Pre-service Teachers

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The goals of this study were to analyze undergraduate elementary school pre-service teachers' conceptions of physical and virtual manipulative materials using an interpretive *framework*, and present lessons learned from presenting this framework to them. The *framework* involved the use of the *Concrete*, *Pictorial* and *Abstract cognitive levels* combined with the *Virtual level*, or *CPVA framework*. The *Virtual level* involves *three sublevels*: *virtual-Concrete*, *virtual-Pictorial* and *virtual-Abstract*, which were used to describe the capabilities of apps and applets that incorporate digital and dynamic components (the possibility of virtual movement and manipulation on the screen). The *CPVA instrument* was developed and used to answer two research questions related to participants' understanding of *CPVA levels* using the *framework* during a mathematics for elementary school methods course. Pre-, post- and follow-up test scores were analyzed using a one-way repeated measures *ANOVA*. The participants demonstrated improvement from pre-test to post- and follow-up-tests. However, this improvement was limited by their understanding of the *Virtual level*.

This study involved the analysis of undergraduate pre-service teachers' conceptions of physical and virtual manipulative materials using an interpretive framework. The framework involved the use of the well-known Concrete, Pictorial (Representational) and Abstract (CPA or CRA) cognitive levels (Gujarati, 2013; Sousa, 2008) combined with the Virtual level, which resulted in the CPVA-framework involving virtual-learning tools (such as applets and apps). Also, this level involves three sublevels (virtual-Concrete, virtual-Pictorial and virtual-Abstract), which were used to describe the capabilities of apps and applets involving digital and dynamic components (the possibility of virtual movement and manipulation on the screen). The CPVA-Learning Levels (CPVALL) Instrument (see Appendix A) developed by the investigators was used to answer two research questions:

What initial conceptions of physical and virtual manipulative materials when using the *CPVA-cognitive levels* were undergraduate pre-service teachers bringing to a mathematics for elementary school methods course (*CPVALL Instrument* pre-test scores)?

How did the undergraduate pre-service teachers' conceptions of physical and virtual manipulative materials when using the *CPVA-cognitive levels* change during a mathematics for elementary school methods course (*CPVALL Instrument* pre-, post and follow-up test scores)?

Rationale

Developments in educational technology have provided reasons to reassess *the initial CPA*- or *CRA-cognitive levels* and related research findings. The *Concrete-Pictorial-Virtual-Abstract* (*CPVA*) *framework* presented in this study could be used for this purpose. For example, as part of a learning environment, this framework could help in analyzing the appropriateness of an applet or app to accommodate for a student's developmental level, learning needs or challenges in a learning sequence/progression. In this manner, if a student could benefit more from using physical manipulative materials instead of virtual manipulatives, or vice versa, then the proper selection and understanding of what these learning tools could provide and how they could help or not are very relevant.

As a subpart of the *CPVA framework*, the *Concrete level* involves the use of physical manipulative materials as instructional tools. This selection could involve real pencils, cubes, or apples. Similarly, the *Pictorial level* involves the use of drawings, pictures or images of the physical manipulative materials. This other selection could involve pictures or images of pencils, cubes, tiles, or apples, including non-dynamic-digital images. In this case, non-dynamic indicates the lack of possible movement on paper, printout or computer screen. The *Abstract level* involves the use of symbolic or verbal representations: ten, sixth, IV, 3, 45, 2.5, or 1/3. This alternative includes digital text or audio versions. The combination of these learning levels is necessary to represent mathematical ideas. For example, the use of five physical pencils to represent the numeral 5 involves the combination of the *Concrete* and *Abstract levels*.

The use of Concrete, *Pictorial* and *Abstract levels* is accepted and supported by mathematics education researchers (Clements, 1999; Naiser, Wright, & Capraro, 2004; Suydam, 1985; Suydam & Higgins, 1977). According to Clements (1999), researchers have indicated that students who use manipulatives to learn mathematics: 1. outperformed those students who do not, including the areas of retention of concepts learned and problem-solving skills; 2. benefited from this use no matter what grade level, ability level, or topic; and 3. improved their attitudes toward mathematics as *knowledgeable* teachers use the manipulative materials appropriately.

In this study, as described by Moyer-Packenham and Bolyard (2016) the *Virtual-level* involves "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (p. 3). Also, as Duebel (2010) stated, static (without movement) and dynamic (with movement) virtual models can be found on the Web, but static models are not considered "true" virtual manipulatives. Static models are more like pictures of physical objects or manipulative materials that have traditionally been used in classrooms, but learners cannot actually "manipulate" them. For this reason, in this study, "*Virtual*" is meant to involve both digital and dynamic components (with the possibility of virtual movement and manipulation on the screen). Also, digital video, projections on a screen or printout from computer digital files were not considered options for the *Virtual level*.

Furthermore, besides having digital and dynamic components, the *Virtual level* was considered to have *three sublevels*: *virtual-Concrete*, *virtual-Pictorial*, and *virtual-Abstract* (*virtual-CPA*). These *sublevels* could be used in isolation or in combination with one or more of the other levels. However, it is important to note that in most mathematics teaching activities the *Abstract level* is usually involved because of the need for words and/or symbols to communicate mathematical ideas.

The *virtual-Concrete level* includes digital-dynamic (non-static) versions of the concrete materials or manipulatives, which could be virtually manipulated on the screen in a manner similar to physical objects to represent quantities or patterns (Ortiz, 2017). This process implies that the images on the screen can be virtually moved and relocated. The *Base Blocks* applet (Utah State University, 2017) provides an example of a possible use of *virtual-Concrete level* (see Figure 1). The virtual blocks can be manipulated (dynamic component) and moved on the screen (digital component) to represent place value ideas. This applet combines other cognitive levels as well. For example, the *Pictorial level* is involved digitally (but without dynamic movement) with the images at the top of each column (cube for thousands, flat for hundreds, long for tens and unit for ones); and the *Abstract level* is involved digitally (but without dynamic movement) when the students read the words and numerals on the screen (for example, Clear, Use blocks to represent a number, and the numeral 1248 on the right side of Figure 1). This applet was used as part of item 10 in the *CPVALL Instrument* (see Appendix A).

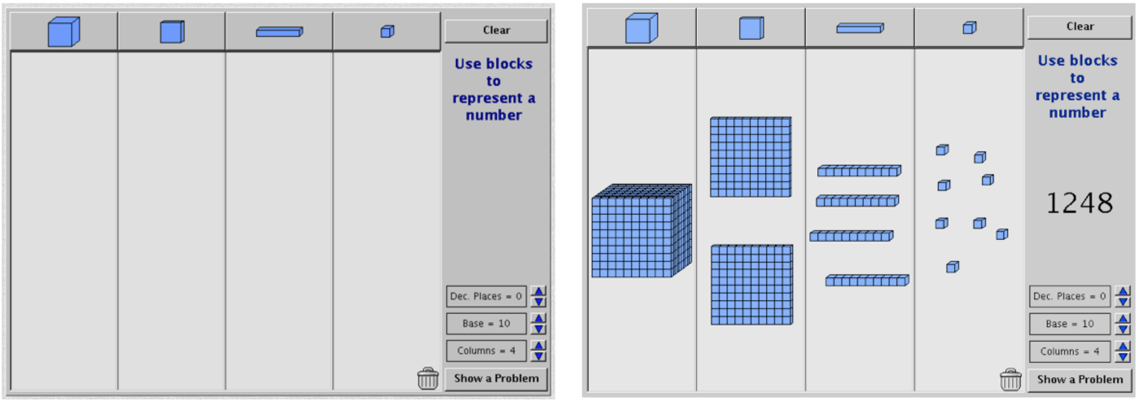

Figure 1. *Base Blocks applet* before adding *Base-ten blocks* (left side), and after adding *Base-ten blocks* virtually (right side) (blocks can be moved from one column to another).

