The Analysis of a Novice Primary Teachers' Mathematical Knowledge in Teaching: Area Measurement

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The purpose of this paper is to investigate a novice primary teacher's mathematical knowledge in teaching on area measurement. Data were collected from a novice primary teacher of fourteen students in a primary school located in Ankara, Turkey using field notes, video recordings of lessons, and audio recordings of interviews before and after her teaching. Her mathematical knowledge in teaching was analyzed by means of the Knowledge Quartet model. The novice primary teacher's mathematical knowledge in teaching on area measurement was adequate regarding some of the codes such as choice of representations, making connections between concepts, and responding to unavailability of resources. However, her knowledge was inadequate in the codes of adherence to textbook, choice of examples, and responding to students' ideas. Implications and suggestions to improve teachers' mathematical knowledge in teaching are presented accordingly.

Since teachers are one of the important factors affecting student learning (Fennema & Franke, 1992), they contribute to students' mathematics achievement gains (Aaronson, Barrow, & Sander, 2007; Hill, Rowan, & Ball, 2005; Sanders, 2000). In fact, Sanders (2000) emphasizes that in order to increase students' achievement, the only thing that should be done is to reduce the number of ineffective teachers. Some researchers mention that there is a strict relationship between teachers' effectiveness and classroom instruction (Ball, Thames, & Phelps, 2008; Ma, 1999) where it can be said that teachers' effectiveness is related to all aspects of teaching (Ball, 1991). Although little is known about what contributes to teachers' effectiveness, Gilbert and Gilbert (2011) state that it is influenced by teachers' knowledge. Therefore, better understanding of teacher knowledge and its development is important in order to improve mathematics instruction (Ball et al., 2008). In this study, a novice primary teacher's mathematical knowledge in teaching on area measurement was investigated. It is believed that findings of this study would be useful for obtaining insights into a novice primary teacher's mathematical knowledge in teaching and extending students' knowledge about area measurement.

Theoretical Background

Teacher Knowledge

Shulman (1986), in his seminal article, defines three categories of knowledge: subject matter, pedagogical content, and curriculum. Subject matter knowledge refers to understanding "facts or concepts of a domain" (Shulman, 1986, p. 9). That is, teachers must not only know but be able to explain why a particular proposition is true. Although Shulman's definition for subject matter knowledge attracted researchers' attention, some researchers argue that subject matter knowledge is not enough for effective teaching and pedagogical content knowledge is also necessary (Ball, 2003; Even, 1993; Hill et al., 2005; Rowland, Huckstep & Thwaites, 2003). Pedagogical content knowledge is defined as

> …the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to

> other…. an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (Shulman, 1986, p. 9).

Furthermore, the pedagogical content knowledge is defined as "a special amalgam of content and pedagogy" (Shulman, 1987, p. 8) and this knowledge distinguishes teachers from specialists of the subject. That is, knowing the content is not enough to be able to teach it. A teacher with strong subject matter knowledge can know basic procedures and facts; however, s/he may not know how to use them to promote student learning. In short, a teacher with strong pedagogical content knowledge knows how to transform his/her subject matter knowledge to make it comprehensible to students. Therefore, it can be concluded that both teachers' subject matter knowledge and pedagogical content knowledge are important for student learning.

Since Shulman's framework is not specific to one subject; most researchers in teacher knowledge accept and use his framework as a base model (Ball, 1991; Ball et al., 2008; Grossman, 1990; Fennema & Franke, 1992; Fernandez-Balboa & Stiehl, 1995; Marks, 1990). Then various models were designed in specific content domains to ensure a better assessment of teacher knowledge (Ball et al., 2008; Rowland et al., 2003). For instance, Ball and her colleagues extended Shulman's framework. They used a new term "Mathematical Knowledge for Teaching (MKT)" and defined it as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395). They state that MKT consists of two main domains: subject matter knowledge and pedagogical content knowledge. According to their model, common content knowledge, specialized content knowledge, and horizon content knowledge are categorized under subject matter knowledge. The first sub-domain of subject matter knowledge, common content knowledge, is described as the knowledge that any well-educated person can have. That is, it is not unique to teaching. On the other hand, specialized content knowledge is defined as knowledge that is specific to teaching. In other words, teachers should have more knowledge unique to teaching mathematics than their students. Lastly, horizon content knowledge is the knowledge of how mathematical content spans over the curriculum. Specifically, a teacher with horizon content knowledge links content not only with previous years but also with following years. The second domain of the MKT model, pedagogical content knowledge, consists of three sub-domains: knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. Knowledge of content and students includes both knowledge of specific content and knowledge of students' thinking and learning process. Teachers with this knowledge are aware of students' understanding, conceptions, and misconceptions for specific content. The second subdomain, knowledge of content and teaching, is "knowledge that combines knowledge about mathematics and knowledge about teaching" (Ball et al., 2008, p. 401). In other words, teachers must pay attention to the sequence of lessons, as well as the advantages and disadvantages of various methods, techniques, procedures, demonstrations, and representations. The last sub-domain, knowledge of content and curriculum, is the knowledge of how a teacher relates mathematics with mathematics curriculum. As seen in the model, Ball's framework is specific to mathematics and has many sub-domains. In other words, Ball and her colleagues examine teachers' mathematical knowledge from multidomainal perspectives. However, there are some drawbacks of the model based on this categorization. Since boundaries between the sub-domains of their model are not clear, they may overlap. For example, being knowledgeable about students' difficulties, misconceptions, and troubles can be classified as sub-domains in both knowledge of content and students and knowledge of content and teaching. It may be difficult to decide whether the given mathematical knowledge is common for everyone or unique to teaching. That is, whether it belongs to the common content knowledge sub-domain or the specialized content knowledge sub-domain. Also, since Ball and her colleagues examine subject matter knowledge and pedagogical content knowledge separately, it is difficult to discriminate whether the given property belongs to subject matter knowledge or pedagogical content knowledge (Ball et al., 2008). Furthermore, Watson (2008) states that discriminating teachers' knowledge as subject matter knowledge or pedagogical content knowledge is not helpful. In fact, it "can veil the essential mathematical activity in which different kinds of knowledge relate and inform each other" (Rowland & Zaskis, 2013, p. 249).

