

An Investigation of Eighth Grade Students' Skills in Problem-Posing

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The purpose of this study was to investigate eighth grade students' skills in different problem-posing situations. Problem-posing situations include free, semi-structured and structured problem-posing situations. The study was conducted using case study methodology, which is within the scope of descriptive research. The study was conducted with 166 eighth grade students (ages 13-15). The Problem-Posing Test was used as a data collection tool. In the analysis of the data, an analytical rubric developed by the researchers was used. The problem-posing assessment criteria were mathematical expression, language, suitability for acquisitions, quantity and quality of data, solvability, originality and presence of the solution. The data were analysed descriptively with a qualitative approach. It was observed that the students had generally low success in different problem-posing activities. In this research, the students were found to be more successful in semi-structured problem-posing situation than free and structured problem-posing situations when each problem-posing situation was examined separately. It was found that the majority of the problems the students posed were routine problems. It was determined that the students mostly used "writing an equation" and "drawing a shape" strategies for the solutions of the problems they posed in different problem-posing situations.

Problem-posing, as one of the concepts that draw attention in mathematics research in recent years, is at the core of mathematics (National Council of Teachers of Mathematics [NCTM], 2000). "Problem-posing and problem-solving" are important components of mathematics teaching and closely tied to each other (Cai, 2003; Cankoy & Darbaz, 2010; Silver & Cai, 1996). In this context, every student should be given the opportunity to identify and pose their own problems (NCTM, 1991). In middle school mathematics curricula applied in Turkey, similar or specific problem-posing processes of students were emphasized in development of problem-solving skills (Ministry of National Education [MoNE], 2013). Problem-posing, in mathematics education programs prepared by the MoNE, is considered as one of the stages of problem-solving and is expressed as "problem-posing" or "expanding problem". Problem-posing activities take place in the form of "... solve and pose the related problems" in objectives related to each learning area (MoNE, 2009, p. 118, 214, 290).

Silver (1994, p.19) stated that "problem-posing refers to both the generation of new problems and the reformulation of given problem." In this context, problem-posing can occur before the problem-solving, during problem-solving, or after the problem-solving. Stoyanova and Ellerton (1996) pointed out that problem-posing based on mathematical experience is the process by which students constructing interpretations of concrete situations and expressing those as meaningful mathematical problems.

When we look at the contributions provided by problem-posing activities to students, even the simplest problem-posing activities seem to have had a positive impact on students (Brown & Walter, 2005; Cifarelli & Cai, 2006; Silver & Cai, 1996; Toluk-Uçar, 2009), allowing opportunities for students to reason, explore, and accurately express mathematical

situations (Akay, Soybaş & Aygün, 2006). Moreover, problem-posing activities are important to provide more permanent learning (Cankoy & Darbaz 2010), develop critical thinking skills (Nixon-Ponder, 1995), reduce anxiety and fears (Brown & Walter, 2005), be able to reveal misconceptions (Tichá & Hošpesová, 2013), and be used as an evaluation tool (English, 1998). Problem-posing and problem-solving were stated by many researchers to have a strong bond between them (Cai, 1998; Christou, Mousoulides, Pittalis, Pitta-Pantazi & Sriraman, 2005; Ellerton, 1986; Kar, Özdemir, İpek & Albayrak, 2010; Kilpatrick, 1987; Silver & Cai, 1996). Silver and Cai (1996) stated that there was a high correlation between problem-solving and problem-posing in the study conducted with more than 500 middle school students. The students who had high problem-solving skills were determined to be able to pose more mathematical problems. In a study with two groups including eight students in each with high success and low success in problem-solving, Ellerton (1986) found that high achieving students used more complex calculations, more complex number systems, and more computations than low-achieving students. Christou et al. (2005) also pointed out that problem-posing is not only related to problem-solving but also with general mathematical achievement. Kar et al. (2010) found that there was a significant relationship between problem-solving and problem-posing skills of participants, problem-solving skills and the number of problems that were posed in their study conducted with mathematics teacher candidates. Therefore, problem-posing and problem-solving can be seen as an integral part of math classes and math activities (Abu-Elwan, 1999; Kılıç, 2013).

Polya (1973) has suggested a four-stage method to solve a mathematical problem. These stages are: understanding the problem, devising a plan, carrying out the plan, and looking back. Looking back stage requires the solver to check the result, derive it differently, and use it for other problems. This means that after solving the initial problem, the solver should think of related problems that can be solved with the same, or similar, solution or method. While this is clearly problem-posing, Gonzales (1998) has added a fifth stage to Polya's problem-solving method: posing a related problem.

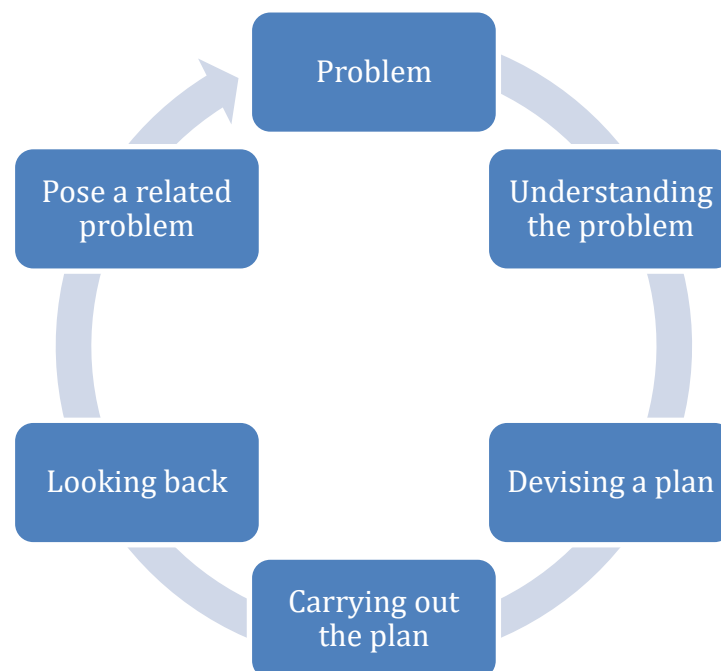


Figure 1. Problem-solving and problem-posing cycle.

