Pre-Service Mathematics Teachers' Pattern Conversion Ability: Generating Figural Patterns Based on Number Patterns

Çiğdem Kılıç
İstanbul Medeniyet Üniversitesi
e-mail:cigdem.kilic@medeniyet.edu.tr

In that current study, pattern conversion ability of 25 pre-service mathematics teachers (producing figural patterns following number patterns) was investigated. During the study participants were asked to generate figural patterns based on those number patterns. The results of the study indicate that many participants could generate different figural patterns effectively, mostly by using geometric shapes. Moreover, most of the participants could generate linear figural patterns successfully compared with non-linear patterns based on number patterns and used different pattern generating strategies. In that study, some of the participants had issues while generating figural patterns.

Keywords
number pattern, figural pattern, linear pattern, quadratic pattern, pattern conversion ability

1.Introduction

Patterns are the heart and soul of mathematics (Zazkis & Liljedahl, 2002) and identifying and extending patterns is an important process in algebraic thinking (Van De Walle, 2004). Patterns can also contribute to the development of functional thinking (Souviney, 1994; Van De Walle, 2004; Warren & Cooper, 2006), in terms of seeing relationships and making connections (Cathcart et al., 2003), problem solving (Bassarear, 2008; Cathcart et al., 2003; Reys et al., 1998), counting (Bassarear, 2008; Frobisher & Threlfall, 1999), and using number systems (Frobisher & Threlfall, 1999) and arithmetical operations (Bassarear, 2008; Frobisher& Threlfall ,1999). Fox (2005) asserted that studying patterns is closely connected to mathematical content areas such as numbers, geometry, measurement, and data. In Turkey, mathematics curriculum, there are learning areas such as numbers and operations, algebra, geometry and measurement and probability. In these learning areas, students are expected to extend number and figural patterns and find algebraic rules of patterns (MONE, 2013).

1.1. Pattern types

There are many definitions of pattern in the literature. Souviney (1994), for example, defined a pattern as a systematic configuration of geometric figures, sounds, symbols, and actions. McGarvey (2012) defined a pattern as an act of perceiving or imposing structural regularities on physical, behavioural, visual, or symbolic phenomena. A mathematical *pattern* may be described as any predictable regularity, usually involving numerical, spatial or logical relationships (Mulligan & Mitchelmore, 2009). Patterns can also be seen in spoken and written words, musical forms and video images, ornamental designs, natural geometry, traffic, and the objects that we create (Reys et al., 1998).

Previous studies have shown many different kinds of pattern types refering to the type of representation system within which the terms in a sequence are expressed as numerical or figural/geometric forms. Stacey (1989) classified patterns as linear and quadratic patterns according to their *n*th terms expressed as $an+b$ and an^2+bn+c, respectively. Smith (1997) indicated that patterns can be numerical (involving numbers) or non-numerical (involving shapes, sounds, or other attributes such as colour and position).In some studies, patterns are classified as repeating or growing (Cathcart et al., 2003; Mulligan & Mitchelmore, 2009; Reys et al., 1998;Van De Walle, 2004; Warren & Cooper, 2006). For example, Zazkis and Liljedahl (2002) classified patterns into numerical patterns, pictorial/geometric patterns, patterns in computational procedures, linear and quadratic patterns, and repeating patterns. *Repeating patterns* have a recognisable repeating cycle of elements, referred to as the 'unit of repeat' (Zazkis & Liljedahl , 2002). This kind of pattern can have one attribute such as the colour, size, shape, or orientation of objects (Threlfall, 1999). The following are examples of repeating patterns: alphabetic letters such as A-B-A-B-A-B, geometric shapes such as ▼●▼●▼● and actions such as stand, sit, stand, sit, stand, sit, stand, sit (Warren & Cooper, 2006). Repeating patterns are particularly important, since they recur in measurement (which involves the iteration of identical spatial units) and multiplication (which involves the iteration of identical numerical units) (Mulligan & Mitchelmore, 2009). *Growing patterns* change over time (Cathcart et al., 2003) and may be linear such as Y B B Y B B B B Y B B B B B B (in this example, only B's are growing) (Reys et al., 1998) or quadratic such as n^2 squares (the third term is 3^2 and 100^{th} is 100^2). Using different representation forms such as shapes, numbers, size, and etc. patterns can be represented.

1.2.Representations in pattern activities

Representations are typically a sign or a configuration of signs, characters, or objects. They stand for (symbolise, depict, encode, or represent) something other than themselves (Goldin & Shteingold, 2001). A representation is something that stands for something else (Duval 2006). There are many different representation forms including diagrams, graphs, tables, sketches, equations, and words (Bassarear, 2008). In the well-known classification of Lesh, Post, and Behr (1987), representations are pictures, manipulative models, oral language, written symbols, and real-world situations.

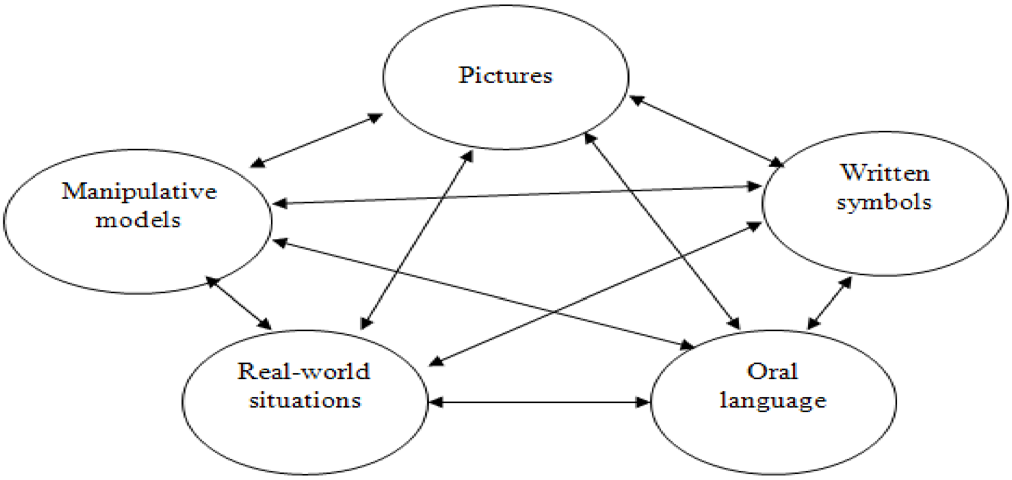


Figure 1. Representation types

Pictures, or diagrams-static figural models, can be internalised as images (Lesh et al., 1987), while manipulative models are objects that students can touch, move, and stack. For example, children might use unfix cubes, base ten blocks, or fraction circles to solve everyday problems (Clement, 2004; Lesh et al., 1987). Spoken language is used in two ways: (i) for students to report their answers and (ii) for students to express their reasoning (Clement, 2004). Finally, written symbols refer to both mathematical symbols and the written words associated with them (Clement, 2004). Furthermore, as shown in the classification in Figure 1, representations are interrelated. Translation refers to the psychological process of going from one form of representation to another (e.g. going from an equation to a graph or vice versa; (Janvier et al., 1987). Duval (2006) indicated that representation transformation is at the heart of mathematical activity and there are two types of transformation such as treatments and conversions. Although treatments happen within the same register like solving an equation, conversions consist of changing a register without changing the objects like passing from the algebraic notation for an equation to its graphic representation. Translations among representations assist mathematical learning and problem solving (Lesh et al., 1987).