Since all of these components of teacher knowledge interact with each other throughout instruction, Rowland, Huckstep, and Thwaites (2003) designed another model, the Knowledge Quartet (KQ), to investigate teachers' mathematical subject matter knowledge and pedagogical content knowledge together. That is, the distinction of teachers' mathematical knowledge between them is of lesser importance. Rowland and his colleagues (2009) use the term "Mathematical Knowledge in Teaching" instead of "Mathematical Knowledge for Teaching" in order to emphasize the difference between their model and Ball's model. Since teachers' mathematical knowledge in teaching comes into play during teaching, they state that it should be studied in the classroom environment (Hegarty, 2000; Rowland & Ruthven, 2011). Rowland and his colleagues argue that "mathematical content knowledge for teaching will be most clearly seen in the action of teaching" (Rowland et al., 2009, p. 25). For this purpose, they videotaped 24 mathematics lessons taught by 12 preservice teachers in the UK. Afterwards, they met with teachers to discuss their lessons and helped these teachers identify and improve on their mathematical knowledge. Their aim is to define the dimensions of the KQ with 'easily remembered labels' that will incorporate important factors in mathematical knowledge in teaching (Rowland & Ruthven, 2011, p. 197).

Foundation, transformation, connection, and contingency are the four dimensions of the KQ model, and each dimension contains more specific codes. The first dimension, foundation, is related to teachers' theoretical background and includes their content knowledge, beliefs about mathematics, and the purposes of mathematics education. Thus, it affects all three dimensions of the KQ model. Teachers' usage of textbook and terminology during their instruction and knowledge of students' errors and misconceptions give some clues about the foundation dimension. Rowland and Turner (2007) explain that foundation is acquired "in the academy, in preparation (intentionally or otherwise) for their role in the classroom" (p. 112). This dimension differs from the other three in that it includes teachers' possessed knowledge. Teachers can possess knowledge, facts, or beliefs, but they do not always need to show it during teaching practice. However, the effect of a teacher's knowledge on his or her planning and teaching practice can be seen in the transformation dimension. This dimension focuses on teachers' capacity to transfer their knowledge to students. If the teacher's transformative knowledge is adequate, they know how to translate their knowledge to help students learn content meaningfully. Teachers' choice of representations, demonstrations, and examples can give information about this dimension. Similarly, teachers' decisions about sequencing, making connections between topics and procedures, and deciding conceptual appropriateness give information about their knowledge in connection, the third dimension of the KQ. The last dimension of the KQ, contingency, is about the events that are almost impossible to plan for and the ways teachers respond to these unplanned events. Effective teachers can manage these unplanned events and change them into opportunities for a better teaching activity (Rowland et al., 2009). In total, there are twenty codes under these four dimensions as shown in Table 1.

Table 1
The Dimensions and Contributory Codes of the KQ Model (Rowland, 2014, p. 25)

Dimension	Contributory Codes
Foundation	awareness of purpose adherence to textbook concentration on procedures identifying errors overt display of subject knowledge theoretical underpinning of pedagogy use of mathematical terminology
Transformation	choice of examples choice of representation teacher demonstration (to explain a procedure)
Connection	anticipation of complexity decisions about sequencing recognition of conceptual appropriateness making connections between procedures making connections between concepts
Contingency	deviation from agenda responding to students' ideas use of opportunities teacher insight responding to the (un)availability of tools and resources

The KQ model emphasizes that since instruction takes place in the classroom and that teachers' subject matter knowledge and pedagogical content knowledge interact with each other during instruction, it is difficult to distinguish them. Therefore, the extent of a novice primary teacher's mathematical knowledge in teaching on area measurement was examined by means of the KQ model in this study.

Measurement

Teachers' mathematical knowledge in teaching is important for every mathematical strand including measurement. Measurement is "the assignment of a numerical value to an attribute of an object" (National Council of Teachers of Mathematics [NCTM], 2000, p. 44). Specifically, Lehrer (2003) emphasizes that "Measurement is an enterprise that spans both mathematics and science but has its roots in everyday experiences" (p. 179). In the same way, the measurement content strand "… offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts and notions of function" (NCTM, 2000, p. 44). Since measurement includes commonly encountered and much needed concepts of daily life and is related to other strands in mathematics, it is accepted as one of the content strands of mathematics curricula and introduced in first grade and then extended throughout elementary school (Chen, Reys, & Reys, 2009; Clements & Sarama, 2007; Ministry of Education, Culture, Sports, Science, and Technology [MEXT], 2008; Ministry of National Education [MoNE], 2005; National Curriculum Board [NCB], 2011; NCTM, 2000).

Area measurement, one of the most commonly taught and essential topics of the measurement content strand (Baturo & Nason, 1996; MEXT, 2008; MoNE, 2005; NCB; 2011; NCTM, 2000), is an essential part of daily life and is related to other mathematical topics (Hiebert, 1981; Hirstein, Lamb, & Osborne, 1978). It is defined as the amount of two-dimensional space or a region enclosed (Strutchens, Martin, & Kenney, 2003; Van de Walle, Karp, & Bay-Williams, 2012). Students in Japan and Australia initially encounter the concept of area in the first grade by comparing area of different plane figures. Unlike these students, the students in Turkey and the US begin to learn the concept of area in the third grade. While the students in Turkey start to learn the concept of area by covering and measuring a plane figure by means of non-standard materials, the students in the US start to learn the concept by recognizing it as an attribute of a plane figure, measuring it by counting, and finding it by formula. Unlike the students in the US, the students in Australia, Japan, and Turkey are expected to calculate the area of a square and a rectangle and to investigate patterns to explore a relationship between area and its side lengths in the fourth grade. In other words, calculation of area measurement by means of formula of length×width, base×height or row×column is introduced in the fourth grade of Australian, Japanese, and Turkish curriculum (MEXT, 2008; MoNE, 2005; NCB, 2011).

