Epistemic and Mathematical Beliefs of Exemplary Statistics Teachers

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Education research has established that teachers' beliefs matter because they can affect classroom practices and student outcomes, and beliefs are viewed as an important construct within statistics education. However, relatively little research about the beliefs of in-service statistics teachers has been conducted. Additionally, little research has been conducted with highly-experienced statistics teachers rather than pre-service teachers or typical teachers. Using two survey instruments, this study describes the beliefs about knowledge and beliefs about mathematical problem-solving for a sample of exemplary statistics teachers. The exemplary statistics teachers in this study responded to the Epistemic Beliefs Inventory and the Indiana Mathematics Belief Scales. Results show that the exemplary statistics teachers in this study had mature epistemic beliefs and strongly positive beliefs toward mathematical problem-solving, but notable patterns of similar responses that run counter to these trends were observed and are discussed.

Statistics has an increasingly prominent role in school mathematics globally (International Commission on Mathematical Instruction, 2011). This same trend has occurred in the United States (Scheaffer & Jacobbe, 2014) as evidenced by the statistics content in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers , 2010) and the growth of the Advanced Placement (AP) Statistics program, a program in which secondary school students take tertiary-level courses in their secondary schools with the opportunity to take a high-stakes exam for university credit. Today, many students are learning statistics: more than 40 states have adopted the CCSSM, and more than 200,000 students take the AP Statistics exam each year. Because it has been operating for more than 20 years – for longer than the CCSSM have been adopted – the AP Statistics program is an important context for providing insights about the teaching and learning of statistics in schools.

While statistics is mathematical science, there are differences between mathematics and statistics that have implications for instruction (Cobb & Moore, 1997; Franklin et al., 2007; Garfield & Ben-Zvi, 2008; Moore, 1988). However, the teachers tasked with teaching statistics are often mathematics teachers whose preparation does not emphasis statistics which may affect their content knowledge and confidence (Lovett & Lee, 2017). To ensure positive statistics outcomes for students, more preparation for and research about statistics teachers is needed.

Teachers' beliefs are important because they may affect their classroom practices, students' beliefs, and play a role in informational literacy (Hofer & Bendixen, 2012; Philipp, 2007). Despite a growing body of literature about statistics teachers, little research about their beliefs has occurred (Pierce & Chick, 2011). There are many different areas about which teachers hold beliefs. With mathematics teachers, beliefs about mathematics have long been of interest (Philipp, 2007). In the broader field of education, there has been increased research about the role of epistemic beliefs, that is beliefs about knowledge (Buehl & Fives, 2016). Because the preparation that many statistics teachers receive is focused on mathematics rather than statistics (Conference Board of the Mathematical Sciences, 2012;

Franklin et al., 2015), this study examines in-service statistics teachers' beliefs about mathematics. Additionally, because knowledge has a central role in both understanding and teaching statistics, this study examines in-service statistics teachers' beliefs about knowledge. A better understanding of the mathematical and epistemic beliefs of AP statistics teachers may eventually lead to a better understanding of the environments in which students are learning statistics.

Background and Literature Review

Philipp (2007), summarizing the extant literature, defines *beliefs* in the context of mathematics education as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 259) and distinguishes them from attitudes and emotions. Research on students' and teachers' beliefs is broad owing to the multiplicity of belief systems of interest to researchers. This study focuses on two types of beliefs: beliefs about knowledge and beliefs about mathematics.

Epistemic beliefs

The term *epistemic beliefs* refers to beliefs about knowledge (e.g., Wildenger, Hofer, & Burr, 2010), and research about epistemic beliefs spans decades and numerous distinct research programs (Hofer & Pintrich, 1997). Unlike other epistemic belief frameworks, Schommer's (1990) framework is multidimensional and has been characterized as more analytic in its treatment of beliefs (Hofer & Pintrich, 1997). In Schommer's epistemic beliefs framework, there are five independent dimensions related to "the structure, certainty, and source of knowledge, and the control and speed of knowledge acquisition" (p. 498). Each of the five constructs in Schommer's framework is conceptualized on a continuum of naïve beliefs to sophisticated beliefs and are characterized by statements from a naïve perspective:

- Simple Knowledge: "Knowledge is simple rather than complex" (Schommer, 1990, p. 499)
- Certain Knowledge: "Knowledge is certain rather than tentative" (Schommer, 1990, p. 499)
- Omniscient Authority: "Knowledge is handed down by authority rather than derived by reason" (Schommer, 1990, p. 499)
- Innate Ability: "The ability to learn is innate rather than acquired" (Schommer, 1990, p. 499)
- Quick Learning: "Learning is quick or not at all" (Schommer, 1990, p. 499)

Disagreeing with these characterizing statements – and items included from the naïve perspective on surveys – is associated with more sophisticated epistemic beliefs (Gatsos, 2015; Muis, 2004; Schommer, 1990; Schraw, Bendixen, & Dunkle, 2002). However, the terms naïve and sophisticated have negative and positive connotations which can complicate the discussion of epistemic beliefs (Muis, 2004). While Schommer-Aikins (2002) argues that there are real differences between sophisticated and unsophisticated learners, in this study, the labels *availing* for learning and *nonavailing* for learning are used instead of *sophisticated* and *naïve*, respectively, to avoid the normative connotations associated with the latter pair of terms (Muis, 2004), particularly in the context of interpreting scores for specific scales or responses to particular items. The substitution of terms should be viewed as synonyms and not as substituting a different continuum for the epistemic belief constructs so that interpretations from instruments aligned with the naïve/sophisticated terminology are still appropriate. Nonavailing epistemic beliefs are either not associated with or negatively associated with learning outcomes, and availing epistemic beliefs are those positively associated with learning outcomes (Muis, 2004). Constructivist beliefs are also associated with availing epistemic beliefs (Mason, Boscolo, Tornatora, & Ronconi, 2013).

Schommer's (1990) five-factor epistemological framework has been criticized for its inclusion of two constructs that may not be strictly epistemological in nature. Schommer's Certain Knowledge and Simple Knowledge constructs are consistent with what numerous distinct research traditions into epistemologies have investigated about the nature of knowledge, and Schommer's Omniscient Authority construct is consistent with prior work on the source of knowledge. However, Hofer and Pintrich (1997) note that Omniscient Authority may not sufficiently reflect the complexity and multidimensionality of the source of knowledge more broadly (cf. Belenky, Clinchy, Goldberger, & Tarukle, 1986/1997). Schommer's Fixed Ability and Quick Learning constructs have been critiqued as not being operationalizing aspects of the nature or source of knowledge (Hofer & Pintrich, 1997). Rather, Hofer and Pintrich note that Fixed Ability and Quick Learning may be more closely related to views of intelligence and task difficulty, respectively. However, these potentially non-epistemological constructs are of interest in an educational context, and so their inclusion in the framework is not problematic for this study.

Using several different instruments, researchers have applied Schommer's (1990) framework with university students with consistent, favorable results about the proposed factor structure (Hofer & Pintrich, 1997; Schommer-Aikins, Duell, & Hutter, 2005; Welch & Ray, 2012). Compared with university students, little empirical work has been conducted with non-student adults (Hofer & Pintrich, 1997). However, Schommer's framework has been applied with in studies of non-student adults (e.g., Schommer, 1992, as cited in 1994; Schraw & Olafson, 2002) and the factor structure identified has been consistent with what has been identified in student populations (Schommer, 1998). While beliefs are related to age, applying epistemological frameworks developed with adult university students to other adults later in life is less problematic than attempting to apply the models in populations at markedly different developmental levels (Hofer & Pintrich, 1997).

Teachers' epistemic beliefs are of interest because they have an indirect effect on student outcomes. Differing epistemic beliefs can manifest as different teaching styles (Johnston, Woodside-Jiron, & Day, 2001; Lidar, Lundqvist, & Östman, 2006), though some research has shown that beliefs alone may not be enough to affect classroom practices (e.g., Schraw & Olafson, 2002; Wilcox-Herzog, 2002). Students' epistemic beliefs can be affected by teachers' beliefs (e.g., Johnston et al., 2001; Lidar et al., 2006), and teachers' epistemic beliefs have also been shown to predict (e.g., Muis & Foy, 2010) student beliefs. Teachers' epistemic beliefs can also affect student learning through the ways in which teachers support students in reframing problems (Elby & Hammer, 2010). While the specific ways in and extent to which teachers' epistemic beliefs affect student learning outcomes are not yet known, studies in a variety of contexts have demonstrated effects for what is already a plausible hypothesis.