Patterns can be represented in many ways such as *physical materials* (e.g. beads, coloured buttons, cubes), *oral* (e.g. the "do, mi, mi, do" pattern), *numbers or symbolic rules* (e.g. 2, 6, 12, 20 or $y=x^2+3x$), and *figures* (geometric or not) (Van De Walle, 2004). Steele (2008) indicated that "recognizing and understanding the relationship between different external representations of algebraic situations helps students identify and generalise patterns that lead toward understanding variables and functions" (p. 98). As mentioned earlier, representation types are very important in pattern activities. In that study representation types in pattern activity was considered such as *pictures (figures), manipulative models (physical materials), oral language, written symbols (numbers or symbolic rules) and real-world situations*.

In that study pattern conversion ability is described as creating a correct pattern (from one representation type to another different representation type such as from symbolic to figural) and using effective pattern generating strategies. As shown in Figure 2 pattern conversion ability including two interrelated layers refers to use effective generating pattern strategy including translation between representations (from symbolic to figural) and creating a correct pattern type based on the task demand.

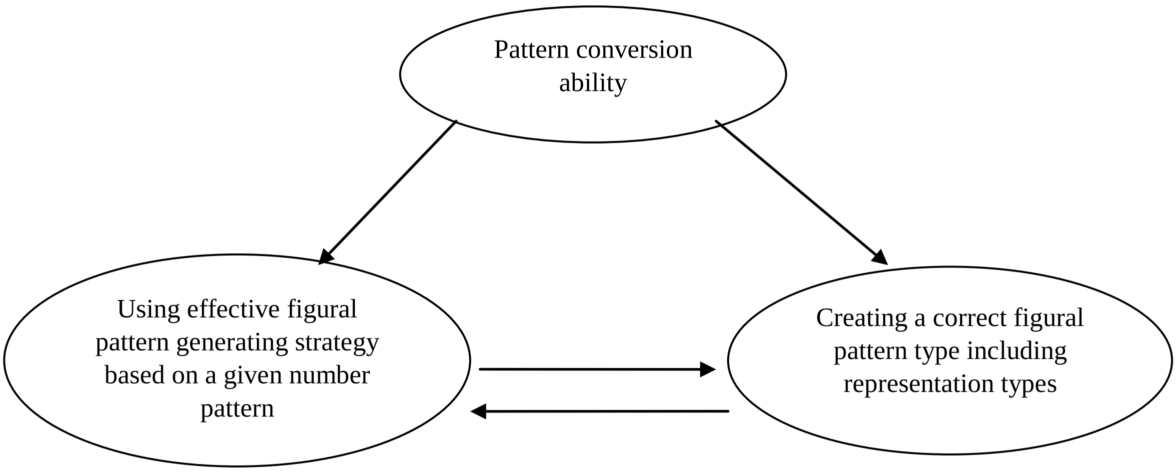


Figure 2. Pattern conversion process

In the literature, studies of patterns conducted with pre-service teachers focus on understanding their pattern generalisation. Zazkis and Liljedahl (2002), for example, focused on the capacity of pre-service elementary school teachers to generalise a repeating visual number pattern. Rivera and Becker (2003) found that prospective teachers analyse induction tasks that contain both figural and numerical cues. Hallagan et al. (2009) asserted that pre-service elementary school teachers encounter more difficulties determining the algebraic generalisation x^2+1 than 4x algebraic generalisations, while Chua and Hoyles (2010) asked pre-service teachers to generalise and represent figural patterns involving the quadratic rule symbolically. Yeşildere and Akkoç (2010, 2011) investigated the algebraic generalisation strategies used by pre-service elementary school mathematics teachers in order to find the general term of number patterns as well as their generalisation performance when forming linear and non-linear shape patterns. Tanışlı and Köse (2011) also examined pre-service teachers' strategies for generalising linear shape patterns.

In summary, while a number of studies have investigated pre-service teachers' generalising figural (i.e. from figural to symbolic representations) or numerical patterns (i.e. from symbolic to symbolic representations) generating figural patterns based on number patterns (i.e. from the symbolic representation to figural) requiring translation between representations is not studied at all. However, given that translation between representation types is important for mathematical learning, problem solving, and algebraic thinking that present study aimed to fill that gap in the literature by examining pattern conversion ability of pre-service mathematics teachers. In this study, the following research questions are addressed:

- What kinds of figural patterns were generated by participants based on linear and quadratic number patterns?

- What kinds of strategies were used by participants while generating figural patterns based on linear and quadratic number patterns?

Moreover, in that study it is also expected to provide insights into the similarities and differences of pre-service teachers' pattern conversion abilities.

2. Method

In this study, data triangulation was considered by supporting its quantitative findings by using qualitative research methods. In triangulation-based research design, "the researcher simultaneously collects both quantitative and qualitative data, compares the results, and then uses those findings to see whether they validate each other" (Fraenkel & Wallen, 2005). Data were collected by using a two-step process. In the first step, a number pattern task was carried out by all participants; in the second step, all volunteer participants participated in task-based interviews.

In that study phenomenography being one of the qualitative research methodology is considered. Phenomenography can provide to get information about peoples' conceptions of a given phenomenon through their speech and actions (Larsson& Holmström 2007). It is the approach for comparing how the participants think about the mathematics in the tasks (Walkowiak, 2014) and it is asserted that although participants are all undertaking the same task, there will be a number of qualitatively different ways of experiencing or understanding the question or problem which can be observed and identified (Fielding, 2014).In phenomenographic studies open-ended interviews are preferred (Larsson& Holmström, 2007; Walkowiak, 2014). In order to understand in greater depth how participants generated figural patterns related to both linear and non-linear number patterns task-based interview was used. Furthermore, the task-based interviews allowed the researcher to find out how figural patterns were generated or not and understand participants' pattern conversion ability by analysing their responses.

2.1.Participants

In phenomenography, the goal is to have variation in the sample rather than large numbers of people (Walkowiak, 2014) for that reason totally 25 participants attended study. Of these participants, 15 were girls and 10 were boys and their ages ranged between 21 and 23 years. That study took place in Turkey in a state university and also they were prepared to teach mathematics in accordance with mathematics curriculum of the state middle schools. All participants were middle socio-economic levels. Participants who took Special Mathematics Teaching Course I and attended Special Mathematics Teaching Course II participated in the study. In Special Mathematics Teaching Course I, participants are taught representation types, number and geometry. In Special Mathematics Teaching II Course, participants are taught basic algebra, patterns, pattern types, and generalising figural and number patterns. It was assumed that all participants already had basic pattern knowledge, geometry and algebra. Moreover participants in this study have not encountered pattern as a mathematical concept during their schooling very often because pattern is a new topic in mathematics curriculums in Turkey since 2005.For interview excerpts the real names of participants were kept confidential participants were thus coded for boys as PB_1, PB_2,...,PB_{10} and for girls as PG_1, PG_2,...,PG_{15}. The code "I" was used for the interviewer (the researcher).

2.2.Data collection

Participants were asked to generate figural patterns by considering a number pattern task consisting of one linear pattern and one non-linear (quadratic) number pattern in a written form. The task demand knowledge on pattern structures that grow according to the number of objects in each stage; thus, participants have the potential to generate several interesting figural patterns. The number pattern task is described as below:

> Task 1(Question 1): Could you generate a figural pattern of first five stages regarding the 3,5,7,9,11,.. number pattern?
> Task 2 (Question 2): Could you generate a figural pattern of first five stages regarding the 2,6,12,20,30,.. number pattern?