In spite of its practical applications to daily life and relations to other mathematical topics, studies indicate that area measurement is one of the topics that students often struggle with. One of these struggles is students' confusion about area and perimeter concepts (Hirstein et al., 1978; Kenney & Kouba, 1997; Chappell & Thompson, 1999). Students tend to think that if two rectangles have the same area, then their perimeters must also be the same (Hart, 1984; Woodward & Byrd, 1983). One of the reasons for students' difficulties with and poor understanding of area measurement can be attributed to teachers' knowledge and their practices in the classroom. If teachers focus on the procedural understanding of area measurement and do not let their students experience an area concept with concrete objects, these students may not understand the meaning of area in a conceptual way (Hirstein, et. al., 1978). However, the most important part of measurement is "knowing what the attribute to be measured is" (Baturo & Nason, 1996, p. 238). Otherwise, the meaning of area for the students would be formula dependent as base×height, row×column or length×width (Clements & Stephan, 2004; Outhred & Mitchelmore, 1996). Then, students would have troubles applying the above formulas to find the area of irregular planes (Lehrer, 2003).

Rationale for the Study

Since "The globalization of markets, the spread of information technologies and the premium being paid for workforce skills all emphasize the mounting need for proficiency in mathematics" [National Research Council, (NRC), 2001, xiii], improving student learning and understanding of mathematics is critical to making them mathematically literate. Hawley and Rosenholtz (1984) state that teachers and their knowledge have the greatest impact among the factors that affect student performance. In the same way, Ball (1991) emphasizes teacher knowledge by stating "Teachers cannot help children learn things they themselves do not understand'' (p. 5). That is, teacher knowledge lies at the heart of factors in improving student learning and understanding. Specifically, mathematical knowledge in teaching is related to the ability of teachers to be aware of purpose, to choose and use examples, representations, and demonstrations, to respond to students' questions, and to connect mathematical concepts and procedures (Brown & Borko, 1992; Van de Walle et al., 2012). In the same way, the National Mathematics Advisory Panel (2008) states that teachers must know both the mathematical content that they are teaching and how this content is related to previous and future topics. Although the mathematical knowledge in teaching on different topics has been the subject of an increasing amount of research (Ball, 1990; Enochs & Gabel, 1984; Even, 1993; Even & Tirosh, 1995; Huang & Kulm, 2012; Isiksal, 2006; Roche & Clarke, 2012; Tekin-Sitrava, 2014), little research on area measurement has been conducted in Turkey and other countries (Baturo & Nason, 1996; Menon, 1998; Murphy, 2010; Simon & Blume, 1994; Tierney, Boyd, & Davis, 1990). In addition, when these studies were examined in detail, it was seen that they were conducted with pre-service teachers. According to our recent research on literature, there was no research studying in-service teachers' mathematical knowledge in teaching on area measurement. This paper critiques an in-service primary teacher's mathematical knowledge in teaching on area measurement, and it may be beneficial to provide pre-service and in-service teachers in Turkey and abroad with awareness of their knowledge which directly influences their teaching practice. Criticizing the poor points of area measurement instruction and giving information about how these poor points turned into sound points may also be helpful for teachers to analyze their lesson plans and teaching practices.

Additionally, the studies mentioned above were conducted by means of interviews and questionnaires, and examined teacher knowledge outside of the classroom. However, since teachers put their knowledge into action in classrooms, separating teacher knowledge from the classrooms may be misleading. When rich and constantly changing classroom environments are imagined, it is accepted that teacher knowledge is also dynamic and highly context-dependent. Thus, to gain a complete understanding of teacher knowledge, teachers' practices in the classroom environment deserve further attention. That is, there may be differences between teachers' plans and their instruction regardless of how well they plan their lessons. Therefore, although teachers can thoroughly and clearly enough answer the questions given in a test or asked during an interview, they may act differently in the classroom from what they presume or answer. For example, the teacher can state that s/he uses the textbook as a guide or does not use every example given in the textbook during her teaching practice. However, s/he may not critically examine the textbook or may not modify the examples according to the students' level, and hence may strictly adhere to the textbook while teaching the topic. Hence, studying teachers' mathematical knowledge in teaching in a classroom environment is important to see how they transform and use their knowledge in a real context. Moreover, knowing how to respond to students' questions or how to use these questions to connect mathematical ideas are important parts of classroom discourse and cannot be observed other than in the classroom environment (Boaler, 2002). Therefore, this study examined an in-service teacher's mathematical knowledge in teaching on area measurement in the classroom environment by means of the KQ model to acquire information about teachers' use of textbook, choice of examples and representations, and responses to students' ideas and questions. In this context, this study seeks answers to the following research question:

What is the nature of a novice primary teacher's mathematical knowledge in teaching on area measurement?

How does a novice primary teacher's use of textbook, choice of representations and examples, connections between concepts, use of resources, and responses to students' ideas give information about her mathematical knowledge in teaching on area measurement?

Methodology

Research Design

A case study is "the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances" (Stake, 1995, p. xi). This case can be an individual, organization, a class or a school on which researcher(s) would like to focus or get in-depth information about. This study used a qualitative case study method to investigate a novice primary teacher's mathematical knowledge in teaching on area measurement. Case study was the most appropriate method for this study as we were primarily interested in understanding the process of how the teacher used the textbook, chose representations and examples, connected concepts, used resources, and responded to the students' ideas during area measurement instruction (Bogdan & Biklen, 1992; Creswell, 2008; Merriam, 1998). Furthermore, in order to see the possible relationships between the case and the context, the novice primary teacher's mathematical knowledge was investigated in its naturally occurring environment (Fetterman, 1988; Dyson & Genishi, 2005) which is the fourth grade classroom. This boundary is also important as the nature of the classroom environment may influence the teacher's mathematical knowledge in teaching and the teacher may reflect her knowledge during area measurement instruction.