The virtual-Pictorial level allows dynamic changes of the colors of the virtual images (such as circles or squares in a dynamic manner) but without allowing for the virtual movement of these images from one place of the screen to another as was possible with the Base Blocks applet. The Bar Chart (Utah State University, 2017) applet is an example of the virtual-Pictorial level. In the Bar Chart, the color of the columns may be manipulated but cannot be moved around the screen (see Figure 2). This applet was used as part of item 9 in the CPVALL Instrument (see Appendix A).

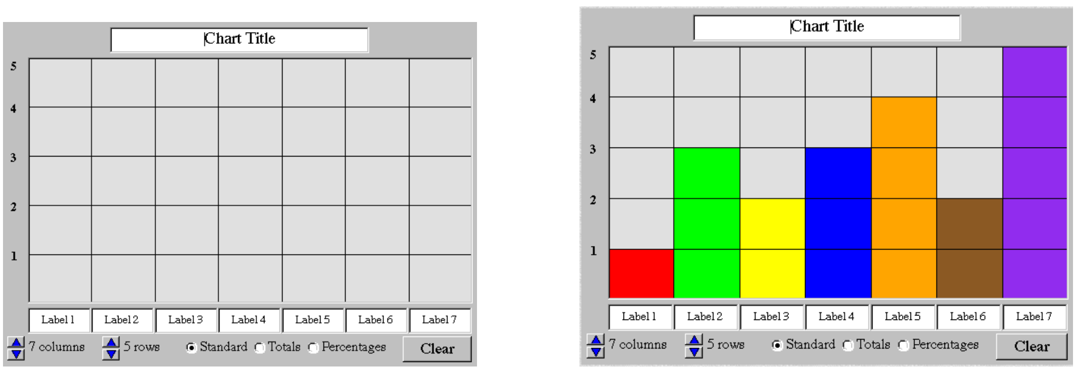

Figure 2. *Bar Chart applet* before adding colors to squares (left side), and after adding colors to squares (right side) virtually (squares cannot be moved around the screen).

The *virtual-Abstract level* allows for the dynamic manipulation of numbers or words. The *Sieve of Eratosthenes* applet (Utah State University, 2017) is an example of this virtual level (see Figure 3). This applet was used as part of example in the treatment section. The numbers in the columns may be dynamically manipulated by pressing any of the numbers and dynamically providing an output of the remaining numbers (not divisible by 2). This process may be repeated by using other prime numbers until only prime numbers are left in the columns. Table 1 presents a summary of the *CPVA-levels* and components.

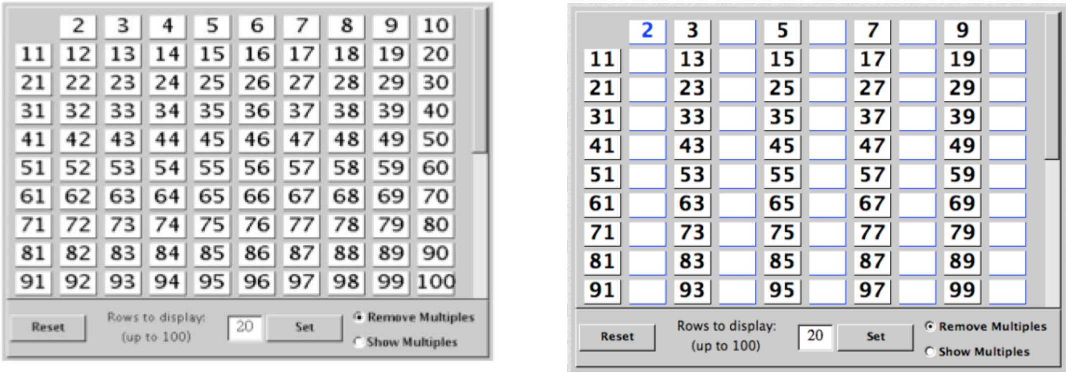

Figure 3. Sieve of Eratosthenes *applet* before pressing on any of the buttons representing numbers from 2 to 100 (left side), and after pressing number 2 as a factor (right side) (the multiples of 2 were eliminated dynamically).

Table 1
Learning levels and component combination

Level/Component	Non-Digital	Digital	Dynamic (with movement)	Non-dynamic (without movement)
Concrete	x		x	
Pictorial	x	x		x
Abstract	x	x		x
virtual-Concrete		x	x	
virtual-Pictorial		x	x	
virtual-Abstract		x	x	

Background

There are encouraging findings related to the use of virtual manipulatives (Moyer-Packenham & Westenskow, 2013; Moyer-Packenham & Westenskow, 2016), but what we have is difficult to generalize across concepts and grade levels (Burns & Hamm, 2011). One possible solution to overcome this limitation is to use *CPVA-framework* to provide more consistent terminology and clarity. When reviewing the literature, different combinations of the *CPVA-levels* were found, but it was not always clear from their descriptions which *Virtual-levels* (*virtual-Concrete*, *virtual-Pictorial* and/or *virtual-Abstract*) were involved and might be more effective. Nevertheless, these studies have shed some light into the possible effectiveness of *virtual manipulatives*. Overall, most studies have compared the effectiveness of *Concrete-level* (physical manipulative materials) versus *virtual-Concrete level* (*virtual manipulatives* based on physical manipulative materials).

Moyer-Packenham and Westenskow (2013) conducted a meta-analysis synthesizing the findings from 66 research reports, which examined the effects of (VMs) on student achievement. They indicated that 32 of the analyzed studies contained data yielding 82 effect size scores with effects of virtual manipulatives on student achievement and contributed to a conceptual analysis of affordances promoting mathematical learning. In this context, affordances are "cues of the potential uses of an artefact by an agent in a given environment [and] cues that *invite* an agent to act upon the artefact" (Burlamaqui & Dong, 2014, p. 13). Also, when compared with other instructional treatments, they reported that the results of the average effect size scores yielded a moderate effect for virtual manipulatives. When compared with physical manipulative materials and textbook instruction and by mathematical domains, grade levels, and study duration, there were additional large, moderate, and small effects for virtual manipulatives. In addition, they reported that the conceptual analysis of the data provided empirical evidence for five specific interrelated affordances of the *VMs* promoted mathematics learning: focused constraint, creative variation, simultaneous linking, efficient precision, and motivation. These findings are supportive of the use of *VMs* for enhancing students' learning in different ways.

In another paper, Moyer-Packenham and Westenskow (2016) revisited the research on *VMs* and synthesized the findings from 104 research reports. They reported that 46 studies yielded 104 effect size scores of *VMs* on student achievement. Consistent with the first meta-analysis, the effect size scores analysis indicated overall moderate effects for *VMs* compared with other instructional treatments. The five affordances categories reported in the first study were revisited and confirmed by the second study.