This task was chosen because it allowed us to assess how participants generated figural patterns based on number patterns. To confirm the suitability of this task, the opinions of a relevant mathematics educator were considered. The educator indicated that the task used in this study was suitable for pre-service teachers. In order to understand the conformity, validity, and reliability of the task-based interview questions, a pilot study was conducted with a pre-service mathematics teacher. As a result of the pilot study, the questions were revised in order to prevent mathematical misconceptions and uncertainties as well as unexpected situations.

In the interviews, participants were asked to read the task first and then to think aloud during the interview. The interview protocol was designed to help them explain how they were generating figural patterns based. The questions used in the task-based interviews were open-ended (Hunting, 1997) in order to assess participants' thinking processes and support certainty and extended of thinking participants. As Fielding (2014) emphasized each participant brings his or her prior experience and learning to the task and this affects the way in which it will be undertaken. The interview questions included the following: *Could you explain how you generated such a situation step by step?, Do you think that the answer you generated is a pattern? Why? Could you explain?, Do you think that the pattern you produced is in accordance with the number pattern? Why? Could you explain?, Could you generate a different pattern based on the same number pattern? How?, Which type of pattern did you find easier to produce? Why?.* The task-based interviews lasted for about 20–30 minutes. All interviews were recorded and fully transcribed and follow-up analyses.

2.3.Data analysis

The data obtained from the number pattern task were analysed at two levels: (i) semantic analysis and (ii) descriptive analysis. In the semantic analysis, first of all the figural patterns or situations generated by participants were analysed in accordance with the linear or non-linear number pattern and issues. The produced figural patterns or situations and issues were first listed and classified according to their semantic structures. Generated patterns or situations were then coded as linear or quadratic patterns and issues were noted and responses of participants related to the 3,5,7,9,11,.. number pattern and issues are presented in Table 3 and responses of participants related to the 2,6,12,20,30,… number pattern and issues are presented in Table 5. Secondly, in semantic analysis process the strategies while generating figural patterns for number patterns and their frequencies and issues are presented in Table 4 and Table 6. The strategies used by participants coded based on the open-ended interviews. During the interview participants explained step by step the process of generating figural patterns. After the semantic analysis of the generated patterns and the strategies, their frequencies and percentages were calculated. The descriptive analysis then provided descriptive information to offer an overall picture of the figural patterns generated by participants and the strategies they used.

After the main data collection process, the interview transcripts were transcribed verbatim by the researcher. The interview data were analysed by using Miles and Huberman's (1994) data analysis model, which consists of three phases: data reduction, data display, and conclusion drawing/verification. In the data reduction phase, the researcher coded the data that were considered to be important concepts and patterns for the study. The generated figural patterns were analysed by using the figural pattern diagram presented in Figure 3, including the categories and subcategories developed by the researcher based on previous studies of figural patterns related to the 3,5,7,9,11 and 2,6,12,20,30,… number patterns. Interviews excerpts are given after these categories.

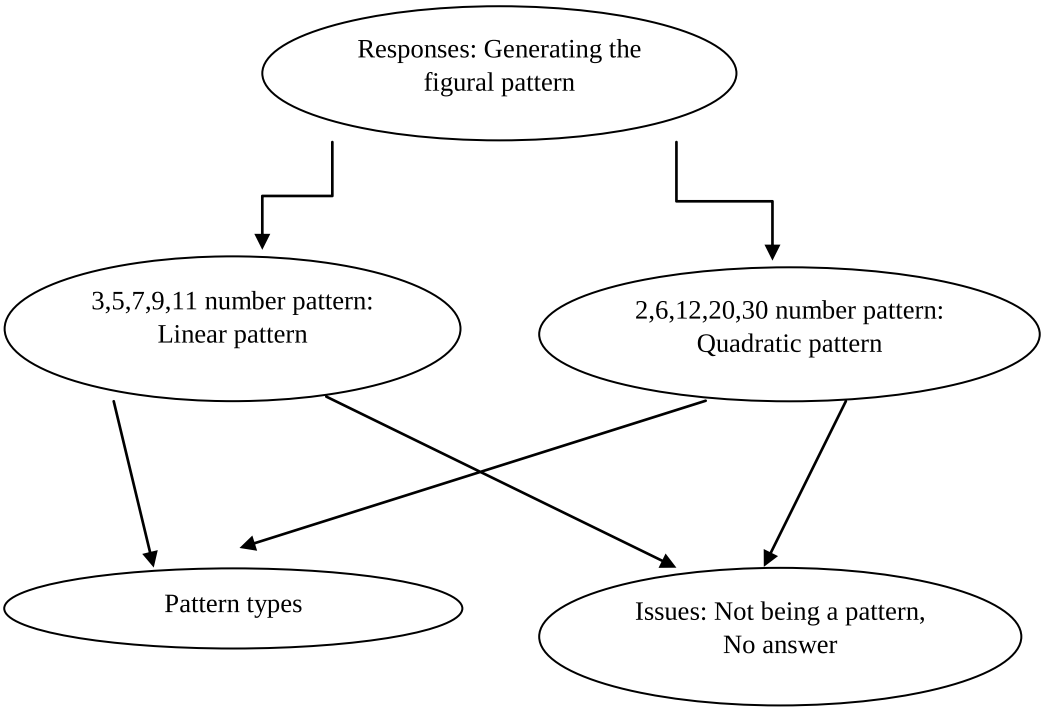
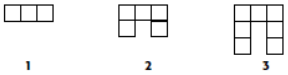
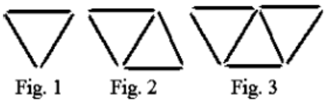
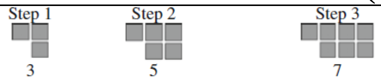
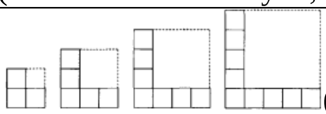
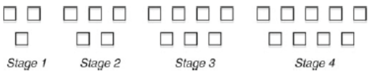
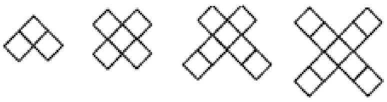
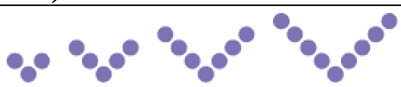
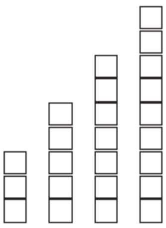


Figure 3. Framework for analysing the generated figural patterns

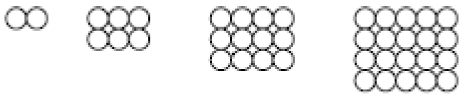
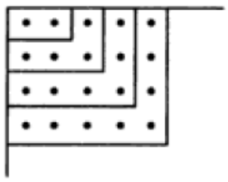
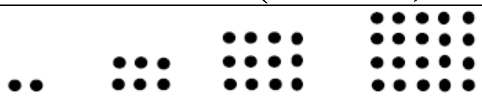
The figural representations of the 3,5,7,9,11 and 2,6,12,20,30 number patterns were coded as types (Type 1, Type 2, etc.) separately to analyse the data obtained from the study effectively. Table 1 and Table 2 provide the pattern types related to for both number patterns in the literature. In the results section, besides those pattern types from the literature, some of the new pattern types created by participants for the linear and quadratic pattern are presented.