In addition to problem-posing activity, it is thought that an evaluation of the posed problems is also important (Katrancı, 2014; Turhan & Güven, 2014). In a study performed by Silver and Cai (2005), the necessity of criteria for the evaluation of the problems that students pose was mentioned, and they determined three basic criteria within the scope of this evaluation. These are the number of posed problems and the originality and the quality of the problems. In a study conducted with pre-service teachers, Gonzales (1994) used a guideline that could be used as a guide to both solve and evaluate the problems. This guide, consisting of 16 items in total, included criteria such as clear expression of the problem; the appropriateness of the language used; the creativity, applicability, and originality of the problem; and the appropriateness of the problem in terms of mathematical level and the concepts. In the study focused on the arithmetic problems posed by the middle school students, Silver and Cai (1996) categorized the problem-posing products as mathematical problems and non-mathematical problems or statements. In the second stage, they classified mathematical problems according to their solvability and complexity. English (1998) applied a 16-week problem-posing program to the participants in a study conducted with third grade students. During this program, students were asked to solve problems that they posed themselves or solve ones posed by their friends. In general, it is seen that the criteria used have been mathematical expression (Gonzales, 1994; Stoyanova, 2005), linguistic knowledge and expression (Arıkan & Ünal, 2013; Cankoy & Özder, 2017; Ekici, 2016; Gonzales, 1994), suitability for acquisition (Gonzales, 1994; Şengül-Akdemir & Törnüklü, 2017), quality (English, 1998; Kaba & Şengül, 2016; Kılıç, 2013), solvability (Cai, 1998; Cankoy & Özder, 2017; Çelik & Özdemir, 2011; Silver & Cai, 1996; Yuan & Sriraman, 2011), originality (Gonzales, 1994; Yuan & Sriraman, 2011), and situation of problem-solving by the student (Cai, Moyer, Wang, Hwang, Nie & Garber, 2013; English, 1998; Şengül-Akdemir & Törnüklü, 2017).

Another purpose of the current study was to determine the types of problems posed by students. Mathematical problems can be classified as routine and non-routine problems. Routine problems are related to situations that are common in real life, and most of the problems that can be solved by one or more of the four operations are routine problems. Most of the non-routine problems are related to the disclosure of relationships, orders, and patterns, and a number of strategies are needed for their solutions (Altun, 2014). A problem that is routine for one student may not be routine for another. Whether a problem is routine or not depends on both the content of the problem and the person who will solve the problem (MoNE, 2013). According to Polya (1973), a problem is a routine problem if it can be solved by taking advantage of previously solved problems. Non-routine problems are those that require more skill than routine problems and the solution method of them is not clear. According to Baki (2008), mathematical questions can be classified as questions in the type of exercise, problem, application, and research. Another classification classifies problems into closed and open-ended problems (Akay et al., 2006). In the current study, the problems posed by the students were examined in terms of whether they were routine or non-routine problems.

Another focus of the study was problem-solving strategies that students use while solving the problems they posed. Some basic problem-solving strategies are systematic listing, prediction and control, drawing a figure, making a table, searching for relation, using variables (writing an equation and non-equation), backward working, eliminating the problem, simplifying the problem, animating, and reasoning (Altun, 2014; Altun & Arslan, 2006; Van De Walle, Karp & Bay-Williams, 2014; Yazgan & Bintaş, 2005). There is a

possible relation between problems posed and problem-solving strategies used by students (Cai, 1998; 2003).

When the related literature is examined, it is seen that there are various situations used in problem-posing activities (Christou et al., 2005; English, 1998; Silver, 1994; Silver & Cai, 1996; Stoyanova & Ellerton, 1996). In a theoretical framework presented by Stoyanova and Ellerton (1996), problem-posing situations have been classified into three groups as free, semi-structured, and structured problem-posing. In free problem-posing situations, students are asked to produce problems from an artificial or natural situation given to the students. Semi-structured problem-posing situations require investigating and identification of an open-ended situation with knowledge, skills, and past experiences. In structured problem-posing situations, students are required to pose problems based on an existing problem (Stoyanova & Ellerton, 1996). The problem-posing activities used in this study consisted of free, semi-structured, and structured problem-posing situations as in the classification of Stoyanova and Ellerton (1996). There are some research studies that have used this classification (Abu-Elwan, 2002; Kırnay-Dönmez, 2014). However, there are only a limited number of studies on different problem-posing situations in the literature (Kırnay-Dönmez, 2014; Ngah, Ismail, Tasir & Mohamad Said, 2016; Silber & Cai, 2017). Furthermore, the relationship between problem-posing and problem-solving also suggests that problem-posing activities may be related to problem-solving strategies and problem types (Akay et al., 2006; Cai, 1998). In the current research, it was aimed to investigate the skills of the eighth grade students in different problem-posing situations. For this purpose, the following are the sub-problems of the present research:

What are the skills of students in different problem-posing situations?

What are the types of problems that students posed in different problem-posing situations?

What are the problem-solving strategies students use to solve the problems they have posed?

Method

In this study, we aimed to examine problem-posing skills of eighth grade students in different problem-posing situations. Alongside the problem-posing skills, we wanted to determine types of problems (routine, non-routine) they posed and strategies they used to solve their own problems. Therefore, the case study method was chosen for the current study.

Case study is an in-depth description and examination of a limited system (Merriam, 2013) and the most important advantage of this method is that it gives the researcher a chance to concentrate on a very specific topic or situation (Çepni, 2010). Also, the case study method is widely used to make judgements (Yin, 2017). Since we aimed to determine the problem-posing skills of eighth grade students in detail and in a short period of time, the case study method was considered suitable for the study.

Participants

The research was carried out with a total of 166 eighth grade students (ages 13-15) in two different metropolitan cities during the second semester of the 2015-2016 academic year in Turkey. The participants consisted of 89 male and 77 female students. The participants had no prior problem-posing experience. The students were mostly socio-economically disadvantaged and had low to average academic success.

Data Collection Tool

The Problem-Posing Test developed by the researchers was used to collect the data required for the study. The developed Problem-Posing Test (PPT) is composed of six open-ended items including two items for each problem-posing situation. The data collection tool was applied to the students in two sessions. The PPT was prepared by considering the Middle School Mathematics Curriculum and selecting different mathematical subjects. When the literature is examined, it is seen that tests used in problem-posing studies are often from a single subject area for a limited number of objectives (English, 1998; Işık & Kar, 2012; Kılıç, 2013; Silver & Cai, 1996). On the other hand, there are also studies using activities for different learning areas (Arıkan & Ünal, 2015; Cai, 2003).

In this study, the items in the PPT were chosen from different learning areas and objectives in the Middle School Mathematics Curriculum. A pilot study was conducted to ensure the validity of the test. After the pilot study, minor language changes were made to eliminate some vague expressions. Final version of the data collection tool was developed by making necessary corrections in the light of pilot study results and expert opinions. There were questions including the Pythagorean Theorem and the square root extraction process for free problem-posing situation in the test. Inequality and number problems were asked for semi-structured problem-posing situation. For structured problem-posing situation, there were problems including the first-degree equations with two unknowns and triangle inequality. As a result, there were six questions in total: two free, two semi-structured, and two structured. The students had two hours to complete the PPT.

Analysis of Data

A graded scoring scale (rubric) developed by the researchers was used to assess the problems posed by the participants (see Appendix-1). The rubric was developed based on evaluation criteria used in some research related to problem-posing in the literature through making the necessary adaptations and additions. In a study with sixth and seventh grade students, Silver and Cai (1996) classified responses of students primarily as mathematical problems and non-mathematical problems or expressions and then evaluated mathematical problems according to their solvability. In the last stage, the posed problems were examined in terms of mathematical and linguistic complexity. Stoyanova (2005) assessed the problems posed by the students as correct answers, partly correct answers, and answers that would not be analysed. Gonzales (1994) developed an assessment guideline to score problems posed by themselves in his study with mathematics teacher candidates. In this guide, participants scored their problems according to criteria such as expression of the problem, appropriateness of the used language, the level of mathematics, originality of the problem, conformity to mathematical concepts, and solvability. Kaba and Şengül (2016) evaluated the problems according to four basic criteria such as problem text, compatibility of the problem with the mathematical principles, type of the problem, and the solvability of the problem. In the light of similar examples, we defined seven criteria to evaluate the problem-posing products: use of mathematical language (symbol, notation), appropriateness of grammar and expression, suitability for acquisitions of the posed problems, quantity and quality of the data, solvability of the posed problem, originality of the problem, and presentation of a solution by a student.