The Context and Participant

This study took place in a fourth grade classroom at a primary school located in a small urban district of Ankara, Turkey. The school housed two classes at kindergarten through eighth grade students whose families were mostly low-income. At the time of this study, the school was composed of approximately 250 students and the average class size was between 10-15. As mentioned previously, this research intended to investigate the mathematical knowledge in teaching on area measurement of a novice primary teacher, Ebru (pseudonym). Ebru graduated from one of the four-year teacher education programs in the Department of Primary Teacher Education, Turkey. Before entering into a pre-service primary education program, pre-service teachers have to pass a national university entrance exam. During university education, they take mathematics and mathematics education courses in addition to other content courses and educational courses. Furthermore, they enroll in a school experience course and two teaching practice courses before their graduation. While the school experience course is based mostly on observation of instruction, teaching practice courses are based on both observation and practice. Pre-service teachers are also required to enroll in a nationwide examination in order to be recruited by the government following their graduation. Following their recruitment, primary teachers normally work with same students from the first grade through the fourth grade. However, due to various circumstances, primary teachers may start to teach students from the second, third, or even fourth grade. Ebru, the participant of this study, passed all the above-mentioned stages successfully and started her teaching career in a public school.

Ebru was selected as the participant of this study for several reasons. First of all, the focus of this study is to investigate a novice primary teacher's mathematical knowledge in teaching on area measurement; therefore, it is critical that the teacher is eager to participate. That is, Ebru was chosen because of her willingness to participate and to meet with researchers before and after her instruction. Furthermore, showing a primary teacher who had an exemplary record and who had taught the same students from the first grade are the other selection criteria. Teaching the same students starting from the first grade is important as the teacher may know their difficulties or misconceptions, previous knowledge, or their learning style, and be able to plan his/her instruction considering these points. Ebru was in her fourth year of teaching and was teaching the same students from the first grade. Furthermore, Ebru was recognized by the principals as effective and committed to the continuous improvement of her instruction.

In Turkey, the objectives of the primary mathematics curriculum are nationwide and prepared by curriculum developers. All primary schools and their teachers have to plan their instruction with regard to these objectives. Furthermore, the textbooks used by teachers are also written according to the objectives of the curriculum and teachers are requested to use these textbooks as a guide. These textbooks have many sections for each topic including objectives, warm-up, presentation, application, and evaluation. While some teachers use the textbooks during instruction starting from the first part, warm up, and ending with the last one, evaluation, some others use them as a guide but organize their instruction themselves. The objectives of area measurement offered in the fourth grade of the primary mathematics curriculum are:

Estimate the area using non-standard area measurement units and check it by counting the units.

Recognize that a planar region's area is equal to the number of square units covering it.

Calculate square and rectangular regions' areas using square units (MoNE, 2005).

Data Collection and Analysis

Multiple data sources were used in order to investigate the novice primary teacher's mathematical knowledge in teaching on area measurement. These sources were observations, video-recordings of Ebru's area measurement instruction and stimulated-recall interviews with Ebru, the first researcher's field notes, and informal conversations before and following Ebru's instruction. Ebru was observed and video-recorded for five lesson hours, each lesson hour taking 40 minutes, as the primary mathematics curriculum of Turkey dedicated five lesson hours for the topic of area measurement. Although each of these five lessons was observed and video-recorded, in order to analyze the lesson in depth, the present study focused on recognition of a planar region's area is equal to the number of square units covering it. This objective corresponded to two lesson hours. The data were gathered by the first author and during the instruction; she sat at the back of the classroom and did not interact with the teacher or the students. In order to secure Ebru's area measurement lessons for later transcripts and to use them during the stimulated-recall interviews to help remind her of the lessons, lessons were video-recorded. The video camera was placed in the back of the classroom in order to not disrupt the natural learning environment and to record the teacher's behaviors and the board. The first researcher also took field notes while sitting close to the video camera to adjust the camera's focus if necessary. The first purpose of these field notes was to document what happened related to the codes of the Knowledge Quartet model, at what time these events happened, and any other noteworthy events. Shortly, these notes served to accurately represent Ebru's mathematical knowledge in teaching on area measurement. All video-recordings and field notes were transcribed and made available to Ebru for review.

Patton (1990) states that researchers can understand "what is in and on someone else's mind" by means of the stimulated-recall interviews. Furthermore, Bloom (1954) emphasizes that if researchers help participants of their study recall what they did by means of video or audio-recordings, participants can give more accurate and detailed information during these interviews. Therefore, to get a clearer understanding of Ebru's mathematical knowledge in teaching on area measurement, some informal conversations before the class and stimulated-recall interviews following the class to take further information about the instruction were conducted, and these were also video-recorded. The interview questions were prepared according to each day's instruction, the informal statements made by Ebru during informal conversations, and the field notes. These questions were open ended to add depth to Ebru's actions during the instruction. Following the interviews, the video-recordings were transcribed verbatim. They were also made available to Ebru to check their accuracy.