Table 1. Examples of Figural Representations of The 3,5,7,9,11 (2n+1) Pattern in the Literature

Types	Structures	Characteristics
Type1	 (Wickett et al., 2002).	In Type 1, three squares ares used in first term of the pattern and pattern extended adding two squares into corners.
Type 2	 (Radford, 2008).	In Type 2, in first term of pattern is a triangle constructed using three toothpicks and next terms two toothpicks added.
Type 3	 (Jurdak & El Mouhayar ,2014).	In Type 3, pattern has been represented by squares.
Type 4	(Waring et al., 1999).	L- shapes type pattern has been represented by squares.
Type 5	 (Radford, 2008).	In Type 5, pattern has been represented by circles having two rows of circles.
Type 6	 (Radford, 2010).	In Type 6 each stage has two rows of squares, a top row and a bottom row.
Type 7	 (Warren, 2005).	In Type 7, pattern has been represented by squares.
Type 8	 (Gregg, 2002).	In that type, pattern constructed of circles like V model.
Type 9	 (Gregg, 2002).	In Type 9, pattern has been constructed by squares in vertical order.

As seen in Table 1, the figural pattern examples given for 3,5,7,9,11 number pattern has different arrangements and shapes (e.g. square, circle, line). However, while, for instance, Types 4,6,7 and 9 use the same geometric shapes, the construction of those patterns are different because of the arrangement.

Table 2. Examples of Figural Representations of 2,6,12,20,30 (n.(n+1)=n^2+n) Pattern in the Literature

Types	Structures	Characteristics
Type1	 (Van de Walle, 2004).	That pattern has been represented by circles representing a rectangular.
Type 2	 (Orton et al., 1999).	In overlapping rectangles pattern, the first stage contains 2, second 6, third 12 dots, etc.
Type 3	 (Vogel, 2005).	In Type 3 sequence of rectangular numbers is presented as an array of dot patterns

As seen in Table 2, the representation types of 2,6,12,20,30,.. seem to differ in terms of arrangement. Geometric figures such as lines and circles are used to represent those types of patterns. Although it has been uses the same geometric shapes for Types 2 and 3 (circles), the construction of those patterns differ because of the arrangement.

The figural pattern generating strategies that participants used were analysed based on categories was found in previous studies and categorized by Barbosa &Vale (2015). Although, those categories were for generalization strategies applied to visual patterns in this current study same categories were considered for generating figural patterns. Those strategies are explained as below;

- *Counting -Visual;* drawing a figure and counting its' elements.
- *Whole-object (no adjustment)-Non-visual*; considering a term of the sequence as unit and using multiples of that unit.
- *Whole-object w/visual adjustment-Visual;* considering a term of the sequence as unit and using multiples of that unit. A final adjustment is made based on the context of the problem.
- *Whole-object w/numeric adjustment-Non-visual;*considering a term of the sequence as unit and using multiples of that unit. A final adjustment is made based on numeric properties.
- *Recursive-Non-visual;*extending the sequence using the common difference, building on previous terms (numeric relations).

- *Recursive-Visual;*extending the sequence using the common difference, building on previous terms (features of the figures).
- *Difference rate (no adjustment)-Non-visual;* using the common difference as a multiplying factor without proceeding to a final adjustment.
- *Difference rate w/adjustment-Visual;*using the common difference as a multiplying factor and proceeding to an adjustment of the result.
- *Explicit-Non-visual;* discovering a numerical rule that allows the immediate calculation of any output value given the correspondent input value.
- *Explicit-Visual;* discovering a rule, based on the context of the problem, that allows the immediate calculation of any output value given the correspondent input value.
- *Guess and Check- Non-visual;*guessing a rule by trying multiple input values to check its' validity.

2.4.Validity and reliability

In order to increase the reliability and validity of the study, the member checks technique was used, as suggested by Lincoln and Guba (1985). Furthermore, the researcher asked for the opinion and assessment of one colleague who was blinded to the data and unbiased regarding the code list and research findings. In order to examine inter-rater reliability and increase the reliability of the qualitative results, another colleague who has a mathematics education background independently classified the generated figural patterns. The formula of Miles and Huberman (1994) was used to calculate inter-ratter reliability and this was determined to be 95% for Task 1 and 92% for Task 2. The pilot study also contributed to the validity and reliability of the pattern task and interview questions.

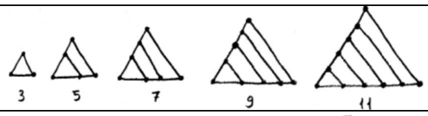
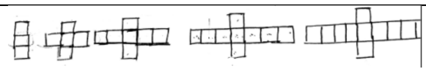
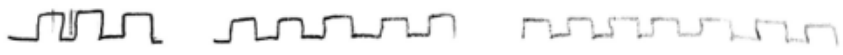
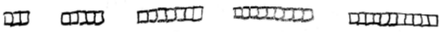
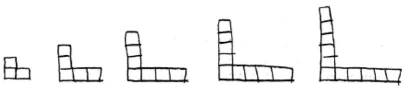
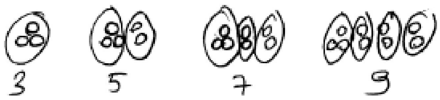
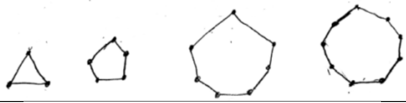
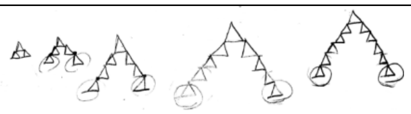
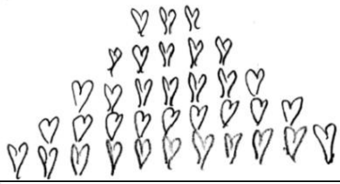
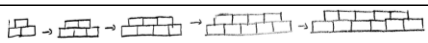
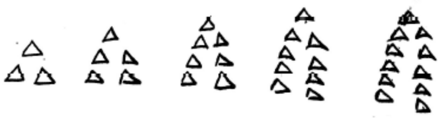
3.Results

The ability of participants related to generating figural patterns is now discussed. In particular, the first five stages regarding the 3,5,7,9,11 number pattern are presented in Table 3 and performance on the 2,6,12,20,30 number pattern is presented in Table 5. The strategies used by participants for linear number pattern is given in Table 4 and for non-linear pattern in Table 6. All responses are given in these tables in accordance with their frequency from high to low. For 3,5,7,9,11,.. number pattern Type 4 and Type 5 being in the literature emerged in the study so the rest of the pattern types created for 3,5,7,9,11,.. number pattern numbered from 10 to 21 type. For 2,6,12,20,30,.. number pattern Type 1 placing in the literature emerged in the study so the rest of the pattern types emerged in the study numbered from 4 to 12.

3.1.Pattern types created by participants related to 3,5,7,9,11,... number pattern

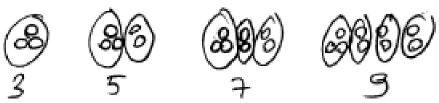
Table 3 presents the frequency of responses related to the linear number pattern as well as the types of patterns and issues encountered by participants. Altogether, 23 participants could produce figural patterns and 2 could not. As seen in Table 3, 14 different figural patterns were created by participants for the 3,5,7,9,11 number pattern during the study, with Type 1 created by 6 participants, Type 2, Type 3 and Type 4 by 2 participants. The rest of the 10 patterns created by 1 participant. The structures of the figural patterns created by participants consist of mostly geometric structures such as dot, line, square, triangle, rectangular and one non-geometric shape.