Several research studies involving *Virtual-tools* indicate positive results in student achievement, understanding and attitude, and some of them did not find differences in students' learning when comparing *Concrete*- and *virtual-Concrete-levels*. Some of these studies were reviewed in the next part of this section, including the areas of problem solving, fractions, geometry, and integers.

In problem solving, Ainsa (1999) carried out a study using *Concrete* and *virtual-Concrete levels* to introduce numerical problem solving in early childhood classrooms and found that children responded favorably to both types of tools and suggested combining them during learning activities. Moyer-Packenham, Salkind, and Bolyard (2008) carried out an initial exploration of teachers' current use the *Virtual level* in K-8 classrooms. The researchers indicated that it was common for teachers to use the *virtual-Concrete level* alone and *Concrete level* before using the *virtual-Concrete level* during the main portion of lessons.

Other studies investigated the area of fractions. Reimer and Moyer (2005) found statistical significance when a class of third-grade students used the *virtual-Concrete level* to learn about fractions, high improvement in students' post-test of conceptual knowledge scores, and high positive correlation between students' scores on the post-tests of conceptual and procedural knowledge. Moyer-Packenham and Suh (2012) examined the influence of using the *virtual-Concrete level* on different achievement groups during a teaching experiment in four fifth-grade classrooms. They found statistically significant gains following the treatment for all groups, for students in the low achieving group, average and high achieving groups (using follow-up paired samples individual t-tests on the low group), and for students in the average and high achieving groups (only with numerical concepts). Burns and Hamm (2011) examined the effectiveness of the *Concrete level* compared to the *virtual-Concrete level* on student review of fraction concepts in third grade and introduction of symmetry concepts in fourth grade. The *virtual-Concrete level group* was directed to

Visual fractions (Rand, 2013), *NLVM* (Utah State University, 2017) and the fraction bars activity *Number Sense* (Ventura Educational Systems, 2014). Results of the post-test suggested that student learning was unchanged by lesson condition. The use of the *CPVA-framework* could have helped in analyzing the specific effect of the variety of *Virtual sublevels* used in this study.

In terms of geometry, Yuan (2007) found positive results in a study involving the *virtual-Concrete level*. In another study, Steen, Brooks, & Lyon (2006) concluded that although *virtual-Concrete*- and *Concrete-level* groups pre-test to post-test point differences between these groups were not statistically significant, both improved over the control group (no manipulatives).

Bolyard and Moyer-Packenham (2012) investigated how the use of the *Virtual level* impacted student achievement for addition and subtraction of integers. They found that students demonstrated achievement gains in integer computation; facility at the *Pictorial* and *Abstract levels* when written representations were involved; and had more difficulties at the *Abstract level* with creating symbolic representations and making connections among representational forms.

Another study involved several *VMs* and mathematical domains for fifth grade (division, geometry and fractions). In an exploratory study, Anderson-Pence and Moyer-Packenham (2016) examined the influence of different *VM* types on the nature of student pairs' techno-mathematical discourse (*TMD*). The study included three pairs of fifth graders using *VMs*, and three *VM* types: 3 linked, 3 pictorial and 3 tutorial. Anderson-Pence and Moyer-Packenham (2016) defined linked *VMs* "as open-ended *VMs* that present multiple representations of mathematical concepts (e.g., pictorial images, number line models, and numeric symbols) that change simultaneously as they are manipulated," and "reflect the user's actions and choices without dictating solution strategies" (p. 8). *Pictorial VMs* were defined "as visual representations of mathematics concepts like physical manipulatives" and "reflect the user's actions and choices, but they do not include numeric symbols associated with the visual representation" (p. 8). Students' levels of discourse in generalization, justification, and collaboration were measured while working with each *VM* type. One-way *ANOVAs* indicated statistically significant differences in the quality of student discourse when using the different *VM* types. When working with linked *VMs*, students' discussions reflected consistently higher levels of discourse than when working with pictorial or tutorial *VMs*. However, pictorial *VMs* were associated with the largest amount of student-to-student discussion. Anderson-Pence and Moyer-Packenham (2016) suggested that to encourage meaningful *TMD*, teachers should select *VMs* with features that link multiple representations, and tutorial *VMs* did not encourage meaningful student-to-student mathematical discourse. This study did attempt to provide distinctions between the different cognitive levels involved in applets and apps. However, the present study proposes a different set of terms and definitions.

In summary, the findings of these studies support the use of the *Virtual level* alone or in combination with *Concrete, Pictorial* and *Abstract levels* to help students learn mathematics. It is evident that clarity in the selection and use of the *CPVA-levels* in these studies could help in the interpretation and use of their findings (Burns & Hamm, 2011), which is related to the research questions of the present study.

Methodology

The *CPVA framework* was presented to pre-service teachers in an elementary education bachelor's degree program and used throughout the semester. The sample included 93 students enrolled in a mathematics methods course required for this program. They also participated in field experiences carried out concurrently with the course at different public schools in the Florida Metropolitan area. They also had taken a content course with an emphasis on using manipulative materials to learn mathematics. They completed the *CPVALL Instrument* as *pre-* (first class), *post-* (after an introduction during the first class) and *follow-up tests* (last class at end of the semester). Also, some participants had previous experiences using different materials to learn mathematics; such as, base-ten blocks, color cubes, fraction tiles, color chips, hands-on equations, pattern blocks, tangrams, Geoboards, and apps/applets. Similarly, some of them had previous experiences using different materials to teach mathematics; such as base-ten blocks, color cubes, fraction tiles, color chips, hands-on equations, pattern blocks, tangrams, virtual manipulatives, Geoboards, and apps/applets. They had a lower level of prior experiences with using learning materials to teach mathematics. During the semester, they were introduced to several physical and virtual manipulative materials, involving whole numbers, fractions, decimals, and geometry, and were encouraged to use them during field experience activities.

Treatment

After completing the *CPVALL Instrument* (see Appendix A) as a pre-test, the students were more formally introduced to the *CPVA framework* during a 45-minute presentation of the *CPVA framework*. During this introduction, the students were also asked to identify and discussed their selection of *CPVA-learning levels* involved in seven different learning situations; for example:

1. Identify the level or levels represented by this graph made of physical cubes to represent the number of students who prefer each color (see Figure 4).

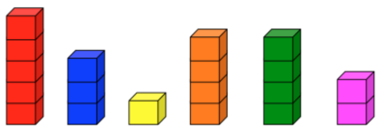

Figure 4. Graph made of physical cubes representing preferences.

The expected answer for this problem was the combination of *Concrete* and *Abstract* levels. The physical cubes were at the *Concrete level*, and the use of quantities and name of colors were at the *Abstract level*.

2. Identify the level or levels represented by *Sieve of Eratosthenes* (see Figure 3 presented earlier) (Utah State University, 2017). This virtual manipulative displays a grid containing numerals from 2 to 200. You can use it to explore patterns and relationships involving multiples. Using this virtual manipulative, you may: remove multiples of a number, show multiples of numbers, reset the workspace, and choose how many rows to display.

The expected answer for this problem was *Pictorial level* because of the table format to organize data, *Abstract level* because of the numerals and number names, and *virtual-Abstract level* because of the digital and dynamic manipulation (possible movement) of numbers to get factors and multiples.

Finally, the students were asked to describe an effective mathematics activity that they had experienced as a student or teacher, match the selected activity with of one of the cognitive levels or combination of cognitive levels, be ready to share with the class the specific learning activity, and ask others to find the level(s) represented in the activity. The *CPVALL Instrument* was administered again as a post-test after this introduction.