Recently, researchers examined the personal epistemologies of six academic statisticians in the United Kingdom (Diamond & Stylianides, 2017) using an epistemological framework consistent with Schommer's (1990; Hofer, 2000). Using data collected from a series of interviews, Diamond and Stylianides (2017) determined that the statisticians held beliefs that could be characterized as constructivist (or availing). The statisticians in Diamond and Stylianides's study held different epistemic beliefs for different aspects of statistics, for example their beliefs about the mathematical foundations of statistics were characterized by objectivism, their beliefs about statistical modelling were characterized by constructivism, and their beliefs about communicating statistical results were characterized by sociocultural factors (Diamond & Stylianides, 2017). Diamond and Stylianides's research suggests that beliefs about knowledge germane for statistics are not characterized by a single paradigm such as objectivism but instead may differ based on specific contexts.

Mathematical beliefs

Within the already broad research area of beliefs, mathematical beliefs – "those belief systems held by teachers on the teaching and learning of mathematics" (Handal, 2003, p. 47) – are a broad area for research. Much of the extant research has focused on beliefs, sometimes called *conceptions* (Philipp, 2007), on a continuum with the label *traditional* at one end and labels such as *progressive* or *reform* at the other (Handal, 2003; Philipp, 2007). Research on teachers' beliefs is important because multiple studies have shown that teachers' beliefs influence both students' beliefs (Givvin, Geller, & Stigler, 2019) and students' achievement (e.g., Fennema et al., 1996; Staub & Stern, 2002). Understanding what teachers believe is critical to understanding what students will achieve.

Studies with pre-service teachers have tended to show that they hold more traditional beliefs, while studies with in-service teachers have documented a wider variety of beliefs including more progressive beliefs (Handal, 2003; Philipp, 2007). In studies seeking to document teachers' mathematical beliefs, qualitative approaches such as interviews have been most common, though some rating scale instruments have been developed (Philipp, 2007). Much of the research on teachers' mathematical beliefs has been conducted with pre-service teachers or in-service primary school teachers rather than in-service teachers (Handal, 2003; Philipp, 2007).

There is considerable variation in mathematics teachers' beliefs (Handal, 2003; Philipp, 2007) across different backgrounds and preparation (e.g., Lester, McCormick, & Kapusuz, 2004; Martínez-Sierra, García-García, Valle-Zequeida, & Dolores-Flores, 2020; Smith, Swars, Smith, Hart, & Haardörfer, 2012), experience levels (e.g., Perry, Tracey, & Howard, 1999), grade levels (Nathan & Koedinger, 2000; Ren & Smith, 2018), and cultural contexts (e.g., Barkatsas & Malone, 2002; Ní Fhloinn, Nolan, Hoehne Candido, & Guerrero, 2018). Because of this considerable variation, documenting the similarities and differences among teachers in different educational contexts can provide critical information about what factors might affect teachers' beliefs and how teachers' beliefs might affect educational outcomes.

Beyond documenting what teachers' mathematical beliefs are, researchers have investigated the connection between teachers' beliefs and their classroom practices and ways in which teachers' beliefs develop and change. That teachers' beliefs affect their classroom practices is certainly plausible and evidence suggests that such a link does exist (Philipp, 2007). Case studies with teachers who were adopting a reform-oriented curriculum have shown that, when teachers' mathematical beliefs conflicted with the beliefs to which the curriculum was aligned, they learned less from the new materials, relied more on their prior teaching materials, and implemented the curriculum less thoroughly than teachers who beliefs were aligned (Collopy, 2003; Remillard & Bryans, 2004). Because studies have shown that teachers' classroom practices may change before their beliefs and that there is a complicated, iterative relationship between beliefs and practices (Knapp & Peterson, 1995; Peterson, Fennema, Carpenter, & Loef, 1989), teachers whose beliefs hinder their implementation of new pedagogical resources and methods may benefit from iterative changes in their practices and beliefs. However, as with epistemic beliefs, there are numerous examples of inconsistency between teachers' beliefs and their classroom practices (Philipp, 2007), for example a teacher with progressive, student-centered beliefs who teaches in a teacher-centered manner. To resolve these inconsistencies, researchers have turned to viewing beliefs as being situated within particular contexts (Philipp, 2007).