Table 3. Responses of participants related to the 3,5,7,9,11,.. number pattern and issues

Type		f	Type		f
10.		6	15.		1
11.		2	16.		1
12.		2	17.		1
4.		2	18.		1
5.		1	19.		1
13.		1	20.		1
14.		1	21.		1
			Total		23
			Issues		
				Not being a pattern	2
			Total		2

f= Frequency

Examples of the figural patterns created and interview excerpts are given as following. Participant PG_3 declared that she created a figural pattern for 3,5,7,9,11,… number pattern using circles. When I asked participant whether it is a pattern or not she declared that it is a pattern because there is an order and represent the number pattern. An interview excerpt and pattern example as below:



I: Do you think that the pattern you produced is in accordance with the number pattern? Why?

PG_3: Yes it is. Because the terms of the pattern are growing two in every stage in accordance with 3,5,7,9,11,… number pattern.

Participant PG_3 could convert that number pattern to a correct figural pattern using circles effectively.

3.2.The strategies that used by participants for 3,5,7,9,11,.. number pattern and issues

Six strategies while generating figural patterns for 3,5,7,9,11,… number pattern and their frequencies and issues are presented in Table 4.

Table 4. Strategies of participants related to the 3,5,7,9,11,…number pattern and issues

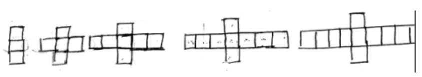
Strategies	Descriptions	f
Explicit and recursive	Finding a general rule+ focusing on the changes between consecutive numbers	10
Counting	Considering shapes in accordance with numbers	5
Explicit	Drawing shapes with finding a general rule	3
Recursive	Using the previous figure to build up to a new figure adding 2 each time	3
Determining the type of pattern+Explicit	Determining the type of pattern and then finding a general rule	1
Focusing on nature of numbers+ Recursive	Focusing on the nature of numbers and then considering the accrual between two terms	1
Total		23
Issues		
Not being a pattern		2

Frequency=f

The strategies that emerged in the study and interview excerpts as below;

Explicit and recursive strategy

10 participants used both explicit and recursive strategy while creating figural pattern. They found a general rule and then focused on the changes between consecutive numbers.



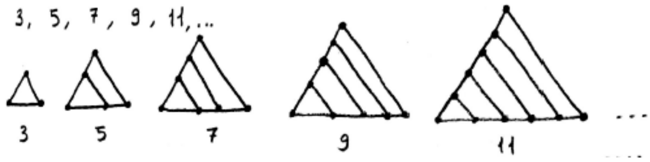
I: In the first task, I asked you to generate a figural pattern based on the 3,5,7,9,11 number pattern. You created that pattern. Could you explain how you generated step by step?

PG_6: First of all, I found the general rule of 3,5,7,9,11,… number pattern as 2n+1 then I focused on the changes between consecutive numbers. I focused on two because the pattern is increasing by two in every stage. I started with three squares for creating the first stage of the figural pattern and then used this to generate the remaining stages of the pattern, adding two in every stage.

Participant PG_6 firstly found a general rule and focused on the changes between consecutive numbers two strategies while creating figural pattern effectively.

Counting strategy

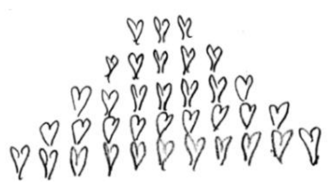
Five participants preferred counting strategy. They considered shapes in accordance with numbers. In that strategy participants wrote numbers firstly and created figures focusing on the numbers.



During the interview, participant PG_4 declared that she generated shapes in accordance with numbers in the pattern. Furthermore, she mentioned that she thought the numbers as shapes and tried to take into consideration the systematic structures. She said she used the corners of the triangle for number 3 and noticed an increase in the numbers in the corners.

Explicit strategy

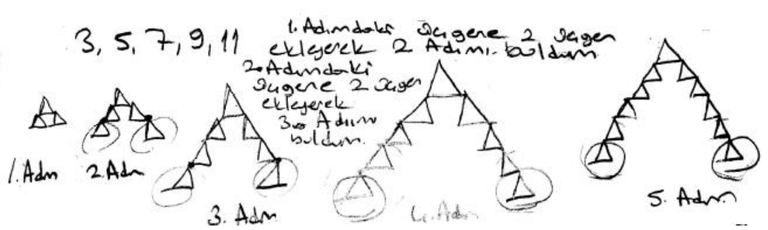
Three of the participants used explicit strategy. They created figural patterns with finding general rule for numbers.



Participant PB_{10} mentioned that he created the figural pattern with the finding general rule of number pattern and used non-geometric form such as heart.

Recursive strategy

Three of the participants considered that recursive strategy while creating a figural pattern. They used the previous figure to build up to a new figure adding 2 each time.



Participant PG_5 created figural pattern consisting of triangles. An interview excerpt as below:

I: In the first task, I asked you to generate a figural pattern based on the 3,5,7,9,11,.. number pattern. You created this. Could you explain how you generated this pattern step by step?

PG_5: First of all, I created the first stage of the figural pattern with triangles and then used this to generate the remaining stages of the pattern, adding two triangles in every stage. I focused on the accrual between two terms. I focused on two because the pattern is increasing by two in every stage. For that reason I used the previous figure to build up to a

new figure adding two each time. I then created the first, second, third, fourth, and fifth stages of the pattern.

I: Very good. You said you created that figural pattern using triangles? Could you explain why?

PG_5: When I saw the first stage of number pattern it reminded me triangle and I decided that I could add two triangles in every stage of figural pattern easily. First, I put one triangle at top and I added two triangles bottom corners of the the triangle.

Participant PG_5 used the previous figure to build up to a new figure adding 2 each time.

Determining the type of pattern+ explicit strategy

One participant preferred explicit strategy. He determined the type of pattern firstly and then found a general rule of pattern.


Participant PG_7 declared that first of all she determined the type of pattern. When I asked what is the type of pattern?. She said that type pattern is growing pattern because the terms of pattern are growing-increasing two in every step. Furthermore, she indicated that she secondly generalised the pattern rule as 2n+1 and then generated the remaining stages of the pattern based on the type of pattern and general rule.

Focusing on nature of numbers+ recursive strategy

One of the participant mentioned that he focused on the nature of numbers in pattern and then generated the pattern considering the accrual between two terms. Example of the figural patterns created by participant PB_5 and an interview excerpt is given below:


I: In the first task, I asked you to generate a figural pattern based on the 3,5,7,9,11,… number pattern. You created this. Could you explain how you generated this pattern step by step?

PB_5: First of all, I focused on the nature of numbers in 3,5,7,9,11,… number pattern. They all odd numbers and I focused on two because the pattern is increasing by two in every stage. I created the first stage of the figural pattern with triangles focusing on the nature of numbers and then used this to generate the remaining stages of the pattern, adding two in every stage. I focused on the accrual between two terms.

Participant PB_5 created a figural pattern using triangles correctly. He firstly focused on the nature of numbers and considered the accrual between two terms while creating figural pattern.