Furthermore, throughout the semester, the framework was demonstrated and connected to other topics, using physical and virtual manipulatives. The following physical manipulative materials were used during the semester: color tiles, pattern blocks, attribute blocks, Base-ten blocks, Cuisenaire rods, two-color counters, geoboard and fraction tiles. The following Apps were discussed during the semester for virtual manipulatives: Color Tiles (Brainingcamp, LLC, 2017), Number Rods (Cuisenaire rods) (Brainingcamp, LLC, 2017), Attribute Blocks (Ventura Educational Systems, 2014), Pattern Blocks (Brainingcamp, LLC, 2017), Place Value MAB (Base-ten blocks) (Kondys, 2017), TanZen Lite (Tangrams) (Little White Bear Studios, LLC, 2015), and Geoboard (Clarity Innovations, 2017). These apps have the digital and dynamic components required for the *Virtual level*. Table 2 shows the learning levels and mathematics content for each app. The *CPVALL Instrument* was administered again as a follow-up-test at the end of the semester.

CPVA Learning Levels Assessment

The *CPVALL Instrument* was developed by the principal investigator to assess participants' ability to identify learning cognitive levels (*Concrete*, *Pictorial*, Abstract, *virtual-Concrete*, *virtual-Pictorial*, and *virtual-Abstract*), and included a two-page test and a one-page answer sheet (see Appendix A). The *test* included a short description of the *CPVA-cognitive levels*, 11 items, and two questions about the participants' past experiences with using different tools for learning and teaching, respectively. On the top of the answer sheet, the participants provided general information regarding date, gender, course section and mathematics methods courses taken in the past. They also marked their answer for each question in the answer sheet by selecting the one or all the ones that they thought were appropriate. The items included in this test were reviewed by a group of graduate students and faculty with expertise in the area of mathematics education during a pilot test and the items were found to demonstrated adequate face validity.

Table 2
Learning levels and mathematics content for each apps

App	Topics	*C*	*P*	*A*	*vC*	*vP*	*vA*
Color tiles	Counting, operation concepts		x ten frames	x read/write/ type	x		
Number Rods	Counting, operations concepts		x number line/ images	x read/write/ type	x		
Attribute blocks	Sorting, classifying, logic, Venn Diagrams		x Venn diagram/ images	x read	x		
Pattern blocks	Shapes, sorting, classifying, patterns, variables, expressions		x images	x read/write/ type	x		

Place Value MAB	Place value, computation, whole numbers, decimals	x images/ place value chart	x read/write/ type/ calculator	x
TanZen	Shapes, fractions, decimals, area, perimeter	x images/ puzzles	x read	x
Geoboard	Shapes, fractions, decimals, area, perimeter	x	x read/write/ type	x

Note: C = Concrete, P = Pictorial, A = Abstract, vC = virtual-Concrete, vP = virtual-Pictorial, and vA = virtual-Abstract

The objective of the *CPVA Instrument* (11 items) was to assess participants' ability to identify the learning level or combination of levels (*Concrete*, *Pictorial*, *virtual-Concrete*, *virtual-Pictorial*, *virtual-Abstract*, and *Abstract*) involved in instructional situations. The maximum score was 50 points. Appendix B describes the expected answers for the items in this instrument and the scoring rubric used to assess the students' answers.

Data Analysis

To determine if there were significant statistical differences between the means, data sets were analyzed using analysis of variance (*ANOVA*), repeated measures. This analysis was used to detect any overall differences between related means. In this study, we used a one-way repeated measures *ANOVA* to analyze changes in mean scores over three time points or conditions based on participants' completion of the *CPVALL Instrument* as *pre-* (first class before introduction of *CPVA framework*), *post-* (after an introduction during the first class) and *follow-up-tests* (last class at end of the semester). The same participants were measured three times on the same dependent variable (scores in each administration of the *CPVALL Instrument*) and the independent variable was **time**.

The repeated measures *ANOVA* is particularly susceptible to the violation of the assumption of sphericity (Lund & Lund, 2018). The sphericity condition is met when the variances of the differences between all combinations of related groups (levels) are considered as equal, and a violation of this condition occurs when the variances of the *differences* between all combinations of related groups are not considered equal. When violations of sphericity occur, corrections can be made to produce a more valid critical *F*-value (i.e., reduce the increase in Type I error rate) by estimating the degree to which sphericity has been violated and applying a correction factor to the degrees of freedom of the *F*-distribution. Mauchly's Test of Sphericity is a formal way of testing the assumption of sphericity (Lund & Lund, 2018)). In other words, the Mauchly's Test of Sphericity is used to test the null hypothesis that the variances of the differences are equal. **The Greenhouse-Geisser correction** is used to combat the violation of the assumption of sphericity, which relies on estimating sphericity (Lund & Lund, 2018).

Findings

Appendix C includes a summary of students' selections of cognitive levels in the *CPVA Instrument*, and Appendix D provides the frequency of participants' scores for pre-, post-and follow-up tests. Most students consistently selected the correct answer for item 1 (*Concrete level*). Similarly, but at a lower level, students selected the correct answer for item 3 (*Pictorial level*) in the different tests (*pre-, post,* and *follow-up*). For items 2 (*Concrete and Abstract levels*), 4 (*Pictorial and Abstract levels*), and 11 (*Pictorial and Abstract levels*), students demonstrated improvement from the pre-test to the post- and follow-up-tests. There were some improvements for item 5 (*Concrete, Pictorial and Abstract levels*). The scores went from 20 participants selecting the correct answer to 56 and 45 participants, respectively.

Selection of correct answers for items 6 (Pictorial and Abstract levels), 7 (Abstract level), 8 (Pictorial, Abstract and virtual-Abstract levels), 9 (Pictorial, Abstract, virtual-Pictorial and virtual-Abstract levels), and 10 (Pictorial, Abstract, virtual-Concrete and virtual-Abstract) were consistently low for all tests. The main problem for item 6 seems to be the use of the phrase "given on a computer screen with digital pictures" in the item. Participants equated this phrase as meaning "virtual" and missed the need for a non-static/dynamic component for virtual manipulatives. Similarly, in item 7, students tend to equate the phrase "given on a computer screen" with the Virtual levels (virtual-Concrete, virtual-Pictorial and/or virtual-Abstract). Again, they tended to disregard the need for both digital and dynamic components and selected the Virtual levels.

Items 8, 9 and 10 were also problematic for students. For item 8, only one student selected the correct answer in the post-test. This item provided the use of the non-static, digital and dynamic nature of the *virtual-Abstract level*. However, many students identified the *virtual-Abstract level* as part of item 8 (34% for pre-test, 78% for the post-test, and 24% for the follow-up-test), which implies a partial understanding.

Item 9 provided a similar limitation. None of the students selected the correct answer for the pre-test. Then, eight students selected the correct answer in the post-test (*Pictorial, Abstract, virtual-Pictorial and virtual-Abstract level combination*), but it went back to zero for the follow-up administration. Retention of information could be a concern because of this decrease in score. However, several students selected a partially correct answer (*virtual-Pictorial and virtual-Abstract level combination* without the *Pictorial* and *Abstract levels*): 14% for pre-test, 37% post-test, and 27% follow-up-test, which implies some level of understanding.