Data analysis in case studies is an ongoing process as researchers collect and analyze data simultaneously (Glaser & Strauss, 1967; Merriam, 2002). Accordingly, the analysis of data started with watching the video recordings of the each observed lessons and carefully transcribing these lessons. Following these transcriptions, multiple readings of the verbatims were conducted as to not miss any part of the video-recordings. These transcriptions were separated into episodes to determine critical moments in which Ebru used her mathematical knowledge in teaching. Then, the researchers examined these transcriptions for the relevant codes of the Knowledge Quartet model. This coding was based upon the six codes of the model: use of textbook, choice of representations and examples, connections between concepts, and use of resources and students' ideas. During these processes, an analytical-inductive method was used to determine if there were other relevant codes in addition to ones mentioned by Rowland and his colleagues (2009). The findings of the study were presented under the relevant codes of the Knowledge Quartet model which were adherence to textbook, choice of representations and examples, making connections between concepts, responding to unavailability of resources, and responding to students' ideas, respectively.

Findings

The purpose of this study is to investigate a novice primary teacher's mathematical knowledge in teaching on area measurement. Since it will be helpful to have an idea about Ebru's area measurement lessons before presenting and discussing the findings, an overview of her lessons was presented. Then, the lessons were analyzed by means of the contributory codes of the KQ model to get an idea about her mathematical knowledge in teaching on area measurement. Furthermore, when the interviews between the first researcher and Ebru or dialogues between Ebru and her students' are presented, researchers' interpretations or reasoning behind the statements were also given next to them and in italic type.

Ebru stood at a whiteboard facing her students and showed some square and circle papers to direct the students' attention to area measurement. Then, she started the lesson using a textbook example in which the students were asked to determine and explain which one of two fields was larger. Later, Ebru continued the lesson again with square and circle papers to make her students decide which one is more suitable to measure the regions' areas. A four-minute discussion focused on how to measure a region's area. The classroom was then split into two groups and assigned their tasks; the first group was given square papers, the second one circle papers, and both groups were asked to cover their mathematics textbooks. After covering their textbooks, the students discussed which one of these papers was suitable for area measurement. After students decided that square papers were suitable, Ebru continued the lesson by writing an explanation about area and unit squares. While the students copied the writings from the board, Ebru moved around the classroom to check whether the students wrote correctly or not. Following writing, each of the two groups covered Ebru's file using square papers. Both groups found the number of squares needed to cover Ebru's file. After comparing the groups' answers, Ebru asked the students whether they wanted to cover one more object or not. Because students wanted to cover their tables and there were not enough square papers, they drew squares and cut them. Before starting to cover a table, Ebru wanted students to think and to make a guess about the number of squares needed to cover the table. Ebru wrote each of her students' guesses on the board. After Ebru called all the students in front of the first group's table, the students started to cover the table with squares. During the table covering activity, while some of the students counted the squares one by one, some of them tried to count them in groups of two. Once, when one of the students started to count in groups of 5, Ebru made the other students listen to her counting strategy. While all the students were counting in groups of 5, like 5, 10, 15, 20, 25, Ebru asked her students to answer the question "what can be used instead of this rhythmic counting like 5, 10, 15,…?" Many students stated multiplication as an answer to the question, and one of her students stated that it could be quickly calculated by multiplication. When the bell rang, Ebru concluded that to find the number of squares needed to cover the rectangle, we could multiply the number of squares in the row and the column.

Adherence to Textbook

Adherence to textbook, one of the codes of the foundation dimension, is about teachers' use of textbook. Rowland and his colleagues (2009) state that how teachers critically use the textbook and modify it considering the objectives of a lesson gives information about teachers' mathematical knowledge in teaching. In this study, two moments related to the adherence to textbook code were mentioned.

For the objective of "recognizing a planar region's area is equal to the number of square units covering it", there is an example in the textbook as in the (Figure 1):

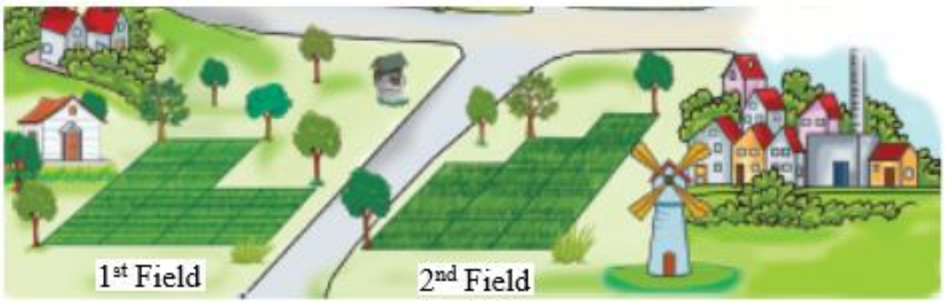

Figure 1. Example for area measurement (MoNE, 4th Grade Student Lesson Textbook, 2005, p. 110).

In this example, the teacher asked the students to compare the first and second fields' area. The task was to find the number of small squares that can be counted one by one, which gives the area of the fields. After comparing their areas, she continued the lesson with circle and square papers to make her students decide why square papers were suitable for area measurement. The following quotes are from the post-interview where the reasoning of why Ebru used circle and square papers was questioned.

Researcher:	You used circle and square papers to make your students discover that some models (circle papers in this example) were not suitable to measure an area. What can be used instead of circle papers? Are there any models?	Interpretation: *Since Ebru used only circle papers during the lesson in order to show her students that they are not suitable to cover the textbook, the researcher asked her if she could mention other non-suitable models.*
Ebru:	Nothing comes to my mind for now. However, ellipse or oval models that have not any corners can be used. To allow for gaps, models that are used must not have any corners.	
Researcher:	You mean that we can measure an area by means of any type of polygons having corners, can't we?	Interpretation: *Ebru's explanation was questioned to get detailed information whether she really thought that all the polygons were suitable for area measurement.*
Ebru:	Yes.	
Researcher:	What can we use instead of square papers?	
Ebru:	Rectangle.	
Researcher:	Any other?	
Ebru:	Pentagon, hexagon,…	

As can be seen from the above dialogue, Ebru thinks that all the polygons can be used to measure an area. Thus, she automatically states polygons without thinking whether they are really suitable or not. Furthermore, since Ebru was not sure about why a square model was appropriate, she could not explain whether or not there were other planar models that can be used to measure an area. In fact, when asked the reason for her choice of circle and square papers, she said that "these models were in the textbook and so I did not think of an alternative model". That is, Ebru did not question why circle and square models were given in the textbook, she accepted this information without any modification. Furthermore, the explanation that Ebru wrote on the board for why square models were suitable and the definition of unit square was again from the textbook:

> A surface can be covered using square tiles without leaving gaps. Hence, squares are used to measure an area. This square, which is used to measure an area, is called a unit square (MoNE, 4th Grade Student Lesson Textbook, 2005, p. 111).