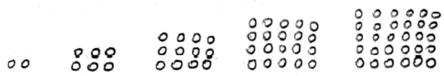
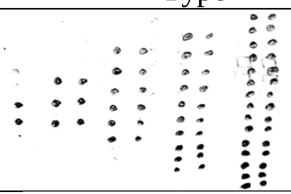
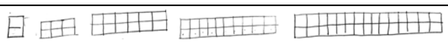
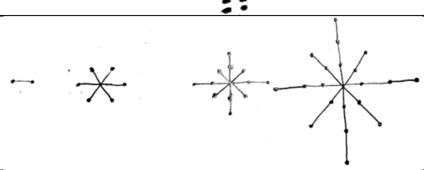
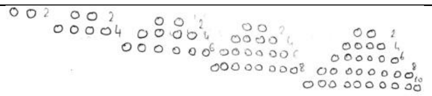
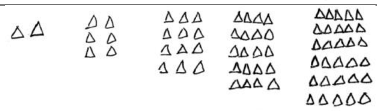
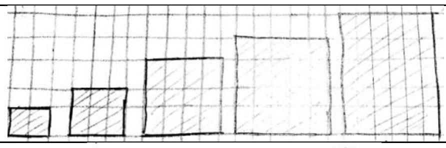
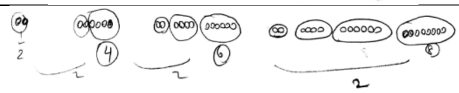
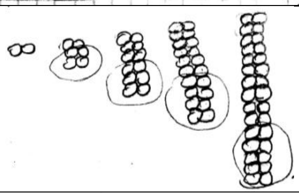
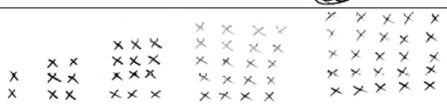
Issues

Two of the participants did not create any figural patterns related to 3,5,7,9,11,.. number pattern. They found the general rule but did not create any figural patterns. Participant PB_3 indicated that he could find general rule easily but could not create any figural patterns considering the 3,5,7,9,11,.. number pattern. Furthermore, he mentioned that he did not think any shapes for the pattern rule.

3.3.Pattern types created by participants related to 2,6,12,20,30,.. number pattern and issues

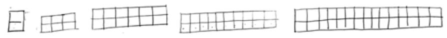
Table 5 presents the frequency of responses related to the quadratic number pattern as well as the types of patterns and issues encountered by participants. As shown in Table 5, 10 participants could produce figural patterns and 15 could not.

Table 5. Responses of participants related to the 2,6,12,20,30,… number pattern and issues

	Type	f		Type	f
1.		1	9.		1
4.		1	10.		1
5.		1	11.		1
6.		1	12.		1
7.		1		Total	10
8.		1		Issues	
				No answer	8
				Not being a pattern	7
				Total	15

f= Frequency

During this activity, 10 different figural patterns were generated by participants. They carried out transformation including conversions correctly. The figural pattern example created by one participant and an interview excerpt is shown below:



I: In the second task, I asked you to generate a figural pattern based on the 2,6,12,20,30,… number pattern. You created that kind of situation. Do you think that this is a pattern or not? Why? Could you explain?

PG_6: Yes, it is a pattern. There is a relation and those figures are made of squares set out in arrays that grow by a constant number. There is an order.

I: Do you think that the pattern you produced is in accordance with the number pattern? Why?

PG_6: Yes it is. Because the terms of the pattern are growing. I mean increasing not constantly but becoming greater.

3.4.The strategies that used by participants for 2,6,12,20,30,… number pattern and issues

Four strategies used by participants while generating figural patterns for 2,6,12,20,30,.. number pattern and their frequencies and issues are presented in Table 6.

Table 6. Strategies of participants related to the 2,6,12,20,30,…number pattern and issues

Strategies	Descriptions	f
Explicit	Finding a general rule+ constructing figural pattern based on the rule	5
Explicit and recursive	Finding a general rule+ focusing on the changes between consecutive numbers	2
Recursive	Using the previous figure to build up to a new figure adding accrual each time	2
Recursive and explicit	Considering the accrual between two terms + finding a general rule	1
Total		10
Issues		
Not being a pattern		11
No answer		4
Total		15

Frequency=f

The strategies used by participants and interview excerpts are given as below;

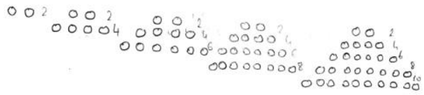
Explicit strategy

Five participants first of all found a general rule and then created a figural pattern based on that rule.

Participant PB_6 created a figural pattern regarding 2,6,12,20,30,… number pattern using finding a general rule+ constructing figural pattern based on the rule strategy. He mentioned that he found the general rule first as n.(n+1) and tried to think what kind of order should be based on that general rule. Furthermore, he mentioned that general rule represent a rectangular which has n and (n+1) sides for that reason he constructed such a pattern.

Explicit and recursive strategy

Among the participants two participants used both explicit and recursive strategy together. They created figural patterns using finding a general rule and focusing on the changes between consecutive numbers strategy. The example of figural pattern and an interview excerpt was given as below;



I: In the second task, I asked you to generate a figural pattern based on the 2,6,12,20,30,.. number pattern. Could you explain how did you generate such a pattern step by step?

PB_2:First of all, I tried to find the general rule of number pattern and then I focused on the accrual between two terms. I created a figural pattern considering both general rule and accrual.

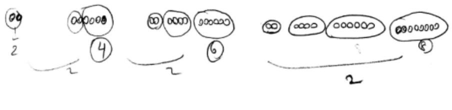
I: How did you decide the shapes that you used while creating that figural pattern. Moreover, you wrote number beside the figures. Could you explain the reasons?

PB_2: I realized the general rule n.(n+1) and focused on the accrual. The accrual between the two terms also increasing such as 4,6,8,10, and so on not constant. So I thought I should use figures in accordance with those increasing. For that reason I started with two circles and then generated the remaining stages of the pattern, adding those accruals in every stage.

Participant PB_2 first of all found a general rule and then focusing on the changes between consecutive numbers while creating figural pattern.

Recursive strategy

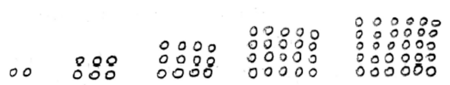
Two participants preferred recursive strategy. They used the previous figure to build up to a new figure adding accrual each time while creating figural pattern. Example is as below;



PG_2 mentioned that she generated a figural pattern considering the former shapes adding accrual each time. Moreover she indicated that she realized the accrual in that pattern like 4,6,8,10 and that systematic structure helped her to generate a figural pattern.

Recursive and explicit strategy

One participant used recursive and explicit strategy while producing figural pattern based on 2,6,12,20,30,…number pattern. Participant PG_{15} created a figural pattern considering the accrual between two terms and then finding a general rule. Interview and example was given as below;



I: I asked you to generate a figural pattern based on the 2,6,12,20,30,.. number pattern. Could you explain how did you generate such a pattern step by step?

PG_{15}: First of all, I tried to find accrual between two terms of the number pattern. It is a growing pattern in fact it is increasing like 4,6,8,10,… and then I found the general rule of the number pattern.

I: How did you decide the shapes that you used while creating a figural pattern and the order between shapes. Could you explain those?

PG_{15}: I could see the accrual between terms of pattern and realized the general rule n.(n+1). The rule reminded me like a rectangular which has sides like n and (n+1). So I thought that I could generate a rectangular having (n+1) and n dimension. While creating that rectangular I preferred circles being easy to organise on the paper.

Participant PG_{15} generated like a rectangular figural pattern correctly using circles.