For item 10, only one student selected the correct answer in the post-test (*Pictorial, Abstract*, *virtual-Concrete and virtual-Abstract*). This item was challenging because students had to visualize the dynamic movement of the Base-ten blocks. Some students identified the *virtual-Concrete level* (41% for pre-test, 7% for post-test and 16% follow-up-test), and others the *virtual-Abstract level* (10% for pre-test, 4% for post-test and 1% follow-up-test). Similar to item 9, several students selected a partially correct answer (*virtual-Concrete and virtual-Abstract level combination* without the *Pictorial* and *Abstract levels*): 3% for pre-test, 19% post-test, and 15% follow-up-test. In general, it might help to provide more examples of this type of apps and applets and make sure the students can understand what is involved with them.

Item 11 was less problematic, but still presented some challenge for the students. In this item, 13% of the students selected the correct answer (*Pictorial and Abstract levels*) in the pre-test, 44% in the post-test and 42% in the follow-up-test. The students also selected *Pictorial level* (25% for pre-test, 6% for post-test and 14% follow-up-test) and *Abstract level* (21% for pre-test, 8% for post-test and 13% follow-up-test) as partially correct answers. Many students did not visualize the table as *Pictorial level* representation in this item.

It was determined that the *CPVALL Instrument* had good internal consistency with Cronbach's alpha of 0.845 for the pre-test, 0.807 for the post-test, and 0.792 for the follow-up-test. The Cronbach's alpha was improved by deleting items 1 and 8. This provided revised Cronbach's alpha of 0.891 for the pre-test, 0.823 for the post-test, and 0.804 for the follow-up-test. Based on the findings, revisions will be made to improve its validity and reliability of the instrument.

The mean, standard deviation, and minimum and maximum values are included in Table 3. Mauchly's test of Sphericity is significant (0.878, 2 d.f., $p < 0.05$). This means the variance of the differences are significantly different, so the assumption of Sphericity was not met due to Mauchly's being statistically significant. We used Greenhouse-Geisser to report values.

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Table 3
Statistics for pre, post and follow-up test scores

Statistics	Pre-test	Post-test	Follow-up Test
Mean	19.897	27.247	25.144
Standard Deviation	5.559	7.036	4.946
Minimum Value	2	8	12
Maximum Value	34	45	37
Overall Mean	24.096		

We ran a Repeated Measures *ANOVA* and found that the means were statistically significant $F(1.8, 171) = 64.3$ ($p < 0.001$) with $\boldsymbol{\eta}^2 = 0.401$, which is a large effect size (see table 4). This means that approximately 40% of the variation in means is accounted for by time. Ellis (2010) defined the power of any test of statistical significance as the probability that it will reject a false null hypothesis and is inversely related to beta (β) or the probability of making a Type II error (or $1 - \beta$). In the present study, the statistical power was equal to 1, which is considered a strong power. Since the statistical power was high, the probability of making a Type II error, or concluding there was no effect when, in fact, there is one, was very low. A larger sample could have offered greater test sensitivity (Ellis, 2010).

Table 4
Results of Repeated Measures ANOVA: Tests of Within-Subjects Effects
Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
factor1	Sphericity Assumed	2780.296	2	1390.148	64.299	0.00	0.401	128.598	1.000
	Greenhouse-Geisser	2780.296	1.782	1559.787	64.299	0.00	0.401	114.612	1.000
	Huynh-Feldt	2780.296	1.814	1532.774	64.299	0.00	0.401	116.632	1.000
	Lower-bound	2780.296	1.000	2780.296	64.299	0.00	0.401	64.299	1.000

Error (factor 1)	Sphericity Assumed	4151.038	192	21.620
	Greenhouse-Geisser	4151.038	171.119	24.258
	Huynh-Feldt	4151.038	174.134	23.838
	Lower-bound	4151.038	96.000	43.240

[a]Note: Computed using alpha = ?

The pairwise comparisons show that the mean difference for the pretest (M = 19.9, SD = 5.6) is statistically significant ($p < 0.001$) from the posttest (M = 27.2, SD = 7.0) and follow up (M = 25.1, SD = 4.9) (see table 5). The mean difference for the posttest was statistically significant ($p = 0.001$).

Table 5
Results of Pairwise Comparisons

Measure: MEASURE_1

(I) factor1	(J) factor1	Mean Difference (I-J)	Std. Error[b]	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
1	2	-7.351*	0.775	0.000	-8.889	-5.812
	3	-5.247*	0.597	0.000	-6.433	-4.061
2	1	7.351*	0.775	0.000	5.812	8.889
	3	2.103*	0.616	0.001	0.880	3.326
3	1	5.247*	0.597	0.000	4.061	6.433
	2	-2.103*	0.616	0.001	-3.326	-0.880

Note: Based on estimated marginal means.
[b]Std. Error = Standard Error, Sig. = Significant
[b]Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).
*The mean difference is significant at that identified level.

Discussion

As part of a mathematics methods course for elementary school undergraduate pre-service teachers, the results of this study have provided further understanding of these students' conceptions of physical and virtual manipulative materials when using the *Concrete, Pictorial, Virtual and Abstract* (*CPVA*) *framework* at three points during one semester. Similar to Ortiz (2017), the participants demonstrated good overall understanding of the *CPVA levels*, but had difficulties related to the identification of learning levels in learning activities. The main difficulty was related to the identification *Virtual sublevels* (*virtual-Concrete*, *virtual-Pictorial* and *virtual-Abstract*). In general, it is important that the participants have an appropriate understanding of the *Virtual level* since it could provide additional resources to help their students of all ability levels "to develop their relational thinking and to generalize mathematical ideas" (Moyer-Packenham, Salkind, & Bolyard, 2008, p. 204).

The participants demonstrated improvement from pre-test to post- and follow-up-tests. However, this improvement was limited by their understanding of the *Virtual sublevels*. One of the many instructional decisions the teachers make in the selection of physical and virtual manipulatives. If teachers are not able to understand and select the appropriate resources to facilitate students' learning and meet their needs at a given time, then the teacher's effectiveness and impact on their students' learning could be diminished. As knowledgeable pre- and in-service teachers use instructional materials appropriately (Clements, 1999), more research is needed to understand the impact their ability to identify *CPVA levels* has on their students' learning. At the same time, the research in this area also needs a consistent framework. Otherwise, the understanding of these findings could be hard to generalize.

There were some lessons learned in terms of how to improve the presentation and implementation of the *CPVA-framework* during this study. In the future, the researchers are planning to present a more detailed definition and more examples of the *Virtual levels*. Also, it is recommended that the pre- and post-tests are administered with access to the app or applets involved in the virtual manipulative items. The availability of the apps or applets will facility the understanding of the items and will provide a more accurate evaluation of the pre-service teacher or in-service teachers' understanding of the *framework*. These efforts will help to make sure the students understand the framework and use it more effectively when implementing learning activities. It also necessary that pre- and in-service teachers have opportunities to implement the framework in learning activities. This type of activity will increase their changes of understanding the framework. They need to take appropriate advantage of the many apps and applets available for free or very affordable prices from different sources, and not blindly incorporate them just because it is assumed to be a *VM*. As indicated in the review of literature, this selection process could prove essential when using this type of learning materials (Bolyard & Moyer-Packenham, 2012; Clements, 1999).