Statistics teachers' beliefs

Beliefs about mathematics have been named as an important consideration in statistics education research (Gal, Ginsburg, & Schau, 1997), though other affective constructs (such as attitudes) are more commonly studied (Pierce & Chick, 2011). In their review of the literature on statistics teachers' beliefs, Pierce and Chick (2011) note that studies have generally found a mixture of favorable and unfavorable views of statistics (e.g., Begg & Edwards, 1999; Chick & Pierce, 2008). Due to the limited research about teachers' statistical beliefs, Pierce and Chick suggest that they may be similar to the beliefs of undergraduate students which tend to be more favorable as with the completion of more tertiary-level statistics courses (Reid & Petocz, 2002), though using undergraduate students' beliefs as a proxy for teachers' beliefs is less defensible for later career teachers than pre-service or early-career teachers.

Using a mixed-methods approach, Duffy, Muis, Foy, Trevors, and Ranellucci (2016) studied the epistemic climate – "facets of knowledge and knowing that are salient in a learning environment" (p. 38) – of statistics classes. Using a survey instrument, interviews, and classroom observations to investigate two collegiate statistics instructors' beliefs about learning and teaching mathematics and statistics, Duffy et al. (2016) found that one instructor, Gilbert, held a mixture of both constructivist and traditional views while the other, Sophia, largely held constructivist views. However, based on the classroom observations, Gilbert's approach was characterized as being more traditional than Sophia's approach, and Sophia's approach was itself characterized as a mixture of teacher-centered and student-centered (Duffy et al., 2016), providing evidence of inconsistent teaching beliefs and practices noted in mathematics teachers (Philipp, 2007). Based on a pre/post beliefs survey administered to students, Duffy et al. found that the students in Gilbert's class had a decline in their beliefs about statistics while the students in Sophia's class held beliefs that were more stable over time.

Beliefs about self-efficacy – "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (Bandura, 1986, p. 391) – have been studied more in the statistics education literature. Numerous instruments have been developed to measure self-efficacy beliefs with undergraduate students (e.g., Finney & Schraw, 2003) and teachers (e.g., Harrell-Williams, Sorto, Pierce, Lesser, & Murphy, 2014, 2015). However, self-efficacy beliefs are grounded in a highly specific context: self-efficacy for learning statistics (e.g., Finney & Schraw, 2003) or self-efficacy for teaching statistics (e.g., Harrell-Williams et al., 2014, 2015). While an important construct within beliefs, self-efficacy is distinct from other types of beliefs such as epistemic or mathematical. Other instruments that have been used to study beliefs in statistics education have done so with a focus on curricular and assessment beliefs (e.g., Zieffler, Park, Garfield, delMas, & Bjornsdottir, 2012). This study contributes to the growing body of statistics education belief literature by examining the epistemic beliefs and mathematical beliefs of in-service statistics teachers.

Methods

Research Questions

This study focuses on in-service statistics teachers who were identified as exemplary by experts in the community. By focusing on a high-achieving subset of teachers, common features shared by that group can be identified (e.g., Philipp, Flores, Sowder, & Schappelle, 1994). Additionally, Peters's (2009, 2011, 2014) work with Advanced Placement (AP) Statistics teachers' understanding of variability motivated this study. In Peters' study, five teachers classified as having a robust understanding of variability could be viewed as a homogenous group based on their personal characteristics, but the study focused on beliefs about variability rather than more general beliefs.

The research questions studied in this study are:

1. What are exemplary statistics teachers' epistemic beliefs?
2. What are exemplary statistics teachers' mathematical beliefs?
3. If possible, what groupings of teachers can be identified based on their beliefs?
4. What are the relationships between exemplary statistics teachers' mathematical and epistemic beliefs?