Issues

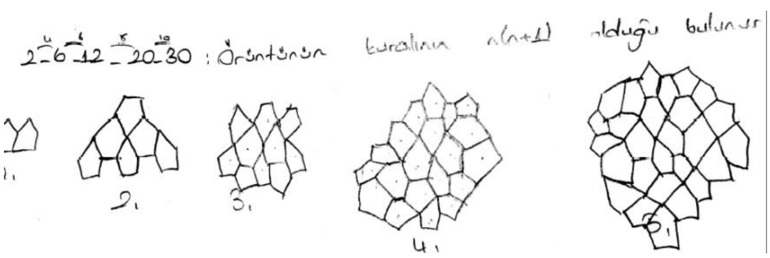
15 of the participants could not create any non-linear figural patterns correctly. The issues participants encountered such as finding a general rule+ not constructing the first stage of pattern, finding a general rule+ constructing irrelevant figures and constructing irrelevant situations. 11 of the participants created not being a pattern situation and 4 of the participants even did not do any think. The examples as below;

Finding a general rule+ not constructing the first stage of the pattern

Six participants found the general rule for 2,6,12,20,30,… number pattern but did not create any figural patterns. Participant PG_7 declared that she found the general rule and the type of pattern but could not create any figures related to number pattern. When I asked the reason she mentioned that because of the type of pattern. She said she could not imagine any figures considering 2,6,12,20,30,… numbers.

Finding a general rule+ constructing irrelevant figures

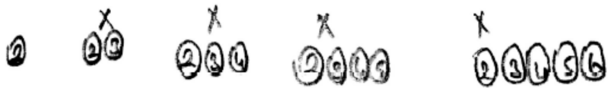
Three of the participants found a general rule and constructed irrelevant figures for 2,6,12,20,30,… number pattern.



Participant PG_7 found the general rule of the number pattern but constructed irrelevant figures. She mentioned that she firstly found general rule and created such situations not being aware of whether they represent figural patterns or not. Moreover she declared that generating a figural pattern for first task was easier than it was for second task because of the numbers in the second number pattern.

Constructing irrelevant situations

Two of the participants constructed irrelevant situations regarding the number pattern. Participant PG_{12} constructed irrelevant situations as below;



She mentioned that she tried to construct figural situations related to 2,6,12,20,30,… number pattern and did not think any figures.

No answer

Participant PB_5 did not give any answers related to 2,6,12,20,30,… number pattern. During the interview he declared that he had a difficulty during that activity because he did not consider any figures or configurations because of the increasing numbers. Moreover, he mentioned that it is a growing pattern not constantly but continuously increasing pattern and did not think any order.

4.Discussion and conclusion

Pattern activity is a very important mathematical activity in primary, middle, and high schools as well as in teacher education programmes. For that reason, patterns are included in the mathematics curriculums. In order to develop students' knowledge of patterns, it is important to educate pre-service teachers to the desired level. In the literature, as in the current research, many pattern studies have focused on how pre-service teachers generalize figural or number patterns (Chua & Hoyles, 2010; Hallagan et al., 2009; Rivera &Becker, 2007; Tanışlı & Köse, 2011;Yeşildere & Akkoç, 2010, 2011; Zazkis & Liljedahl, 2002), whereas studies of pattern conversion ability including using effective generating pattern strategy and conversions between representation types (i.e.from symbolic representation to figural representation) are rare. As Duval (2006) indicated representation transformation including conversions is very important for mathematical activity. Moreover, translations among representations assist mathematical learning and problem solving (Lesh et al., 1987) and recognising and understanding the relationship between different external representations of algebraic situations help identify and generalise patterns Steele (2008). Linking representations is thus very important for algebraic instruction (Brenner et al., 1997). Therefore, this study examined the ability of pre-service mathematics teachers to generate figural patterns based on number patterns (for both linear and non-linear situations) requiring conversion being one of the transformation types (i.e. from symbolic to figural representations).

The data obtained from the current study indicated that different types of figural patterns were generated by participants according to both growing number patterns and could see the growing structure in both types of patterns and could convert number patterns to figural patterns. Most examined pre-service mathematics teachers generated different types of figural patterns that followed the linear number pattern and non-linear number patterns using different figures being either geometric or non-geometric forms. Moreover, some new figural patterns representing the 3,5,7,9,11,… number pattern and 2,6,12,20,30,… number pattern emerged in that study. Some of the figural patterns created by participants using geometric forms such as rectangles, lines, dots, circles and squares predominantly during the generating activity for both the number patterns; thus, performance here can be related to their geometry ability and their pattern activities in class. Furthermore, some participants used not only geometric forms but also they used hearts and multiplication sign. In the future studies pre-service teachers should be motivated to produce different figural patterns based on number patterns and then different figural patterns regarding number patterns placed in the literature should be taught to extend their pattern conversion ability.

During the interviews, participants also asserted that creating a figural pattern and finding a general rule for 3,5,7,9,11,… number pattern was easier than for 2,6,12,20,30,… number pattern. Furthermore, the findings of this study indicated that pre-service mathematics teachers were more successful at creating figural patterns according to the linear number pattern compared with the non-linear (quadratic) number pattern. That situation can be explained patterns types being either linear or non-linear. In Hallagan et al.'s (2009) study of elementary school pre-service teachers, who encountered more difficulties determining the algebraic generalisation for quadratic patterns than they did for linear patterns. Similar results emerged in that current study in terms of pattern types.

In that study, participants used different generating figural pattern strategies. Six strategies for 3,5,7,9,11,.. such as explicit and recursive, counting, explicit, recursive , determining the type of pattern+ explicit and focusing on nature of numbers+ recursive and four strategies for 2,6,12,20,30,.. number pattern like explicit, explicit and recursive, recursive and recursive, and explicit. For 3,5,7,9,11,..number pattern most of the participants preferred explicit and recursive and for 2,6,12,20,30,.. number pattern explicit strategy was most preferred strategy. That situation can be explained participants' pattern activity experience. During teacher education pre-service teachers should encourage to use different pattern conversion strategies.

In that study, more than half of the participants were able to make conversions consisting of changing a register without changing the objects like passing from the symbolic representations (number pattern) to its figural representation (figural pattern). During generating figural patterns participants used more strategies for creating linear patterns than non-linear patterns. That situation can be explained in accordance with the results of the first research question of the study. –It can be concluded that nearly half of the participants during generating figural pattern for non-linear number pattern were not able to make translations from number patterns to figural patterns (i.e. they did not link symbolic representations to figural ones) and moreover some participants could not create any figural patterns based on the non-linear number pattern. Because considering pattern is a new topic in the mathematics curriculum in Turkey, this lack of familiarity with the topic affected their pattern-generating performance. Further, Fox (2005) found that patterns are closely connected to mathematical content areas such as geometry, which can be affect participant performance. It can be concluded that less than half of the participants struggled to select and construct an appropriate representation (could not translate from symbolic representations to figural representations) when generating figural patterns. For that reason, teacher education programmes should include courses including generating figural patterns following number pattern in order to develop pre-service teachers' algebraic thinking. As emerged in the study by Houssart (2000), some teachers had a wider and more sophisticated view of patterns. For that reason, pattern activities that can develop the perceptions of pre-service teachers should be designed for teacher training courses. In addition, generating figural pattern is related to participants' ability to visualise pictures, shapes or forms and thus this may have affected their performance. Moreover, other variables related to learners' performance such as learning style, spatial reasoning and mathematical thinking style might have influenced their figural pattern conversion ability. For that reason, in order to understand pre-service mathematics teachers' pattern conversion ability more in depth, correlation studies and mixed method researches may be conducted.