Conclusion

The *CPVA framework* could provide consistency and clarity when selecting learning materials, especially when it comes to apps and applets. As indicated before, Anderson-Pence and Moyer-Packenham (2016) suggested that teachers' need to be able to select virtual manipulatives with features that link multiple representations and Clements (1999) added that they also need to be knowledgeable about the available teaching tools. Teachers' understanding of the proposed framework could help with facilitating a proper and effective selection of learning tools.

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Appendix A

CPVA Learning Levels Instrument (CPVALL), Answer Sheet and Answer Key CPVALL used as Pre-, Post-, and Follow-up-tests

Please do not write on this copy of the test. You should use the Answer Sheet Provided for answers. Thank you. Within this assessment, the CPAV learning levels are defined in the following manner:

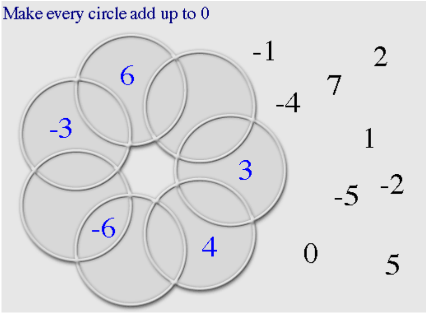
- **Concrete:** This cognitive level involves the use of real concrete objects or manipulative materials (non-digital); for example, apples, sticks, cubes, or marbles.
- **Pictorial:** This learning level involves the use of pictures, drawings or digital images (which are non-dynamic or static versions used in electronic files, including Webpages, Interactive Whiteboard files, Apps or Applets) of concrete objects or manipulative materials.
- **Abstract:** This learning level involves the use of written words or symbols, verbal communication, Braille, sign language, or digital text (which may appear in electronic files like Webpages, Interactive Whiteboard files, word-processed files, databases, spreadsheets, Apps or Applets).
- **Virtual-Concrete (*vConcrete*):** This learning level involves the use of electronically interactive-dynamic-digital versions of objects or manipulative materials (e.g., Apps or Applets) that may be manipulated on a computer screen or other electronic device (e.g., tablets, iPads, iPhones or Interactive Whiteboards). Video shown using Interactive Board or other digital media is not considered Virtual-Concrete.
- **Virtual-Pictorial (*vPictorial*):** This learning level involves the use of electronically interactive-dynamic-digital versions of pictures of graphs, or objects (e.g., Apps or Applets) that may be manipulated on a computer screen or other electronic device (e.g., tablets, iPads, iPhones or Interactive Whiteboards). Video shown using, Interactive Board or other digital media is not considered Virtual-Pictorial.
- **Virtual-Abstract (*vAbstract*):** This learning level involves the use of electronically interactive-dynamic-digital versions of the abstract level (e.g., Apps or Applets) that may be manipulated on a computer screen or other electronic device (e.g., tablets, iPads, iPhones or Interactive Whiteboards). Video shown using Interactive Board or other digital media is not considered Virtual-Abstract.

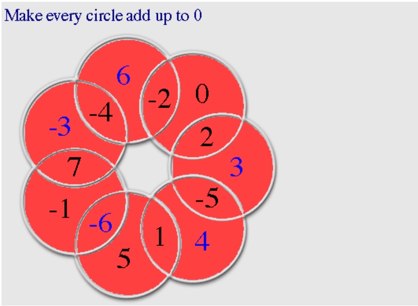
Answer each of the following questions: Select the learning level or levels best represent this situation: Select the answer or answers that are appropriate: Mark your answers in the Answer Sheet provided:

1. A student is playing with physical cubes building a random shape with these cubes, without drawing, verbalizing or writing ideas (words or numerals).
2. A student solved the computation problem “345 + 49 = ?” using only numerals written on paper and physical Base-ten blocks, and then wrote the solution using written symbols on the paper (numerals) (no drawings of the Base-ten blocks were made).
3. A student is freely drawing a random figure or shape without verbalizing or writing ideas (words or numerals).
4. A student is solving the computation problem “345 + 49 = ?” given by using drawing or pictures of Base-ten blocks and then writing symbols on the paper to illustrate the solution (no physical or virtual manipulatives).
5. A student used physical Base-ten blocks to solved the computation problem “345 + 49 = ?” that was written on paper. The student also drew pictures of the Base-ten blocks on the paper to illustrate the solution.
6. A student is solving the computation problem “345 + 49 = ?” given on computer screen with static-digital pictures of Base-ten blocks, and then writing symbols on the paper to illustrate the solution.
7. A student is solving the computation problem “345 + 49 = ?” given on a computer screen, and

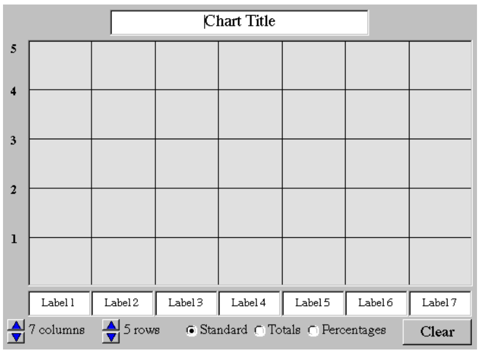
then writing symbols on the paper to demonstrate the solution.

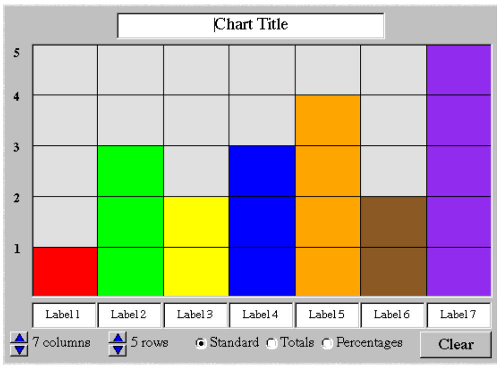
8. A website involves an applet with a puzzle in which students move number values (Integers from ⁻6 to 7) inside of each circle or intersection of circles (like in Venn Diagrams) so that they add up to 0 (see images below; on your left is the puzzle and in your right is the solution). To solve the puzzle, the students drag the black numbers to the blank spaces. They cannot move the blue numbers. When the number values inside any circle or intersection of circles add up to 0, the circle changes color. If the values are correct, the puzzle turns red. If the puzzle is not correct, the student may keep trying until it is solved.



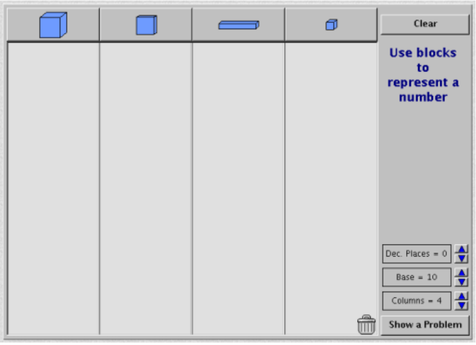


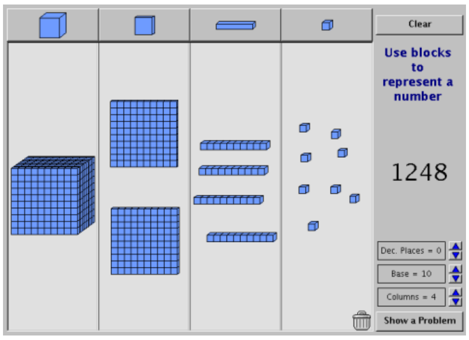
9. A website presents an applet in which students may create a bar chart showing quantities or percentages by labeling columns and clicking on selected values. Colors were added to represent the values for each column. See images below illustrating a chart made with this Applet. The chart on your left is the initial chart without values. The chart on the right was made with the Applet representing different frequencies for each column: red = 1, green = 3, yellow = 2, blue = 3, orange = 4, brown = 2, and purple = 5. The values are also indicated in the vertical number line (from 0 to 5 in this case). A chart title and labels for the columns may be added to the chart. The number of columns may be augmented or reduced from 3 to 12 using the blue arrows up or down. Similarly, the number of rows may be augmented or reduced from 5 to 20 using the blue arrows up or down. The chart may be set to show a "Standard" view using a vertical number line (like the on in the example below), "Total" view without the number line, but showing numerical values for each bar, or percentage also without number line, but showing percentage values for each bar. The student may also click on "Clear" to go back to the empty chart.