Participants

The participants in this study were 12 exemplary statistics teachers; the same exemplary statistics teachers were participants in another study that used semi-structured interviews to investigate their development (Whitaker, 2016a, 2016b). The study received ethics clearance from the relevant university institutional review board. To identify exemplary statistics teachers, critical case sampling (Patton, 2002) was conducted by asking experts in the statistics education community (an Assessment Specialist at the Educational Testing Service who was involved with the creation of the AP Statistics program and the American Statistical Association and National Council of Teachers of Mathematics Joint Committee on Curriculum in Statistics and Probability) to nominate teachers who they considered exemplary for their teaching of statistics at the middle or secondary level. These experts were asked to identify "statistics teachers that you consider to be exemplary … primarily at the middle and high school levels" in a broad sense: no restrictions were placed on experience, gender, courses taught, or similar. A list of 24 such teachers was compiled and contacted via email, and of these, 12 agreed to participate in the study. Each of the 12 participants had experience teaching AP Statistics, though some had taught other statistics courses and some no longer taught at the secondary level at the time of the study.

Among the 12 participants, there were five women and seven men. The highest level of education was a bachelor's degree for two participants, a master's degree for seven participants, and a doctoral degree for three participants. While many of the participants earned degrees in mathematics or mathematics education, there were several notable exceptions. One participant's highest education was a master's degree in statistics, and another participant earned a doctorate in statistics after he began teaching. Additionally, one participant earned a doctorate in a science. The participants were primarily mid-career and late-career teachers, having an average of 30 years of teaching experience; the minimum years of teaching experience was 7 years, the first quartile was 19.5 years, the median was 32.5 years, the third quartile was 42 years, and the maximum was 50 years.

Data Collection

Data for this study were collected using existing surveys. The instruments that were chosen for use in this study are the Indiana Mathematics Belief Scales (IMBS) (Kloosterman & Stage, 1992) with the addition of the Fennema-Sherman Usefulness Scale (Fennema & Sherman, 1976) and the 28-item Epistemic Belief Inventory (EBI) (Schraw et al., 2002). Each of these instruments is described below. Because validity is not an intrinsic property of an instrument (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 2014), a justification for their use with exemplary statistics teachers – not population the instruments were originally developed for – is provided.

The IMBS is a 36-item instrument that assesses six mathematical beliefs:

- Difficult Problems: "I can solve time-consuming mathematics problems." (Kloosterman & Stage, 1992, p. 115)
- Steps: "There are word problems that cannot be solved with simple, step-by-step procedures." (Kloosterman & Stage, 1992, p. 115)
- Understanding: "Understanding concepts is important in mathematics." (Kloosterman & Stage, 1992, p. 115)
- Word Problems: "Word problems are important in mathematics." (Kloosterman & Stage, 1992, p. 115)
- Effort: "Effort can increase mathematical ability." (Kloosterman & Stage, 1992, p. 115)
- Usefulness: "Mathematics is useful in daily life." (Fennema & Sherman, 1976; Kloosterman & Stage, 1992, p. 115)

While the constructs measured by the IMBS were chosen because of their assumed relationship with student motivation and mathematical abilities (Kloosterman & Stage, 1992), the constructs themselves are not specific to students. Rather, the six beliefs assessed by the IMBS are general beliefs about mathematical problem-solving and are not operationalized in such a way that requires the respondent to be enrolled in a mathematics course (Kloosterman & Stage, 1992). A few items may be most appropriate for learners of mathematics (e.g., "I can get smarter in math by trying hard" and "I study mathematics because I know how useful it is" (Kloosterman & Stage, 1992, p. 115)) but are not so rigid in their phrasing as to make them inappropriate for teachers. Additionally, the IMBS was developed for use with undergraduate students – including pre-service mathematics teachers (Kloosterman & Stage, 1992). The IMBS instrument has been widely used, including with pre-service teachers (e.g., Benbow, 1995; Lester et al., 2004), in-service elementary teachers (e.g., Akinsola, 2004; Sepúlveda Pérez, 2006), and undergraduate engineering students (e.g., Berkaliev & Kloosterman, 2009), though repeated administrations within these groups are few. The instrument has also been used with secondary school students, including in a study where it was administered with an epistemic beliefs survey aligned with Schommer's (1990) framework (e.g., Schommer-Aikins et al., 2005). The general nature of the scale, its development for and subsequent use with pre-service teachers, and the wide body of literature suggesting an important relationship between teachers' beliefs and their students' beliefs together suggest that the use of the IMBS with in-service teachers is appropriate.