The responses of the students to the problem-posing tests were scored by two researchers and then the scores were reviewed in terms of reliability. The interrater reliability was found to be 81%. A consensus was provided for the items scored differently by the researchers

through reconsideration. The students were classified according to each of the evaluation criteria as Level 1 who scored “0” point, Level 2 who scored “1” point, Level 3 who scored “2” point, and Level 4 who scored “3” point.

Descriptive statistics of the students' answers were used to find answers to the research problems. Direct quotations were also made from the responses of students to support the statistical data. Each quote was coded as "S (Student number) - (Code of problem-posing type)" to indicate which activity belonged to which student and problem-posing situation. “1” was used for free problem-posing activities, “2” was used for semi-structured problem-posing activities, and “3” was used for structured problem-posing activities. For example, the code S124-2 means the quotation was a semi-structured problem-posing response from the student number 124.

Findings

Findings About the First Sub-Problem

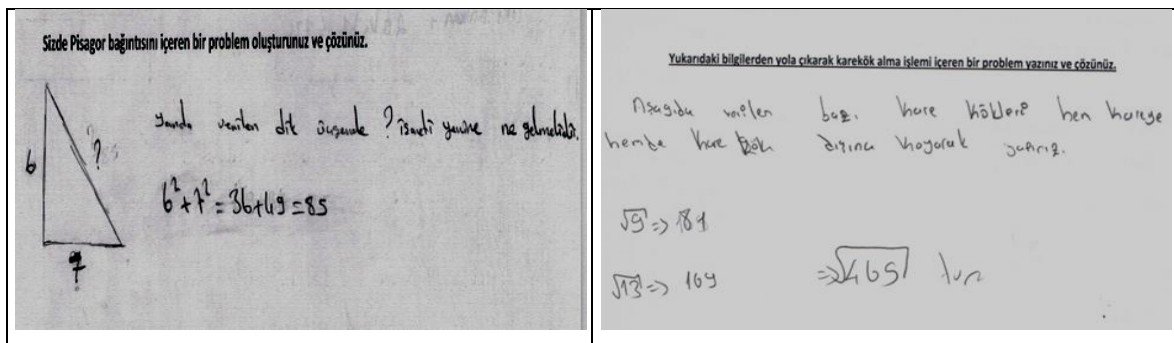
The frequencies and percentages of the scores of the participants in “using mathematical language” criterion are shown in Table 1.

It is seen that the 45.8 % of the problems posed by the students are at the 1st and 2nd levels according to this criterion. It is seen that the ability of students to use the language of mathematics, which is the most basic element that they must possess in order to pose their own problems, seems to be low. Students demonstrated least achievement in using mathematical language is “semi-structured” problem-posing situations. Free problem-posing is another problem-posing situation that is challenging. It is determined that 56.6% of the responses of participants to the structured problem-posing situations appear to be at the 3rd and 4th levels. This demonstrates that participants are more successful in structured problem-posing activities than other problem-posing situations in terms of using mathematics language.

Table 1

Level of ‘Using Mathematical Language’ in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	37.4	9.6	22.0	31.0
Semi-structured	45.8	1.2	16.9	36.1
Structured	33.1	10.3	24.1	32.5
Total	38.8	7.0	21.0	33.2



What should be written in instead of “?” in the given right triangle?	Write the square roots by putting them both in square roots and out of square roots.
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Figure 2. S111-1 Coded Activity

Figure3. S1-1 Coded Activity

The participant S111 used related expressions such as the Pythagorean relation and right triangle but used question marks instead of hypotenuse, and his/her mathematical expressions were not fully appropriate. This problem was scored as a 2nd Level problem according to the criterion of “using mathematics language” since the lack of some mathematical concepts and terms. The answer given by S1 was evaluated as Level 1 according to the criterion of “using mathematics language”. It can be said that the student could not articulate the square root extraction process clearly and that this student might have a misconception about the concept of the square root. In general, it was seen that students did not clearly form mathematical expressions in their problems, and they used missing or incorrect concepts.

The frequencies and percentages of the scores of the participants in “grammar and expression suitability” criterion are shown in Table 2.

Table 2

Level of Grammar and Expression Suitability in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	43.1	14.4	19.3	23.2
Semi-structured	46.4	7.5	16.6	29.5
Structured	40.0	13.9	22.9	23.2
Total	43.2	11.9	19.6	25.3

Approximately 43% of the problems that participants posed are at the 1st Level according to this criterion. This shows that nearly half of the posed problems took 0 points from this criterion. Nearly a quarter of the posed problems (25.3%) are Level 4 problems in terms of grammar and expression. It is seen that participants’ performance showed similar results in different problem-posing situations. In structured problem-posing situations, the percentage of responses at Level 1 (40%) is relatively low compared to other problem-posing situations. From this, it can be interpreted that participants are more successful in structured problem-posing situations. The cumulative percentages of responses at Levels 3 and 4 for free problem-posing situations (42.5%) are lower than semi-structured (46.1%) and structured (46.1%) problem-posing situations. Also, 46.4% of responses to semi-structured problem-posing activities are at Level 1. Therefore, it can be said that the participants were challenged in free and semi-structured problem-posing situations.

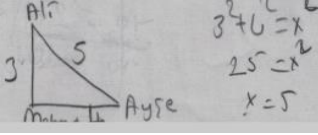
Sizde Pisagor bağıntısını içeren bir problem oluşturunuz ve çözünüz. Ali ile Mehmet'in evlerinin uzunluğu 3 cm, Mehmet ile Ayşe arasındaki uzunluk 4 cm ise Ali ile Ayşe'nin evinin arasındaki uzunluk kaç cm'dir. 	2) $3x - 5 \leq 43$ Yanda verilen eşitsizliği kullanarak bir problem kurunuz ve çözünüz. Bir sayının 3 katının 5 eksiğinin 43'ten büyük de küçük de değil mi? $x = ?$ $3x - 5 \leq 43$ $3x \leq 48$ $x \leq 16$
The house length of Ali and Mehmet is 3 cm, and the length between Mehmet and Ayşe is 4 cm, then what is the length between Ali and Ayşe's house?	If 3 times a number minus 5 is 43 and smaller, $x = ?$

Figure 4. S95-1 Coded Activity

Figure 5. S152-2 Coded Activity

S95 used the wrong terms and did not use the expressions needed to be used in some places in the problem-posing activity and it was scored as the 2nd Level. It is seen that the problem sentence posed by S152 has no integrity in the semi-structured problem-posing activity and has formed the question statement with symbols. This response given by the student was evaluated as Level 3. It is seen in quotations that students use wrong expressions against some concepts and cannot express their opinions in written form. Furthermore, the unit misuse and data incompleteness negatively affect the expression suitability of the posed problems.