10. A website presents an Applet in which students may illustrate addition and subtraction computation in a variety of bases. The decimal places of the number (from 0 to 3 places), base of the number (2, 3, 5, 5 or 10), and number of columns for place values (from 2 to 4 columns) may be changed by pressing the blue arrows up or down respectively (bottom of right-hand corner). Students are asked to use blocks to represent a given number. The students may clear the chart by pressing the "Clear" button and change the number to represent by using the "Show a Problem" button in the bottom part of the chart. Blocks may be eliminated by using the clicking on the block to be deleted and dragging it into the "virtual garbage can." The image on the left shows an empty start. In the image on the right, the student was given 1248 to represent using the blocks with 0 decimal places, in base ten and four columns. In this case, the student would click on the cube once time to represent 1 thousand, twice on the flat to represent 2 hundreds, four times on the flat to represent 4 hundreds, and eight times on the unit to represent 8 ones.





11. Using a word processor, a student made a frequency table to organize the values in the following data set. Data: 6, 3, 6, 4, 9, 4, 6, 4, 8 and 6. No pictures, images, drawings or manipulative materials were used.

Outcome	Frequency
3	1
4	3
6	4
8	1
9	1
Total	10

12. Have you used any of the following for learning mathematics? See answer sheet for choices. **Select all that apply. Write your answer in the space provided in the Answer Sheet:**

13. Have you used any of the following for teaching mathematics? See answer sheet for choices. **Select all that apply. Write your answer in the space provided in the Answer Sheet:**

CPVA Learning Levels Assessment: Answer Sheet

General Information
ID Number: __________________ **Date:** _________ **Gender:** ___ M or ___F **Section:** ______
Have you taken mathematics methods courses before? ___ Yes or ___ No. When? ______________
If yes, what course or courses, and where did you take it or them (UCF or other)? ___ UCF ___
Other: ____

Mark your answer for each question: Select the one or all the ones that you think are appropriate:

#	Concrete	Pictorial	Abstract	Virtual-Concrete	Virtual-Pictorial	Virtual-Abstract
1	☐	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐	☐

12. Have you used any of the following for learning mathematics? **Select all that apply.**

___ Base-ten blocks ___ Attribute blocks ___ Pattern blocks ___ Pan balances
___ Color cubes ___ Two-color chips ___ Tangrams ___ Algebra tiles
___ Fraction tiles ___ Hands on equations ___ Cuisenaire rods ___ Geoboards
___ Calculators ___ Graphing calculators ___ Virtual Manipulatives ___ App or Applet
___ Tablets; e.g., iPads ___ Interactive Whiteboards ___ Spreadsheets Databases ___ Graph Calc.
___ Other technology, software, app or applet? ____
___ Other manipulatives: ____

13. Have you used any of the following for teaching mathematics? **Select all that apply.**

___ Base-ten blocks ___ Attribute blocks ___ Pattern blocks ___ Pan balances
___ Color cubes ___ Two-color chips ___ Tangrams ___ Algebra tiles
___ Fraction tiles ___ Hands on equations ___ Cuisenaire rods ___ Geoboards
___ Calculators ___ Graphing calculators ___ Virtual Manipulatives ___ App or Applet
___ Tablets; e.g., iPads ___ Interactive Whiteboards ___ Spreadsheets Databases ___ Graph Calc.
___ Other technology, software, app or applet? ____
___ Other manipulatives: ____

CPVA Learning Levels Assessment: Answer Key

The following are the expected answer for each question:

#	Concrete	Pictorial	Abstract	Virtual-Concrete	Virtual-Pictorial	Virtual-Abstract
1	☒	☐	☐	☐	☐	☐
2	☒	☐	☒	☐	☐	☐
3	☐	☒	☐	☐	☐	☐
4	☐	☒	☒	☐	☐	☐
5	☒	☒	☒	☐	☐	☐
6	☐	☒	☒	☐	☐	☐
7	☐	☐	☒	☐	☐	☒
8	☐	☒	☒	☐	☐	☐
9	☐	☒	☒	☐	☒	☒
10	☐	☒	☒	☒	☐	☒
11	☐	☒	☒	☐	☐	☐

Appendix B

Description CPVA Scoring Rubric

The following is an item-by-item description of the *CPVA* scoring rubric. It includes the expected answers and points for incorrect or incomplete answers

Item 1.

- 2 points (correct answer: *C- level*),
- 1 point (correct answer plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 2.

- 4 points (both correct answers: *C-* and *A-level combination*),
- 3 points (both correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *C-* or *A-level*)
- 1 point (one of the possible correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 3

- 2 points (correct answer: *P-level*),
- 1 point (correct answer plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 4.

- 4 points (both correct answers: *P-* and *A-level combination*),
- 3 points (both correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *P-* or *A-level*)
- 1 point (one of the correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 5.

- 6 points (all three correct answers: *C-*, *P-* and *A-level combination*),
- 5 points (all three correct answers plus one or more incorrect selections),
- 4 points (two of the possible correct answers: *C-*, *P-* and/or *A-level combinations*),
- 3 points (two of the possible correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *C-*, *P-* or *A-level*)
- 1 point (one of the possible correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 6.

- 4 points (both correct answers: *P-* and *A-level combination*),
- 3 points (both correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *P-* or *A-level*)
- 1 point (one of the correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 7.

- 2 points (correct answer: *A- level*),
- 1 point (correct answer plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 8.

- 6 points (all correct answers: *P-*, *A-* and *vA-level combination*),
- 5 points (all correct answers plus one or more incorrect selections),
- 4 points (two of the possible correct answers: *P-*, *A-* and/or *vA-level combinations*),
- 3 points (two of the possible correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *P-*, *A-* or *vA-level*)
- 1 point (one of the possible correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 9.

- 8 points (all four correct answers: *P-*, *A-*, *vP-* and *vA-level combination*)
- 7 points (a combination of all four possible correct answers plus one or more incorrect selections)
- 6 points (a combination of three of the possible correct answers: *P-*, *A-*, *vP-* and/or *vA-level combinations*),
- 5 points (a combination of three of the possible correct answers plus one or more incorrect selections),
- 4 points (a combination of two of the possible correct answers: *P-*, *A-*, *vP-* and/or *vA-level combinations*),
- 3 points (a combination of two of the possible correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *P-*, *A-*, *vP-* or *vA-level*)
- 1 point (one of the possible correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 10.