The Epistemic Beliefs Inventory (EBI) is a 28-item instrument aligned with Schommer's (1990) five-factor framework for epistemic beliefs (Schraw et al., 2002). The 28-item EBI consists of 5-point Likert type items from Strongly Disagree to Strongly Agree, and responses for each of the five scales on the EBI are summed to create a score for the associated factor (Schraw et al., 2002). While it does not fully assess Schommer's framework (DeBacker, Crowson, Beesley, Thoma, & Hestevold, 2008; Nussbaum & Bendixen, 2002, 2003), it is considered to be among the best instruments available for measuring epistemic beliefs (Wheeler, 2007).

Each of the five constructs in Schommer's (1990) framework and measured by the EBI has a continuum from naïve epistemic beliefs to sophisticated epistemic beliefs (e.g., Barnard-Brak & Lan, 2009; Paechter et al., 2013; Schommer, 1990; Walter, 2009). Each of the statements characterizing Schommer's constructs is written from the nonavailing perspective (e.g., "Knowledge is simple rather than complex", p. 499) so that *disagreeing* with them is associated with *more availing* epistemic beliefs. Similarly, all but five of the items on the EBI (Schraw et al., 2002) are written from the nonavailing perspective (e.g., "Most things worth knowing are easy to understand," p. 275) so that endorsing a negative option (i.e. Disagree or Strongly Disagree) is associated with more availing epistemic beliefs. For the other five items (Items 2, 6, 19, 22, and 28), endorsing a positive option (i.e. Agree or Strongly Agree) is associated with more availing epistemic beliefs, and so these items are reverse-coded (Gatsos, 2015; Schraw et al., 2002). Ultimately, lower scores for each of the five constructs measured by the EBI are associated with more availing epistemic beliefs; this is typical for instruments measuring epistemic beliefs (Ravindran, Greene, & Debacker, 2005).

The EBI was chosen for this study because it is aligned with an epistemic beliefs framework that has been studied with non-student adult populations (Schommer, 1998) and has itself been used with teachers (e.g., Schraw & Olafson, 2002), though without confirmatory analyses. Ravindran et al. (2005) have used the EBI with pre-service teachers and found a similar factor structure, and the intended population of undergraduate students are still adults who are at least somewhat developmentally similar to in-service teachers.

The EBI's 28-item length was desirable to minimize response burden due to length (Dillman, Smyth, & Christian, 2014): other instruments for measuring epistemic beliefs include the 63-item Epistemological Questionnaire (EQ; Schommer, 1990) and the 38-item Epistemological Beliefs Survey (EBS; Wood & Kardash, 2002). The EQ was not used for this study because of its length and because it did not sufficiently align to Schommer's (1990) full five-factor model (DeBacker et al., 2008). The EBS was not used for this study because there is a smaller body of research on which to draw than the EBI (DeBacker et al., 2008), its factor structure is based on selecting a subset from a larger 80-item instrument (DeBacker et al., 2008; Wood & Kardash, 2002), and its length relative to the EBI.

Additionally, the 28-item length and its relative popularity in the literature when compared with other epistemological belief instruments made the EBI an attractive instrument for use among other sources of data. An older, 32-item version of the EBI (Bendixen, Schraw, & Dunkle, 1998; Schraw, Dunkle, & Bendixen, 1995) also appears in the literature (e.g., Bendixen & Hartley, 2003; Mokhtari, 2014; Teo, 2013; Wang, Zhang, Zhang, & Hou, 2013; Welch & Ray, 2012) but was not chosen because the 28-item version was developed by refining the 32-item version and some of the additional four items implicitly position the respondent as a student which was not considered appropriate for this population of teachers. The widespread use of both 28 and 32-item versions of the EBI complicates the reporting of properties for these instruments, in part because they have the same name (see Appendix for details).

Data Analysis

Due to the small, non-random nature of the sample, only descriptive methods are used. All data were analyzed using R version 3.6.0 for Windows (R Core Team, 2019). Distributions of scores for each scale are illustrated using histograms. Relationships among scales are quantified using correlations and displayed visually in a tabular form (corrplot; Wei & Simko, 2017). Spearman's rank-order correlation coefficient is used because the small sample size resulted in each scale having between six and eight unique scores, and treating the distributions of scale scores as bivariate normal was an untenable assumption. Aggregated responses to each item are shown using bar graphs modified to display responses to Likert-type items (sjPlot; Lüdecke, 2019). Individuals' responses to subsets of items are displayed using line plots to enable quick visual comparisons (ggplot2; Wickham, 2009). Additional R packages used in the analysis were psych (Revelle, 2018) and reshape2 (Wickham, 2007). There were no missing data.