The frequencies and percentages of the scores of the problems posed in terms of "suitability to the acquisitions" criterion are shown in Table 3.

Table 3

Level of Suitability to Acquisitions in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	43.4	16.6	18.7	21.3
Semi-structured	47.0	5.1	20.8	27.1
Structured	38.2	21.1	18.4	22.3
Total	42.9	14.2	19.3	23.6

Nearly half (42.9%) of the problems posed by the participants were evaluated as Level 1 in terms of their suitability to the acquisitions. It is understood from this that the problems posed by students are often inadequate in terms of suitability to acquisitions criterion. When the obtained data were examined from different problem-posing situations, it is seen that about 48% of the problems posed in semi-structured problem-posing situations were at the 3rd and 4th Levels. In free and structured problem-posing situations, about 60% of the posed problems were in the 1st and 2nd Levels. Here, the participants were more successful in

posing semi-structured problems than in posing other problem situations according to the suitability criteria for acquisitions.

3) Ahmet elindeki tahta çubukları üçgen şeklinde çerçeveler yapmak istiyor. Biri 8 cm diğeri 12 cm uzunluğundaki çubuklar ile üçgen oluşturabilmek için kullanması gereken 3. çubuğun uzunluğu kaç cm olabilir? (Çubukların uzunlukları tam sayıdır.) <u>Yukarıda verilen probleme benzer bir problem kurunuz ve çözünüz</u> Ali elindeki 4 tane çubuğu bir çerçeve yapmak istiyor. Kenar 4 cm, Alanı 32 olan dikdörtgen çerçevesinin uzun kenarı kaçtır? $\begin{array}{r} 32 \\ 4 \overline{) 32} \\ \underline{32} \\ 0 \end{array}$ Uzun kenar = 8 cm	<u>Yukarıdaki bilgilerden yola çıkarak karekök alma işlemi içeren bir problem yazınız ve çözünüz.</u> Bir karenin bir kenarının uzunluğu 10 cm olduğuna göre alanı kaçtır? $10 \cdot 10 = 100$ $\sqrt{100} = 10$
Ali wants to make a frame with four wooden bars in his hand, if the short side is 4 cm, area is 32, how many cm is the long side?	One side of a square has a length of 10 cm, what's the area?

Figure 6. S125-3 Coded Activity

Figure 7. S117-1 Coded Activity

S125 posed a problem about the area of the rectangle in the structured problem-posing situation for the triangle inequality objective. Although the posed problem was a correct mathematical problem, the response of the students was evaluated as Level 3 according to the criterion of “suitability to acquisitions”. S117 posed a problem about area calculation instead of the square root extraction objective in free problem-posing activity. This answer of the student was scored as Level 2 according to the criteria of “suitability to acquisitions”. It was seen that these posed problems were not fully compatible with the acquisitions.

The frequencies and percentages of scores of the problems posed in terms of “quantity and quality of data” criterion are shown in Table 4.

Table 4

Level of Quality and Quantity of Data in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	48.2	14.5	18.9	18.4
Semi-structured	49.1	6.6	19.9	24.4
Structured	42.2	18.9	18.1	20.8
Total	46.5	13.3	19.0	21.2

It is seen that nearly half (46.5%) of the posed problems appear to be at Level 1 according to this evaluation criterion. This finding shows that majority of the participants were unable to respond to the activities or fail to meet the criteria for the posed problems. In structured problem-posing situations, the responses evaluated at Level 1 are less than other problem-posing situations (42.2%). Semi-structured problem-posing responses at 4th Level (24.4%) are more than other problem-posing situations. It was seen that the participants were scored

better in structured and semi-structured problem-posing situations in terms of “quantity and quality of data” criterion where they were more challenged in free problem-posing situations.

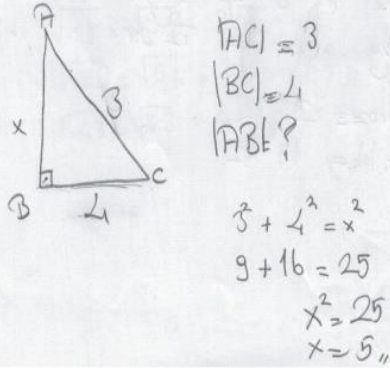
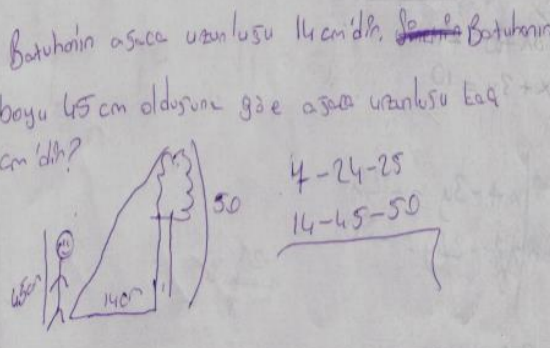
Sizde Pisagor bağıntısını içeren bir problem oluşturunuz ve çözünüz.  $AC = 3$ $BC = 4$ $AB = ?$ $3^2 + 4^2 = x^2$ $9 + 16 = 25$ $x^2 = 25$ $x = 5$	Sizde Pisagor bağıntısını içeren bir problem oluşturunuz ve çözünüz. Batuhan'ın ayağı uzunluğu 14 cm'dir. Batuhan Batuhan'ın boyu 45 cm olduğuna göre ayağı uzunluğu kaç cm'dir?  $4^2 + 24^2 = 25^2$ $16 + 576 = 600$ $x^2 = 24$ $x = 24$ The distance between Batuhan and tree is 14 cm. Since the height of Batuhan is 45 cm, what is the length to the tree?
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Figure 8. S129-1 Coded Activity

Figure 9. S92-1 Coded Activity

S129 did not write a problem sentence and gave incorrect information about a right triangle thus this response was coded as Level 2. S92 presented unreasonable data and some data were unnecessary. In the above examples, it was seen that the students do not fully acquire the mentioned objectives and as a result of this they created unsolvable problems. Students ignored that the hypotenuse should be the longest side in a right triangle. Moreover, it was seen that they were used wrong concepts and units. In addition to the wrong expressions in the problems, the abundance of missing data was obvious.

The frequencies and percentages of scores of the problems posed in terms of “solvability” criterion are shown in Table 5.

Table 5

Level of Solvability in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	54.2	10.8	17.5	17.5
Semi-structured	49.4	8.7	22.9	19.0
Structured	47.0	16.3	22.6	14.1
Total	50.2	11.9	21.0	16.9

According to this criterion, nearly half (50.2%) of the problems are evaluated as Level 1. It can be understood from this that one out of every two problems posed by the participants is a problem that does not have any possible solution. In terms of solvability of the problem, it was seen that the best results were obtained in semi-structured problem-posing situations, and the worst results were obtained in free problem-posing situations. It was seen that only 16.9% of the problems posed by the participants in problem-posing activities are fully solvable problems.