Considering generating patterns based on representations has the potential to improve problem solving and algebraic thinking of participants in the future, as indicated by Van De Walle (2004), different types of patterns such as physical materials, oral, numbers, or symbolic rules could be given participants in order to reveal their pattern conversion ability. That study conducted with a small sample and in the future the same study can be conducted with a large sample to see the big picture.

References

Bassarear, T. (2008). *Mathematics for elementary school teachers.* CA: Brooks/Cole.

Brenner, M. E., Mayer,R. E., Moseley, B. , Brar, T., Durán, R. , S., B. Reed &Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal, 34*(4), 663-689.

Cathcart, W. G., Pothier, Y. M., Vance, J. H. & Bezuk, N. S. (2003). *Learning mathematics in elementary and middle schools.* Englewood Cliffs, N.J.: Merrill/Prentice Hall.

Chua, L. B. & Hoyles,C. (2010). Generalisation and perceptual agility: how did teachers fare in a quadratic generalising problem?, *Research in Mathematics Education,12*(1), 71-72.

Clement, L. (2004). A model for understanding, using, and connecting representations. *Teaching Children Mathematics, 11*(2), 97-102.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61,103–131.

Fielding, H. (2014). An exploration of primary student teachers' understanding of fractions. Pope, S. (Ed.), *Proceedings of the 8th British Congress of Mathematics Education.* (pp.143-150). UK, University of Nottingham.

Fox, J. 2005. Child-initiated mathematical patterning in the pre-compulsory years. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the*

International Group for the Psychology of Mathematics Education. (Vol. 2 , pp. 313-320). Melbourne, Australia: University of Melbourne.

Fraenkel, J., R. & Wallen, N. E. (2005). *How to design and evaluate research in education*. New York, NY: Mc Graw Hill.

Frobisher, L & Threlfall, J. (1999). Teaching and assessing patterns in number in the primary years. In A. Orton (Ed.), *Pattern in the teaching and learning of mathematics* (pp.84-103). London and New York: Casse.

Goldin G. & Shteingold, N. (2001).Systems of representations and the development of mathematical concepts. In Cuoco, A.A & Curcio, F. R. (Eds.), *The Roles of Representation in School Mathematics.* (pp. 1-23). Va: Reston Virginia, NCTM.

Gregg, D. U. (2002).Building students' sense of linear relationships by stacking cubes. *Mathematics Teacher, 95*(5), 330–333.

Hallagan,J. E., Rule, A.C.& Carlson, L. F. (2009). Elementary school pre-service teachers' understandings of algebraic generalizations. *The Montana Mathematics Enthusiast, 6* (1&2), 201- 206.

Houssart, J. (2000). Perceptions of mathematical pattern amongst primary teachers. *Educational Studies, 26* (4), 489-502.

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *Journal of Mathematical Behavior, 16*(2), 145-165.

Janvier, B. D. & Bednarz, N.& Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp.109-122). Hillsdale, NJ: Lawrence Erlbaum.

Jurdak, M. E. & El Mouhayar, R. R. (2014). Trends in the development of student level of reasoning in pattern generalization tasks across grade level. *Educational Studies in Mathematics, 85*,75–92.

Larsson, J. & Holmström, I. (2007). Phenomenographic or phenomenological analysis: does it matter? Examples from a study on anaesthesiologists' work. *International Journal of Qualitative Studies on Health and Well-being, 2,* 55-64.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Lincoln, Y.S.& Guba, E. G. (1985). *Naturalistic inquiry.* Newbury Park, CA: Sage Publication.

McGarvey, L. M. (2012). What is a pattern? Criteria used by teachers and young children, *Mathematical Thinking and Learning, 14* (4), 310-337.

Miles, M.B. & Huberman, A.M. (1994). *An expanded sourcebook qualitative data analysis.* Thousand Oaks, CA: Sage.

MONE (2013). *Ortaokul matematik dersi (5,6,7,8. Sınıflar) öğretim programı*. [Middle School Mathematics Curriculum (5-8. grades)]. Ankara Devlet Kitapları Basımevi.

Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, 21(2), 33-49.

Orton,J. Orton, A. & Roper, T. (1999). Pictorial and practical contexts and the perception of pattern. In A. Orton (Ed.). *Pattern in the teaching and learning of mathematics* (pp.18-30). London and New York: Casse.

Radford, L. (2008). Iconicity and contraction: a semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM Mathematics Education.* DOI 10.1007/s11858-007-0061-0.

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics*, *30*(2), 2–7.

Reys, R. E., Suydam, M. N., Lindquist, M. M. &Smith, N. L. (1998). *Helping children learn mathematics.* Boston, MA: Allyn&Bacon.

Rivera, F. D. & Becker, J. R. (2003). The effects of numerical and figural cues on the induction processes of preservice elementary teachers. Editors: Pateman, N. A., Dougherty, B. J., Zilliox, J. T. *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th Conference of PME-NA.* 4,63-70.

Rivera, F. D. & Becker, J. R. (2007). Abduction–induction (generalization) processes of elementary majors on figural patterns in algebra. *Journal of Mathematical Behavior*, *26*, 140–155.

Smith, S. P. (1997). *Early Childhood Mathematics.* Needham Heights, MA: Allyn & Bacon.

Souviney, R. J. (1994). *Learning to teach mathematics*.New York, NY: Merrill.

Stacey, K. (1989). Finding and using patterns in linear generalizing problems. *Educational Studies in Mathematics, 20,* 147–164.

Steele, D. (2008). Seventh-grade students' representations for pictorial growth and change problems. *ZDM Mathematics Education*, *40,* 97–110.

Tanışlı, D. & Köse, N. (2011). Generalization strategies about linear figural patterns: Effect of figural and numerical clues. *Education and Science*, *36* (160), 184-198.

Threlfall, J. (1999). Repeating patterns in the early primary years. In A. Orton (Ed.). *Pattern in the teaching and learning of mathematics* (pp.18-30). London and New York: Casse.

Van de Walle J. A. (2004). *Elementary and Middle School Mathematics. Teaching Developmentally*. Boston, MA: Allyn &Bacon.

Vogel, R.(2005). Patterns – a fundamental idea of mathematical thinking and learning. *ZDM*, *37*(5), 445-449.

Walkowiak, T. A. (2014). Elementary and middle school students' analysis of pictorial growth. *Journal of Mathematical Behavior, 33*, 56– 71.

Waring, S., Orton, A. & Roper, T. (1999). Pattern and proof. In A. Orton (Ed.). *Pattern in the teaching and learning of mathematics* (pp.18-30). London and New York: Casse.

Warren, E. (2005). Young children's ability to generalise the pattern rule for growing patterns. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference* *of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 305-312. Melbourne: PME.

Wicket, M., Kharas, K.&Burns, M. (2002). *Grades 3-5 Lessons for Algebraic Thinking.* Sausalito, CA: Math Solution Publications.

Warren, E. & Cooper, T. (2006). Using repeating patterns to explore functional thinking. *APMC ,11* (1), 9-14.

Yeşildere, S. & Akkoç, H. (2010). Algebraic generalization strategies of number patterns used by pre-service elementary mathematics teachers. *Procedia Social and Behavioral Sciences, 2*, 1142–1147.

Yeşildere, S. &Akkoç, H. (2011). Pre-service mathematics teachers' generalization process of visual patterns. *Pamukkale University Education Faculty Journal, 30,*141-153.

Zazkis, R. & Liljedahl, P. (2002). Generalization of patterns: The tension between algebraic thinking and algebraic notation. *Educational Studies in Mathematics, 49*, 379–402.