Results

Epistemic beliefs (EBI)

To answer the first research question, the exemplary statistics teachers' responses to the EBI are shown in Figure 1. Items that are reverse-coded are indicated with a * throughout, and reverse coding has already been applied in Figure 1. Participants tended to have low scores for each of the five scales (Figure 2), consistent with more availing epistemic beliefs.

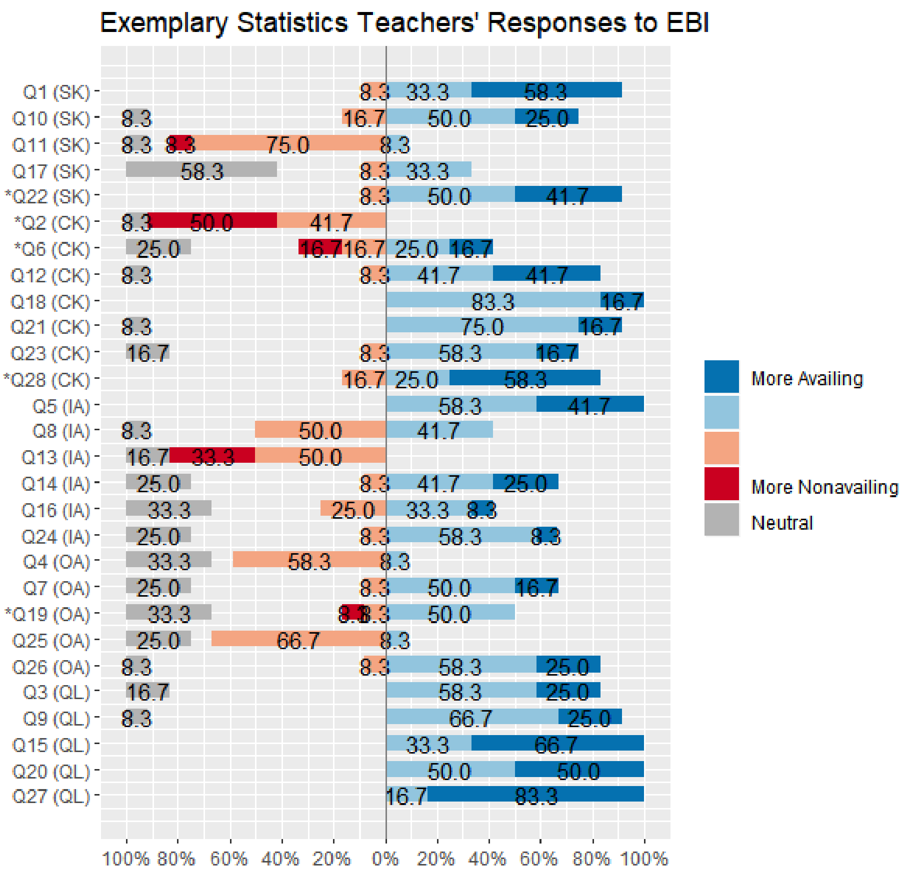

Figure 1. Responses to the EBI instrument with items grouped by construct (indicated in parentheses). The legend represents the continuum for the constructs rather than the Likert-type response options.