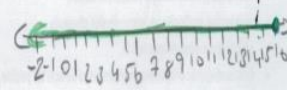
3) "Ahmet 3 kg kiraz ile 4 kg çileğe 12 TL, 1 kg kiraz ile 2 kg çileğe 5TL ödediğine göre 1 kg kiraz kaç liradır?" Yukarıda verilen probleme <u>benzer</u> bir problem oluşturunuz ve çözünüz Merve bir markete gidip 2 kg muz ve 2 kg'da armut 10 TL verdi 1 kg muz ile 1 kg armut ise 6 TL ödediğine göre 1 kg muz kaç TL'dir 2 kg muz = 4 2 kg armut = 2 1 kg muz = 2 1 kg armut = 1	2) $3x - 5 \leq 43$ Yanda verilen eşitsizliği kullanarak bir problem kurunuz ve çözünüz. $3x - 5 \leq 43$ yanda verilen eşitsizliği sayı doğrusu üzerinde açık- tanımlayınız. $3x - 5 \leq 43$ $\frac{3x - 5}{3} \leq \frac{48}{3} = 16$ 
Merve went to a farmers market and paid 10 TL for 2 kg of bananas and 2 kg of pear. If she pays 6 TL for 1 kg of banana and 1 kg of pear, what is the price of 1 kg of banana?	$3x - 5 \leq 43$, show the inequality which has given on the side on the number line.

Figure 10. S96-3 Coded Activity

Figure 11. S122-2 Coded Activity

S96 coded student's response to the structured problem-posing activity was shown in figure 10. It was clear that the solution is not possible in the inequality system posed by this student. For this reason, the response of the student was evaluated as Level 1. It was also seen that the student tried to solve the problem and failed. The response given by the S122 coded student to the semi-structured problem-posing activity was evaluated as Level 4. In this quotation, the student written a solvable problem about showing the given inequality over the number line. It was seen that the student has posed a problem that can be solved easily.

The frequencies and percentages of scores for the problems posed by the participants in terms of "originality" criterion are shown in Table 6.

Table 6

Level of Originality in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	57.2	24.1	17.2	1.5
Semi-structured	50.3	32.5	16.6	0.6
Structured	50.6	33.7	14.2	1.5
Total	52.7	30.1	16.0	1.2

The obtained data show that only 1.2% of the posed problems are original problems. It is understood from the obtained percentages at Levels 1 and 2 that nearly 80% of the posed problems are far from being original. This shows that students are not able to respond to most activities or write responses in the form of exercises. Although there are no significant differences between different problem-posing situations in terms of originality criterion, it has seen that the participants had difficulty in free problem-posing activities as in other criteria.

3) "Ahmet 3 kg kiraz ile 4 kg çileğe 12 TL, 1 kg kiraz ile 2 kg çileğe 5TL ödediğine göre 1 kg kiraz kaç liradır?"

Yukarıda verilen probleme benzer bir problem oluşturunuz ve çözünüz

Müge Manavdan 3kg erik ile 2kg portakala 14tl
4 kg erik ile 2kg portakala 16 tl ödemiştir. Buna
göre 1kg erik ile 1kg portakal ne kadardır?

$$\begin{array}{r} -13x + 2y = 14 \\ 4x + 2y = 16 \\ \hline -3x + -2y = -14 \\ 4x + 2y = 16 \\ \hline x = 2 \\ y = 4 \\ 4 + 2 = 6 \text{ TL} \end{array}$$

$$\begin{array}{r} -13x + 2y = 14 \\ 4x + 2y = 16 \\ \hline -12x + -8y = -56 \\ -12x + 6y = 48 \\ \hline -2y = -104 \\ y = 52 \end{array}$$

Müge has paid 14 TL for 3 kg of plums and 2 kg of oranges, 16 TL for 4 kg of plums and 2 kg of oranges. So how much is 1 kg of oranges and 1 kg of plums?

Figure 12. S138-3 Coded Activity

It was found that the participant with the code S138 posed a clear and solvable problem. There was evidence that the student posed the problem by thinking or trying to solve it. The student also showed the solution to the posed problem. Although the posed problem was accepted as correct, it was very similar to the problem presented as an example in the activity, and therefore, it was far from originality. This answer of the participant was evaluated as Level 2.

The frequencies and percentages of scores of the problems posed in terms of "presenting a solution" criterion are shown in Table 7.

Table 7

Level of Presenting a Solution in Different Problem-Posing Situations

Problem-posing situation	Level (Percentage of responses)			
	1	2	3	4
Free	63.9	9.0	18.7	8.4
Semi-structured	54.2	9.9	21.1	14.8
Structured	61.4	12.1	16.9	9.6
Total	59.8	10.3	18.9	11.0

The students left the solution blank or made it completely wrong for about 60% of the posed problems. The participants could present correct solutions for only 11% of the problems they posed. It seems that students are more successful at solving their own problems in semi-structured problem-posing situations.

Sizde Pisagor bağıntısını içeren bir problem oluşturunuz ve çözünüz. Bir kare piramidin cisim yüksekliği 8 taban kenarı 6, olan piramidin yan yüz kaçtır?  $8^2 + 6^2 = 10^2$ $64 + 36 = 100$ $\sqrt{100} = 10$	2) Futbol maçına bilet almak için sırada bekliyorsunuz. Önünüzde 13 arkanızda 28 kişi var ve bir kişinin bilet alma işlemi 3 dakika sürüyor. Sıra birliğinde kim herkes kaç dk bilet alır? Bu durum ile ilgili bilgileri değiştirmeden bir problem oluşturunuz ve çözünüz. $\begin{array}{r} 28 \\ + 13 \\ \hline 41 \\ + 1 \\ \hline 42 \end{array}$ $\begin{array}{r} 42 \\ \times 3 \\ \hline 126 \end{array}$ toplam 126 dakika sürer cevap=126
The height of a square pyramid is 8 and its base edge is 6, what is the side face?	In how many minutes do everyone get tickets with you?

Figure 13. S117-1 Coded Activity

Figure 14. S151-2 Coded Activity

Findings About the Second Sub-Problem

The types of problems posed by the participants in different problem-posing situations are shown in Table 8.

Table 8

Types of Problems Posed in Different Problem-Posing situations

	Routine problem	Non-Routine problem
Problem-posing situation	%	%
Free	87.9	12.1
Semi-structured	95.2	4.8
Structured	91.5	8.5
Total	91.7	8.3

In terms of the data, most of the posed problems consisted of routine problems in all three problem-posing situations. Non-routine problems correspond to 8.3% of all problems. It has seen that the participants wrote more non-routine problems in free problem-posing activities.