As mentioned in the methodology part of this paper, some primary teachers may prefer to use textbooks as a guide. These teachers consider their students' previous knowledge, misconceptions, or difficulties and plan their instruction accordingly. In other words, they do not directly take examples from textbooks or make activities as they are presented in textbooks. Instead, they question the difficulty and content of activities and examples, they attempt to make their teaching more meaningful for their students. To learn whether Ebru went through these processes or not, she was asked to define area during the interview. She stated that "I went through the textbook, but I could not find any definitions in the textbook, so I thought there is no definition". This explanation indicates that Ebru's adherence to the textbook resulted in a noteworthy hesitance to provide a definition for area. Similarly, Brahier (2016) emphasizes that teachers' strict adherence to lesson plan or textbook may limit flexibility or creativity of their instruction. In short, introducing the area measurement topic with the circle and square papers given in the textbook can be beneficial for directing students' awareness to the area measurement topic and helping them carry out tasks consciously; however, it can be said that adhering to the textbook without reasoning resulted in narrow and limited mathematical knowledge in teaching on area measurement.

Choice of Representations and Examples

Choice of representations and choice of examples are codes of the transformation dimension. While choice of representations is about which representations the teacher will use during instruction to support students' learning and understanding, choice of examples is about which examples that the teacher will select or create to allow the students interact (Rowland et al., 2009).

The first code discussed here is about Ebru's choice of representations. During the lesson, because the objective was to decide which models were suitable for area measurement, students were assigned to two groups. While the first group was given square papers, the second one was given circle papers to cover their mathematics textbooks (Figure 2):

Figure 2. Second group's textbook covering activity.

As mentioned in the adherence to textbook part, the square and circle papers used to cover the textbook were not intentionally selected by Ebru. However, since the students understood that some models were not suitable for area measurement, it can be said that Ebru's choice of representations served the stated objective of the lesson.

The second code that gives information about how Ebru transforms her knowledge is her choice of examples. For the final part of the lesson, Ebru wrote some examples on the board and told her students that they were going to calculate the area of the regions (Figure 3):

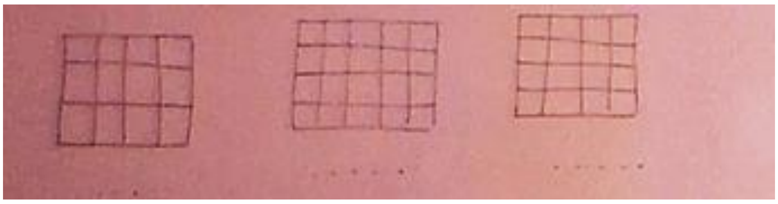

Figure 3. Given examples for area measurement.

After copying the questions from the board, students worked independently for a few minutes. Then, Ebru called one of her students to the board in order to solve and explain his solution to his friends. Although Ebru tried to encourage her students to use the row by column (R×C) formula by saying that 'it would be difficult to count in ones or groups', the students preferred to count one by one for the given examples. To state differently, the unit numbers in the examples were not suitable to make the students prefer to use the R×C formula. So, it can be said that Ebru's mathematical knowledge in teaching on area measurement for the code of choice of examples is not adequate.

Making Connections between Concepts

Making connections between concepts, one of the codes of the connection dimension, is related to both recognizing that different parts of mathematics are connected and knowing how these parts are connected. The most important connection to be made in Ebru's lesson is between repeated addition and the R×C formula. There are examples showing this connection in the 4th Grade Student Lesson Textbook. After the students' group tasks mentioned in the choice of representations code, students were asked to cover their tables (Figure 4):

Figure 4. Table covering activity.

Ebru:	How many papers are there on the table?	Interpretation: *Ebru asked her students identify the number of the squares used to cover the table.*

Students:	1, 2, 3, 4, 5, 6, …	Interpretation: *Ebru's students started to count the square papers one by one.*
Student$_1$:	5-10-15-20…	Interpretation: *One of the students discovered the pattern and started to count by fives.*
Ebru:	She counted very well. What did she make? She counted squares in the column like 1-2-3-4-5, then she continued to count like 5-10-15-20-25-30-35-40-45-50-55-60-65 with her students.	Interpretation: *Ebru and her students started to count forward by fives to 65 which was equal to the total number of square papers.*
Student$_2$	If we multiply 5 with 13…	Interpretation: *When they were counting in chorus to 65, one of her students suggested to multiply 5 with 13.*
Ebru:	What did he say? Repeat it.	Interpretation: *Since making a connection between repeated addition and the R×C formula is in Ebru's mind before starting the area measurement lesson, Ebru asked her students to notice their friend's idea.*
Student$_2$	We multiply 5 with 13.	Interpretation: *The student repeated his idea.*
Ebru:	Why? Where is 5? Where is 13?	Interpretation: *Ebru asked the student to explain his thinking.*
Student$_2$	13 is here (by showing the row) and 5 is here (by showing the column).	Interpretation: *The student showed that there were 13 squares in a row and 5 squares in a column.*
Ebru:	Hmm. What do we do instead of repeated addition by means of short method?	Interpretation: *Ebru tried to make connection between the repeated addition and the multiplication.*
Students:	Multiplication.	Interpretation: *The students (in chorus) answered it.*
Ebru:	We multiply. 1-2-3-4-5 by pointing to the squares in the column and 1-2-3-4-5-6-7-8-9-10-11-12-13 by pointing to the squares in the row. We multiplied	Interpretation: *In order to help her students understand why they multiply 5 with 13, Ebru again showed her students the way to count the*