- 8 points (all four correct answers: *P-*, *A-*, *vC-* and *vA-level combination*)
- 7 points (a combination of all four possible correct answers plus one or more incorrect selections)
- 6 points (a combination of three of the possible correct answers: *P-*, *A-*, *vC-* and/or *vA-level combinations*),
- 5 points (a combination of three of the possible correct answers plus one or more incorrect selections),
- 4 points (a combination of two of the possible correct answers: *P-*, *A-*, *vC-* or *vA-level*),
- 3 points (a combination of two of the possible correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *P-*, *A-*, *vC-* or *vA-level*)
- 1 point (one of the possible correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Item 11.

- 4 points (both correct answers: *P-* and *A-level combination*),
- 3 points (both correct answers plus one or more incorrect selections),
- 2 points (one of the possible correct answers: *P-* or *A-level*)
- 1 point (one of the correct answers plus one or more incorrect selections), and
- 0 points (no correct selection).

Appendix C
Summary of selection of alternatives by item

Item 1	C	P	C-vA	C-P	vA	C-A	A	vC	0					
Pre	92	2	1	1	1	x	x	x	x					
Post	93	x	x	x	x	2	2	x	x					
Follow-up	91	x	x	3	x	x	1	1	1					
Item 2	C-A	A	C	P	P-A	C-P-A	C-P	vP	vA	A-vC	0			
Pre	36	29	15	8	3	2	2	1	x	x	1			
Post	74	6	2	4	4	6	x	x	1	x	x			
Follow-up	67	7	3	1	2	12	4	x	x	1	x			
Item 3	P	A	C	P-A	C-vA	vA	C-A	A-vA	P-vP	C-P	0			
Pre	82	8	3	2	1	1	x	x	x	x	x			
Post	87	4	1	3	x	1	1	x	x	x	x			
Follow-up	83	5	2	2	x	x	1	1	1	1	1			
Item 4	P-A	P	A	C-P-A	C-P	C-A	0	P-vP	vP	P-A-vC	vC	C	CP A-vP	+
Pre	38	29	11	8	5	2	2	1	1	x	x	x	x	x
Post	73	7	1	10	1	x	x	x	1	1	1	1	1	x
Follow-up	73	3	3	12	1	1	x	x	x	1	x	x	x	3
Item 5	C-P-A	C-P	P	C	A	vC	P-vC	PA	C-vC	C-vP	CA	C-A-vP	CA-vC	+
Pre	20	37	15	14	5	1	1	1	1	1	1	x	x	x
Post	56	21	3	2	1	x	x	10	x	x	3	1	x	x
Follow-up	45	36	3	2	3	x	x	1	x	x	3	x	1	2
Item 6	P-A	vP	A-vP	A-vC	P-vP	vC	vPvA	P-vC	vCvP	vA	PA-vP	A	AvP vA	+
Pre	1	19	14	13	11	11	5	5	3	3	3	2	x	8
Post	1	3	24	9	5	2	9	2	2	1	7	x	x	32
Follow-up	2	1	27	8	2	5	5	x	4	x	x	1	14	33
Item 7	A	vA	A-vA	vP	vC	A-vP	A-vC	vC A	P-vC	P-vA	vPA	PA-vA	P	+
Pre	13	22	20	12	10	4	3	2	2	2	1	x	1	5
Post	5	8	36	1	x	x	7	2	1	12	5	x	5	15
Follow-up	6	4	45	x	1	6	5	x	1	12	6	4	x	7
Item 8	P-A-vA	vA	vP	vC	vP-vA	vC-vP	vCvP vA	PvP vA	C	P	P-vP	vCA	A-vC	+
Pre	0	27	20	15	12	6	6	2	1	1	1	1	1	4
Post	1	8	6	5	18	9	22	x	x	x	1	8	4	15
Follow-up	0	6	7	9	17	8	25	x	x	x	x	13		12
Item 9	P-A-vP-vA	vP	vPvA	vA	vC	vCP	vCvP vA	vC-vA	P-vP	A-vP	P-vCvP	C-vCvP	AvP vA	+
Pre	0	40	14	10	10	7	4	2	2	2	1	1	1	3
Post	8	10	37	2	5	8	12	1	2	2	x	x	3	7
Follow-up	0	22	27	2	3	17	14	3	2	1	x	1	x	5
Item 10	P-A-vC-vA	vC	vP	vA	vCP	P-vA	vCvP vA	vCv A	P-vP	A-vP	P-vA	P-vCvP vA	A-vC	+
Pre	0	40	11	10	10	6	5	3	3	1	1	x	1	6
Post	1	7	6	4	13	7	21	19	1	x	x	x	1	x
Follow-up	0	16	7	1	15		28	15	2		5	2	2	9
Item 11	P-A	P	A	vP	vC	vPvA	vA	C	vCvPv A	CP	A-vA	vCv A	All	+
Pre	13	24	20	8	6	6	6	5	2	2	1	1	1	2
Post	43	6	8	2	4	7	10	2	2	x	x	2	1	9
Follow-up	41	14	13	3	2	3	3	x	4	3	1	1	x	8

Notes: C = Concrete, P = Pictorial, A = Abstract, vC = virtual-Concrete, vP = virtual-Pictorial, and vA = virtual-Abstract.
The first alternative to the right of the item number represents the expected correct answer for the item.

Appendix D
Frequency of participants’ scores for pre-, post- and follow-up tests

Item 1/Scale	2	1	0						
Pretest	92	2	3						
Posttest	93	2	2						
Follow-up test	91	3	3						
Item 2/Scale	4	3	2	1	0				
Pretest	36	2	44	5	10				
Posttest	74	6	8	4	5				
Follow-up test	67	12	10	7	11				
Item 3/Scale	2	1	0						
Pretest	82	2	13						
Posttest	87	3	7						
Follow-up test	83	4	10						
Item 4/Scale	4	3	2	1	0				
Pretest	38	0	40	16	3				
Posttest	73	12	8	1	3				
Follow-up test	73	14	3	7	0				
Item 5	6	5	4	3	2	1	0		
Pretest	20	0	39	0	34	3	1		
Posttest	56	0	34	1	6	0	0		
Follow-up test	45	1	40	1	8	1	1		
Item 6	4	3	2	1	0				
Pretest	1	5	2	45	44				
Posttest	1	12	11	51	22				
Follow-up test	2	4	2	66	23				
Item 7	2	1	0						
Pretest	13	30	54						
Posttest	5	51	41						
Follow-up test	6	65	26						
Item 8	6	5	4	3	2	1	0		
Pretest	0	0	1	3	28	22	43		
Posttest	1	2	7	3	9	53	22		
Follow-up test	0	1	0	2	6	63	25		
Item 9	8	7	6	5	4	3	2	1	0
Pretest	0	1	1	0	18	5	50	10	12
Posttest	3	3	8	0	41	12	12	11	7
Follow-up test	0	0	1	1	30	15	25	21	4
Item 10	8	7	6	5	4	3	2	1	0
Pretest	0	0	1	0	6	7	50	20	13
Posttest	1	2	3	7	20	22	12	22	8
Follow-up test	0	0	2	2	17	29	8	22	17
Item 11	4	3	2	1	0				
Pretest	13	1	44	3	36				
Posttest	43	5	14	4	31				
Follow-up test	41	3	27	9	17				