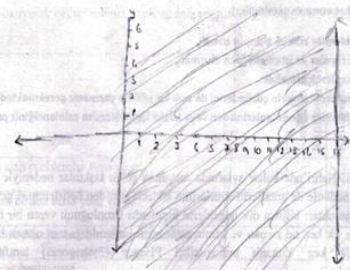
3) "Ahmet 3 kg kiraz ile 4 kg çileğe 12 TL, 1 kg kiraz ile 2 kg çileğe 5 TL ödediğine göre 1 kg kiraz kaç liradır?" Yukarıda verilen probleme <u>benzer</u> bir problem oluşturunuz ve çözünüz Bir çiftlikteki koyunlarla tavukların sayısı toplamı 40 ve bu hayvanların ayak sayısı toplamı 130 ise kaç koyun kaç tavuk vardır? Gösterim: $x + y = 40$ $4x + 2y = 130$ $-2x + 2y = -80$ $4x + 2y = 40$ $2x = 50$ $x = 25$	2) $3x - 5 \leq 43$ Yanda verilen eşitsizliği kullanarak bir problem kurunuz ve çözünüz. $3x - 5 \leq 43$ eşitsizliğini koordinat sistemi üzerinde çözümlü. $\frac{3x - 5}{3} \leq \frac{48}{3}$ $x \leq 16$ 
The total number of chickens and sheep in a farm is 40 and the total number of legs of these animals is 130, how many sheep and (how many) chickens are there?	Draw $3x - 5 \leq 43$ inequality on the coordinate system.

Figure 15. S114-3 Coded Activity

Figure 16. S106-2 Coded Activity

In the quotations given in the figures, it was seen that the problems posed by the students were far from original as they were similar to the problems in the textbooks. In Figure 15, the student was asked to pose a problem similar to the problem in example, it is seen that the student replaced the scenario of the problem in the response. In Figure 16, the response of the S106 coded student appears which can be considered as an exercise rather than a routine problem.

Sizde Pisagor bağıntısını içeren bir problem oluşturunuz ve çözünüz. Boyutları farklı köşeleri birbirine temas edip ortada bir dik üçgen oluşturarak şekilde üç tane kare şeklinde tarlanın en büyüğünün bir kenarı 6m, bir büyüğünün bir kenarı 8m ise en büyük tarlanın alanını bulunuz. $6^2 + 8^2 = x^2$ $36 + 64 = x^2$ $100 = x^2$ $10 = x$	Find the area of the largest field when the different angles contact each other and form a right triangle in the middle, one edge of the smallest of three squares is 6 m long and one edge of a large one is 8 m.
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Figure 17. S32-1 Coded Activity

In Figure 17, the problem created by the S32 coded student was considered as a non-routine problem since the student executed a different product from ordinary problems by taking the advantage of the guidelines in the problem-posing activities.

Findings About the Third Sub-Problem

The problem-solving strategies that the participants have used to solve their own problems in different problem-posing situations are shown in Table 9.

Table 9

Problem-Solving Strategies Used by Students to Solve their own Problems

Problem-solving strategies	Free	Semi-Structured	Structured	Total
	%	%	%	%
Prediction and control	0	0	9.1	2.6
Drawing a shape	37.5	6	18.2	18.3
Seeking a relation	3.1	0	0	0.9
Working backwards	6.3	2	0	2.6
Problem Simplification	3.1	4	0	2.6
Writing an equation	40.6	68	72.7	61.7
Logic Execution	9.4	20	0	11.3
Animation	0	0	0	0
Elimination	0	0	0	0
Making a table	0	0	0	0
Making systematic list	0	0	0	0

It has seen that students used strategies “writing an equation” and “drawing a shape” at most when solving their own problems. Students use more strategies when solving problems in free problem-posing situations while the least number of strategies used is related to structured problem-posing situations.

2) $3x - 5 \leq 43$ Yanda verilen eşitsizliği kullanarak bir problem kurunuz ve çözünüz. Alıcının parasının 3 katının 5 eksiği 43'e eşit veya 43'ten küçük olduğuna göre parası ne kadardır? $3x - 5 \leq 43 + 5$ $3x \leq 48$ $\frac{3x}{3} \leq \frac{48}{3}$ $x \leq 16$	3) "Ahmet 3 kg kiraz ile 4 kg çileğe 12 TL, 1 kg kiraz ile 2 kg çileğe 5 TL ödediğine göre 1 kg kiraz kaç liradır?" Yukarıda verilen probleme benzer bir problem oluşturunuz ve çözünüz Hakan 4 kalem ile 6 silgiye 7 TL, 2 kalem ile 4 silgiye 4 TL ödediğine göre 1 kalem ile 1 silginin toplam kaç TL'dir. $\begin{array}{r} 4 = 4 \\ 5 = 6 \end{array} \left. \begin{array}{l} 7 + 1 \\ 7 + 1 \end{array} \right\} 7 + 1$ $\begin{array}{r} 4 = 4 \\ 5 = 6 \end{array} \left. \begin{array}{l} 7 + 1 \\ 7 + 1 \end{array} \right\} 7 + 1$ olduğuna göre $\begin{array}{r} 1000r \\ 500r \\ + \\ 1500r \end{array}$
How much money does Murat have if its 3 times minus 5 is equal or less than 43?	Hakan has paid 7 tl for 4 pencils and 6 erasers, 4 tl for 2 pencils and 4 erasers so how much do 1 pencil and 1 eraser cost?

Figure 18. S136-2 Coded Activity

Figure 19. S125-3 Coded Activity

In Figure 18, it was seen that the participant S136 uses the strategy of writing an equation when solving the posed problem. In the quotation in Figure 19, it was thought that the strategy used by the student to solve the posed problem was not obvious but the student used the prediction and control strategy.

Discussion, Conclusion and Suggestions

In the present study, the problems posed by the eighth-grade students were investigated in terms of skills in different problem-posing situations, types of the problems posed, and the strategies used when solving posed problems. As a result of the analysis conducted, it has been determined that students generally have difficulty in posing problems. Likewise, Gökkurt, Örnek, Hayat, and Soylu (2015) stated that eighth grade students could not perform the expected performance in problem-solving and problem-posing stages. Korkmaz and Gür (2006) carried out research to determine the problem-posing skills of primary and mathematics teacher candidates. They reported that the vast majority of teacher candidates were unable to fulfil the expressions they thought should be in problem-posing in their own problem-solving attempts and in the implementation process. In contrast to these situations, some studies have also reached the conclusion that students are successful in problem-posing activities (Cai, 2003; Lin & Leng, 2008). It can be said that the problem-solving and posing experiences of the participants are effective in the formation of these differences. It is thought that the students who participated in the study had low success in problem posing activities since they have no previous experience in problem posing.

In the present research, the students were found to be more successful in semi-structured problem-posing situation than free and structured problem-posing situations when each problem-posing situation was examined separately. Although the results were close to each other, it can be said that students were more challenged in free problem-posing situations. A similar result was seen at the research performed by (Ngah et al., 2016) examining the problem-posing skills of middle school students according to different problem-posing situations. Kınap-Dönmez (2014) reached a similar result pointing out that the teacher candidates were very successful in the structured problem-posing situation, but the same proficiency was not seen in semi-structured and free problem-posing situations.