	5 with 13.	*squares.*
Students:	65.	Interpretation: *They all answered.*
Ebru:	65. Then, in order to calculate the area of a rectangle, we can multiply the length and width, can't we?	Interpretation: *Ebru asked her students to decide if a rectangle's area can be calculated by means of multiplication of the length and the width of the rectangle.*
Students:	Yes.	Interpretation: *They all answered in chorus again.*

As seen in the above dialogue, Ebru took her student's suggestion into consideration and tried to convert this table covering activity to an opportunity to teach her students that the area of a rectangle can be calculated by means of multiplication of the length and the width of the rectangle. In other words, this dialogue shows that Ebru attempted to highlight the connection between repeated addition and the R×C formula. Furthermore, the way in which Ebru reminded her students to use the R×C formula for area measurement problems in the remaining area measurement lessons several times provided some evidence about making connections between concepts.

Responding to Unavailability of Resources

Responding to unavailability of resources, one of the codes of the contingency dimension, is related to how teachers critically decide what to do in response to lack or unavailability of planned resources or technology. Although Ebru stated that there were not events that make her deviate from her lesson plan during the post-interview, she made some adaptations resulting from unavailability of the resources. The table covering activity mentioned above can be accepted as an example of this deviation. More specifically, before the area measurement lesson, Ebru had planned to make another covering activity after the mathematics textbook covering activity mentioned under the choice of representations code. However, her students wanted to cover their tables and they would need more square papers. Although this suggestion created a sort of contingent moment, Ebru tried to solve this contingent moment by letting her students draw new squares, cut them out, and cover the table. This was an opportunity for the students to generalize about the relationship between the R×C formula and the number of unit squares in a row and a column.

Responding to Students' Ideas

Responding to students' ideas, one of the codes of the contingency dimension, is related to the teacher's ability to make meaningful, supportive, and informative responses to the students' unexpected ideas in such a way that the students will make sense of the concept. Despite the good example given above, Ebru did not invite and take students' ideas into consideration effectively. For example, before starting the table covering activity, Ebru wanted her students to make a guess about the number of squares required to cover the table and wrote their guesses on the board. However, Ebru accepted all of her students' guesses without exploring how they suggested these guesses. Furthermore, during the covering process, one of her students discovered the R×C formula, found the number of squares by using the formula, and therefore wanted to change his guess from 45 squares to 65 squares. However, Ebru did not allow him to change his guess and did not ask why he wanted to change his guess. When the reason of her behavior was questioned during post-interview, Ebru stated that there was no reason for her different behavior to the student and accepted this moment as a missed opportunity.

Conclusion and Discussion

In this study, a novice primary teacher's mathematical knowledge in teaching on area measurement was examined by means of the codes of adherence to textbook, choice of representations, choice of examples, making connections between concepts, responding to students' ideas, and responding to unavailability of resources. It can be said that while Ebru's mathematical knowledge in teaching on area measurement regarding some of the codes such as adherence to textbook, choice of examples, and responding to students' ideas is not adequate, her knowledge for choice of representations, making connections between concepts, and responding to unavailability of resources is adequate.

Considering the code of adherence to textbook, although Ebru showed some positive findings such as using circle and square papers to introduce area measurement, she could not use the textbook effectively throughout the area measurement instruction. In Turkey, two different mathematics textbooks are used in primary mathematics classrooms, and a teacher handbook is provided by the ministry for each of the two textbooks. Objectives followed by activities, materials necessary for these activities, alternative activities, examples, and assessment practices are given in these teacher handbooks. As mentioned previously, although the teachers in Turkey have to use the textbook suggested by the government, they are expected to critique these activities and materials and modify them according to their students' needs (Ball et al., 2008; Haggarty & Pepin, 2002). However, it might be said that Ebru could not be successful in this regard because she solely adhered to the textbook by following every part without any modification for her students. Her adherence to the textbook resulted in not being able to discuss the reasons why square models were used for the area measurement and not being able to mention any other models that could be used instead of square models. Instead, the circle papers taken from the textbook could well have been a starting point to discuss why some models are not suitable for area measurement. Researchers state that there may be many factors affecting teachers' use of textbook, such as limitation of time, the inability to find good and alternative sources, country policies, and deficiency in subject knowledge (Nicol & Crespo, 2006; Sosniak & Stodolsky, 1993; Remillard, 2005). When the reason of Ebru's adherence to textbook was questioned during the interview, she stated that she had looked for alternative books; however, the textbook suggested by the government had seemed to be more useful compared to those books. Therefore, it can be concluded that Ebru's adherence to the textbook might result from not finding alternative sources. Her first year teaching the fourth grade could be another reason for her adherence to the textbook. In order to overcome these difficulties, professional learning could be set up to provide information about how a textbook can be used as a starting point for instruction or on what kind of modifications can be made according to the students' needs. That is, teachers can be informed that they are not required to do every listed activity or example or spend the exact time on a topic that is given in teacher handbooks and have right to modify the activities in a book according to their students' needs. Furthermore, these teacher handbooks can be more teacher friendly, as they should include numerous activities or examples considering students' levels.