In the current study, when the written responses of the students to the problems were examined, it was seen that the skills of “using the mathematical language” that is the most basic items that they have to possess in order to pose their problems were low. The least achieving problem-posing situation for the students in terms of using mathematical language was semi-structured problem-posing activities. Free problem-posing situations were another situation of problem-posing that students have difficulty in terms of this criterion. Presentation of the ready-made problems in structured problem-posing activities and having the approaches of students to write problems seen in class or textbooks as free problem-posing activities made it easier for students to use mathematics language. Kar and Işık (2015) stated that teachers should pay greater attention to the use of mathematical terminology and relevant verbal expressions to improve mathematical communication skills of students. Students should be encouraged to use the language of mathematics. In order to overcome this problem, problem-posing activities should be carried out in different situations and students should be aware of their mistakes.

It was determined that nearly half of the students were at 1st Level in the criterion of “grammar and expression suitability”. Only about one out of every four problems that were posed according to different problem-posing situations received a full score from the evaluation. It was defined that participants were more successful in structured problem-posing activities for this criterion. The structured problem-posing activities used in the current study were the activities requiring posing problems similar to an existing problem. Therefore, being more successful in these activities was considered natural in terms of expression and statement. As a result, it seems that students have problems not only in using mathematical language but also in using Turkish language. Yıldız (2014) stated that teacher

candidates achieved a medium Level of achievement in problem-posing studies in terms of grammar. In another study, Arıkan and Ünal (2013) stated that second grade elementary school students did not pose appropriate problems and they had misconceptions and could not use Turkish well. Ekici (2016) stated that middle school students were not able to pose problems since they had difficulty to make sentences and question sentences. All these results are in line with the results of in the present study. The inadequacy of students in terms of expressing and writing down what they think is greatly influences the problem-posing successes. This situation can be explained by the achievements of the students in the language and expression courses as well as derived by the educational environment. It is thought that these problems will be reduced in constructive learning environments where learners are active, participatory, and responsible for their learning.

In the current study, when the scores of the problems posed by the students are examined in terms of “suitability to acquisitions” criterion, it can be said that approximately half of them are fully or partially appropriate to acquisitions. There are studies in which similar results have been obtained. Şengül-Akdemir and Türnüklü (2017) determined that 54.5% of the posed problems were curriculum dependent problems in their study performed with sixth graders. It can be said that students are more unsuccessful in free problem-posing situation than in other problem-posing situations in terms of suitability to acquisitions. The lack of information that can be clue to desired acquisitions in free problem-posing situation can lead to this result. It is also possible to make an interpretation that students are not aware of the acquisitions they have learned or that they cannot grasp the relations between the acquisitions.

In terms of “quantity and quality of data in the problem” criterion, nearly half of the posed problems were at Level 1 in the present study. The participants were more successful in semi-structured problem-posing situations in terms of this criterion. The problem-posing situation with the lowest evaluation percentage was the structured problem-posing situation. Kılıç (2013) examined the difficulties of the classroom teacher candidates experienced during problem-posing in different problem-posing situations. She defined difficulties such as lack of problem state for the posed problem, having difficulties about the structure of problem-posing case, using missing data during problem-posing, and incomplete consideration of the problem-posing case. Ekici (2016) stated that the middle school students misused the units in most (majorities) of their responses to problem-posing activities. The cases in this study also revealed that the students misused the units. Most of the students may not be able to write problems including adequate and qualified data in problem-posing activities because of being inadequate in terms of mathematical content and conceptual knowledge. It can also be said that being not familiar with problem-posing activities and having insufficient experience of the participants in this issue lead to this result.

In terms of the solvability criterion, nearly half of the problems posed in the current study by the students were at Level 1. Solvability of the posed problems and controlling whether the posed problem includes logical errors are the most important factors to be considered in the problem-posing process (Kırnap-Dönmez, 2014). In terms of the solvability of the problem, it is seen that the best results are obtained in semi-structured problem-posing situation, and the worst results in free problem-posing situation. Students desired to pose suitable problems to the given inequality in one of the two semi-structured problem-posing activity. So, the necessary data for the solution of the posed problem has already been presented. It is believed that this data presented in the activity contributed to the ability of the participants to pose solvable problems. In addition to studies that achieve similar results in the literature, there are also studies that contradict to the results of this study. Çelik and

Özdemir (2011) investigated the relationship between the proportional reasoning ability and the ability to pose proportion-ratio problem of the seventh and eighth graders in primary education. It has been seen that about one quarter of the posed problems by students were insolvable quality or did not include the type of proportion presented in problem instruction. Silber and Cai (2017) found that most of the participants posed problems were solvable problems in their study with elementary school teacher candidates. In this study, the failure of students to pose a solvable problem can be attributed to their inadequate problem-solving skills.

Silver and Cai (1996) examined the problems in terms of solvability, language, and mathematical complexity in their study in which they carried out the analysis of the posed problems by middle school students. They found that a large majority of the students posed solvable problems, certain part posed complex problems, and nearly half of the students posed related problems. There can be many reasons of students having difficulty in posing solvable problems. According to Yuan and Sriraman (2011), the content knowledge of students has a great influence on the problem-posing successes. Moreover, it can be said that the academic proficiencies of the students influence the success of posing problems (Ekici, 2016). From this point of view, the failure of students about posing solvable problems can be attributed to lack of mathematical content knowledge and academic proficiency levels in this study. Failure of students about grammar and suitability of expression can also obstruct posing solvable problems. Furthermore, it can also be interpreted that problem-solving skills of students are effective on the success of posing solvable problems.

The very few of the problems posed in this study were original problems when evaluated in terms of originality. Approximately 80% of the posed problems were at Levels 1 and 2 Tertemiz and Sulak (2013) examined problem-posing skills of fifth grade students according to the techniques they used. At the end of the study, they reached the result that most of the students changed the value of data given without changing the technical conditions and the subject used to pose problem. Korkmaz and Gür (2006) stated that classroom teacher candidates predominantly used verbal four arithmetical operations problems in their problem-posing and adhere to problems in textbooks. As can be seen in the performed studies, the students generally try to pose problems that they consider easy for them. This case leads them to grow out of creativity, and therefore, original problems do not arise since original problems come with creative thinking. In order to solve this problem, students should be faced with interesting or daily life related problems.

According to the criterion “solving the problem by the student” in the current study, the students left the solution blank or made it completely wrong in most of the posed problems. The solutions nearly for a tenth part of the posed problems were totally made true by the students. It was seen that students were more successful in solving the posed problems in the semi-structured problem situation. In other words, every student who can pose problems cannot show the same success to solve the problem. It has been reported in the literature that the problem-solving success will be increased through teaching with problem-posing approach (Akay, 2006; Dickerson, 1999; Lavy & Shriki, 2007; Turhan & Güven, 2014). Taking the studies done into consideration, it can be said that appropriate teaching according to problem-posing approach will increase the problem-solving performance of students.

Another research topic in the current study was the type of problems posed by students in different problem-posing situations. For this purpose, problems in each problem-posing situation were classified as routine and non-routine problems. When the obtained findings were taken into consideration, a great majority of the posed problems comprised routine problems in all three problem-posing situations. Non-routine problems formed only a small

proportion of all posed problems. Moreover, most of the posed problems were in the form of routine exercise problems without creativity. Akay et al. (2006) investigated the effect of using short open-ended questions and the problem-posing approach in mathematics teaching to understand and learn mathematical concepts. At the end of the research, it was identified that most of the students posed problems were in the form of routine exercise problems without creativity. The problems in the textbooks or problems without real life context solved by teachers during the class period might have caused this result. In the present research, the majority of the problems posed by the students are usually the problems that they have faced before. In this sense, it is clear that skills of the students to pose non-routine problems are quite low. The primary way to change this is to encounter students with non-routine problems. The participants were found to be relatively more successful in free problem-posing situation. It is thought that there is lack of limitations in free problem-posing situation so that the student can use the imagination and cognitive skills more effectively.