Similarly, Ebru's mathematical knowledge in teaching on area measurement was not adequate in terms of choice of examples and responding to students' ideas. Researchers emphasize that using appropriate examples is an essential and critical element of mathematics (Leinhardt, 2001; Rissland-Michener, 1978). During the area measurement lessons, Ebru intended to make her students understand the R×C formula by means of examples. However, as given under the code of choice of examples, since the given rectangles were 3×4, 4×5, and 4×4, the number of unit squares could be quickly counted by the students. That is, the examples were not challenging enough to encourage students to discover alternative ways to find the solution. Therefore, the students did not prefer to use the R×C formula. As Rowland and his colleagues (2009) emphasize, the numbers in the examples given here may not be suitable for the objectives of the lesson. Ebru did not write these examples intentionally to make her students discover that they can find a rectangle's area by multiplying the number of unit squares in a row with the number of unit squares in a column. Instead, Ebru could have selected numbers such as 12×13 or 25×8, where counting the unit squares one by one is not a preferable method and checked students' possible solutions in calculating area. By this way, students would have opportunity to discover another way to find the area and develop conceptual understanding of the R×C formula.

As mentioned in the code of responding to students' ideas, one of Ebru's students wanted to change his estimate during table covering activity; however, Ebru did not let him. Ryan and Williams (2007) emphasize that encouraging students to explain their thinking and using these explanations is important in helping students' learn content meaningfully. However, the ability to respond to students' ideas is one of the most challenging roles that teachers have (Brown & Wragg, 1993; Delaney, 2008). Rowland, Huckstep, and Thwaites (2005) say that time restraints, students' levels of knowledge, or lack of teacher confidence may prevent teachers from responding to students' ideas. Furthermore, Rowland and his colleagues (2009) state that teachers can respond to these ideas by ignoring, acknowledging but putting aside, or acknowledging and incorporating. Ebru responded to her student's idea by ignoring and when Ebru was asked about this contingent moment during the post-interview, she could not state any specific reason. Therefore, it can be concluded that Ebru did not respond to her student's idea properly. Instead, Ebru could have used this contingent moment as an opportunity to explain why the student wanted to change his idea and make other students discuss whether his idea was right or not. The reason for Ebru's behavior can be resulted from having difficulty posing questions to probe his thinking. Until we investigate how Ebru respond to her students' ideas, we cannot know how to help her. Watching video-recordings of her area measurement instruction shows the richness of conversations between Ebru and her students and among the students themselves. Therefore, watching these recordings can help her determine what she notices or attends to, and how she responds to her students. Similarly, having teachers analyze these points is important as they can determine the parts of their teaching that need improvement, which results in improved student learning. That is, giving an opportunity for reflection on such recordings engages teachers in thoughtful consideration of their students' ideas. If at all possible, watching these recordings with other teachers may improve collaboration, may help teachers see and consider others' perspectives, and may help them notice the points that they did not before. Therefore, video-recordings can play an important role in analyzing and improving teacher practices.

As opposed to the codes of adherence to textbook, choice of examples, and responding to students' ideas, Ebru's mathematical knowledge in teaching on area measurement regarding making connections between concepts could be accepted as adequate. It is known that there is a connection between repeated addition and the R×C formula, and Ebru started her lesson with this connection in her mind. Therefore, Ebru tried to make the connection between repeated addition and the R×C formula. During the table covering activity, while most of the students counted the unit squares one by one, one of her students counted in fives and found the answer faster than her friends. Ebru noticed her student's strategy and used it to conceptualize how the students could find the area of a rectangle more easily. When finding an area of a rectangle is not easy by counting the unit squares one by one, the students concluded that they could find it by multiplication of the number of squares on adjacent sides of the rectangle.

In conclusion, as mentioned in the methodology, since this paper reports on a single case study, generalizations cannot be drawn from the results. However, it is possible to raise some questions and to suggest some implications for primary teachers and teacher educators. In this study, it was found that the novice teacher had limited mathematical knowledge in teaching on area measurement in terms of adherence to textbook, choice of examples, and responding to students' ideas codes. Therefore, professional learning opportunities could be organized to help teachers develop their knowledge for these codes. Discussions about these points may help teachers be aware of their strengths and weaknesses in their classroom practices. For example, by means of this professional learning, teachers can realize and accept that their classrooms consist of students with varying abilities and prior knowledge. By taking these issues into consideration, teachers can modify the textbook and choose examples according to their students' levels. Furthermore, teachers can learn to create a positive and welcoming learning environment for their students' ideas and questions that will allow students to discuss their thinking.

Based on the findings of the study, teacher educators who are responsible for ensuring that pre-service teachers have the essential competences required for effective teaching should also reconsider the content of their lessons. Providing them with competences is important as they learn what mathematical knowledge in teaching is and how it is used in the primary school level; in short, they develop and use their mathematical knowledge in teaching. In doing so, teacher educators should introduce mathematics in a way that provides an appropriate model for pre-service teachers' future teaching life, as they have a significant impact on the quality of their instruction. Teacher educators can also use the KQ model as an instrument to give feedback on their students' teaching practices. For example, using the KQ model can help teacher educators discuss how their instruction went, what examples they solved, how these examples were suitable for the students, what possible connections could be made between the examples, what misconceptions occurred, how they overcame or failed to overcome these misconceptions. In short, using the model can direct both teacher educators and pre-service teachers' attention to the most important points of their instruction.

The analysis of Ebru's mathematical knowledge in teaching on area measurement has provided us with an insight into one Turkish primary teacher's mathematical knowledge in teaching considering the use of textbook and resources, choice of representations and examples, and responses to students' ideas. The findings of this study reaffirm the importance of the teacher's pedagogical decisions as they affect the teacher's instruction. Therefore, it can be accepted as a starting point for similar studies with larger groups of teachers in order to see how they use and modify the textbook or prepare their examples according to their students' levels and needs. In the same way, teachers' response types can also be examined to see what kind of responses enhance or inhibit students' understanding of mathematics. Studies involving experienced primary teachers in addition to novice primary teachers would extend our knowledge of primary teachers' mathematical knowledge in teaching. In short, these studies would be an interesting and informative way to see the larger picture of primary teachers' mathematical knowledge in teaching both in Turkey and abroad.

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