Finally, problem-solving strategies that the students used in solving their own posed problems were determined in the present research. It was found that the students mostly used equations and shape-drawing strategies during problem-solving. The most frequent different strategies used in problem-posing activity were free problem-posing situation while the least number of the strategies used in the activities were structured problem-posing situation. Some of the strategies were never used. Cai (2003) determined that fourth-, fifth-, and sixth-grade students at different achievement level schools in Singapore can choose appropriate solution strategies when solving problems and that many students can pose problems using shapes and patterns. Students who succeed in problem-posing and solving the problem can choose the appropriate ones from the strategies they have learned, but they often tend to specific strategies in problem-solving.

In the current study, it was seen that most of the students were not at the desired level of problem-posing skills. Moreover, it was determined that they also had difficulty in solving the posed problems. According to these results, it is suggested that teachers should have activities in the courses for the students to develop both problem-solving skills and problem-posing skills and to create discussion environments about the student made mistakes through defining these mistakes done during the problem-solving and problem building process. Furthermore, problem-posing activities in mathematics classes should be adapted to daily life situations in order to obtain original problem-posing products. Different solution strategies should be included in problem-solving activities. In the courses, different problem-posing situations should be used according to different purposes. In future research, the skills of students in different situations of problem-posing should be examined in other mathematical concepts and on the same concept. Moreover, the reasons for the difficulty of the students' skills in different situations of problem-posing skills examined in this study should be addressed through experimental and comprehensive research.

The current study examined the skills of eighth grade students in different problem-posing situations. Similar studies can be conducted with students from all grades of middle schools, as well as high school students and mathematics teachers. We also approached the issue in terms of the relationship between problem-posing and problem-solving. Future studies may focus on other attributes and relationships of problem-posing skills with other mathematical abilities or attitudes.

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Appendix-1. Rubric towards Evaluation of Problem-Posing Skills

	0 Point	1 Point	2 Points	3 Points
Ability to use mathematical language (symbol, notation, and so on) correctly	Null	There is an error in the use of the mathematical language (or concepts).	The mathematical language is/ (or concepts are) used correctly but incompletely.	The mathematical language is/ (or concepts are) used precisely and correctly.
Compliance of the text of the question with grammar rules, whether it contains an incoherency or spelling mistake	Empty, no text, or incoherency or misspelling.	There is no mistake in writing, but there is incoherency.	There is no incoherency, but the writing is wrong.	There are no incoherency and spelling mistakes.
The suitability of instructions used while referring to the operations to be done in problem or stating the problem to the acquisitions	Empty or unclear how the problem will be solved.	The operation to be done for the solution of the problem is suitable for the acquisitions but it is incomplete/wrong.	The operation to be done for the solution of the problem is not suitable for the acquisitions but it is complete/error free.	The operation to be done for the solution of the problem is suitable for the acquisitions and it is complete/error free.
In order for the problem to be solved, the amount of data and expressions contained in the problem, the logical/operational suitability, and the significance of the result	Empty, cannot be understood because it is not clear how to solve it, or there is no data available because there is no shape-text transfer.	There are both invalid and missing data or too much data-expression.	The data is incompatible or there is missing/more data-expression.	The data are adequate and appropriate.
Accessibility of the problem to the desired result (Solvability)	Empty or not be solved because data in the figure cannot be mathematically expressed in text form	Cannot be solved because it is not appropriate or sufficient data, or lack of expression	Although the data are appropriate and sufficient, they cannot be solved because of writing errors and incoherency.	Solvable.
The scenario of the problem text, the originality in terms of the operation steps in order to reach a solution	Empty or cannot be detected	The problem is pretty ordinary (Type of always been to).	The problem is partly original (so unique that it can be distinguished from the ordinary/classical question type).	The problem is largely original (a type of question whose originality is kept on the front line when it is produced, but not in textbooks or other sources).
Case of solving student posed problem	Empty	Could not apply the givens and desired to the solution	The problem is understood correctly and solved but there is an operation error.	The problem solved correctly.

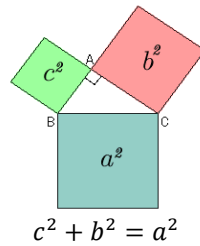
Appendix-2. Problem-Posing Test

Problem-Posing Test -1

Implement the below guidelines and write down a problem for each one through using the desired cases.

Things to take care while posing the problems!

- Problems you will write should be clear and understandable.
 - Problems should consist of given and desired statements.
 - They must contain an open question sentence.
 - Problems you will pose must also clearly describe solutions.
 - The written problems may be the problems inspired by your teachers or other sources.
- 1) “In Egypt, the land boundaries are constantly changing due to the floods in the Nile River during spring, so the boundaries of the land have to be re-determined frequently. For this purpose, a correlation has been used which gives the length of the hypotenuse of the right triangles whose vertical edge lengths are known. Although it is not known exactly when and how this connection was first used, it is thought that this correlation was first proved by the Greek mathematician Pythagoras (Pythagoras).”



- 2) Pose and solve a problem involving the Pythagorean correlation.
(Free Problem-posing Activity)
- 3) Pose and solve a problem through using the given inequality $3x - 5 \leq 43$.
(Semi-structured Problem-posing Activity)
- 4) “Ahmet has paid 12 TL for 3 kg cherry and 4 kg strawberry, 5 TL for 1 kg cherry and 2 kg strawberry, what is the price for 1 kg cherry?”

Pose and solve a problem similar to above given problem.
(Structured Problem-posing Activity)

Problem-Posing Test - 2

Implement the below guidelines and write down a problem for each one through using the desired cases.

Things to take care while posing problems!

- Problems you will write should be clear and understandable.
 - Problems should consist of given and desired statements.
 - They must contain an open question sentence.
 - Problems you will pose must also clearly describe solutions.
 - The written problems may be the problems inspired by your teachers or other sources.
1. “The operation of square root extraction is used to find the side length of square if its area is known.”
If the area of a square is a^2 unit-square, the side lengths of a square are found to be as $\sqrt{a^2} = a$ unit.
Pose and solve a problem including extraction of a square root through the above information.
(Free Problem-posing Activity)
 2. You are waiting in order to get tickets for a football match. You have 13 people in front of you and 28 people behind, and it takes 3 minutes for one person to get a ticket.
Pose and solve a problem without changing the information about this case. (Semi-structured Problem-posing Activity)
 3. Ahmet wants to make triangular frames with wooden bars. If one bar is 8 cm long and the other is 12 cm long to make a triangle, what is the length of the third bar? (The lengths of the bars are integers.)
Pose and solve a problem similar to the above given problem.
(Structured Problem-posing Activity)