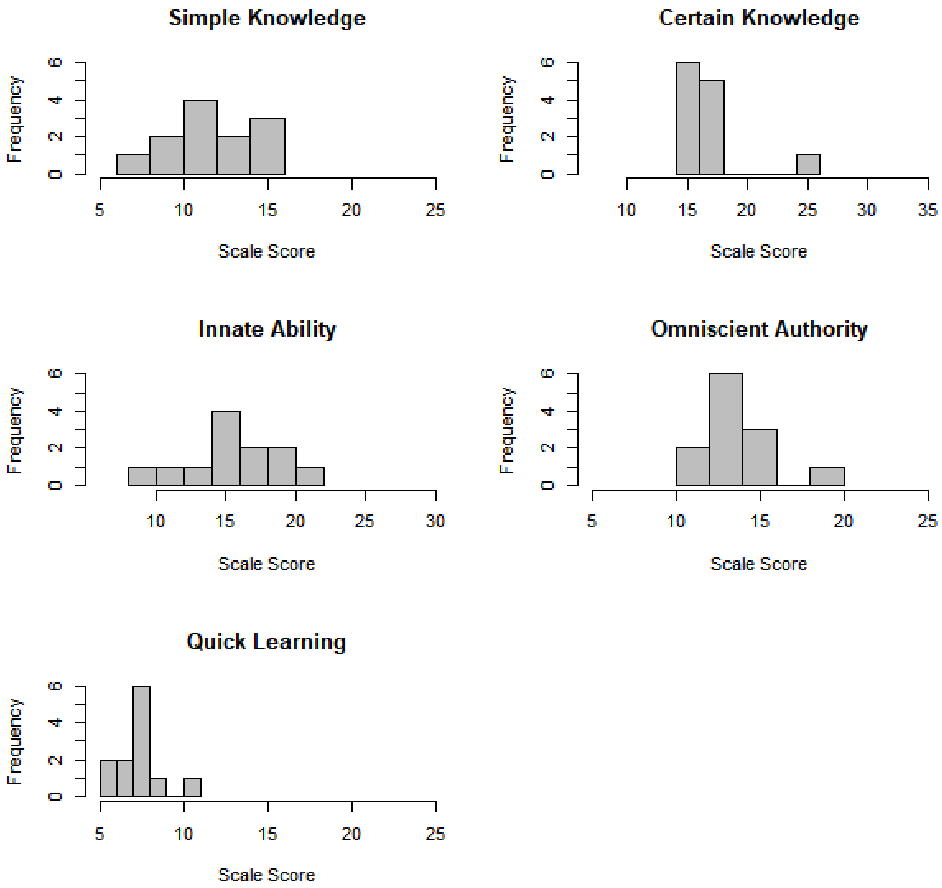

Figure 2. Histograms showing the distribution of scores for each of the EBI's scales.

Notably, the results shown in Figure 1 show that the exemplary statistics teachers were quite consistent in their responses to 25 of the 28 items. That is, on 25 of the items, participants tended to agree or disagree as a group. On only three items did at least one quarter of the participants respond in a direction different from the other participants:

- Q6: Absolute moral truth does not exist. (4/12 disagreed; 5/12 agreed)
- Q8: Really smart students don't have to work as hard to do well in school. (5/12 disagreed; 6/12 agreed)
- Q16: Some people just have a knack for learning and others don't. (5/12 disagreed; 3/12 agreed)

Disagreed indicates a response of Disagree or Strongly Disagree, and *agreed* indicates a response of Agree or Strongly Agree; other responses were neutral.

On the following items, more than half of the participants responded in the direction associated with nonavailing epistemic beliefs:

- (SK) Q11: The best ideas are often the simplest. (1 disagreed; 10 agreed)
- (CK) Q2*: What is true is a matter of opinion. (11 disagreed; 0 agreed)
- (IA) Q13: Some people are born with special gifts and talents. (0 disagreed; 10 agreed)
- (OA) Q4: People should always obey the law. (1 disagreed; 7 agreed)
- (OA) Q25: When someone in authority tells me what to do, I usually do it. (1 disagreed; 8 agreed)

Disagreed indicates a response of Disagree or Strongly Disagree, and *agreed* indicates a response of Agree or Strongly Agree; other responses were neutral.

Mathematical beliefs (IMBS)

To answer the second research question, participants responses to the IMBS (Kloosterman & Stage, 1992) were analyzed. Participants' aggregate responses to each item of the IMBS instrument, grouped by construct, are shown in Figure 3; reverse-coded items are indicated with an asterisk (*) throughout. Histograms showing the scale scores for each of the six beliefs are shown in Figure 4. Overall, the exemplary statistics teachers had quite positive beliefs about mathematics with nearly all participants' scores for each scale above the 3.0 midpoint (Figure 4), and responses to individual items also illustrate widespread positive beliefs about mathematics for this group.

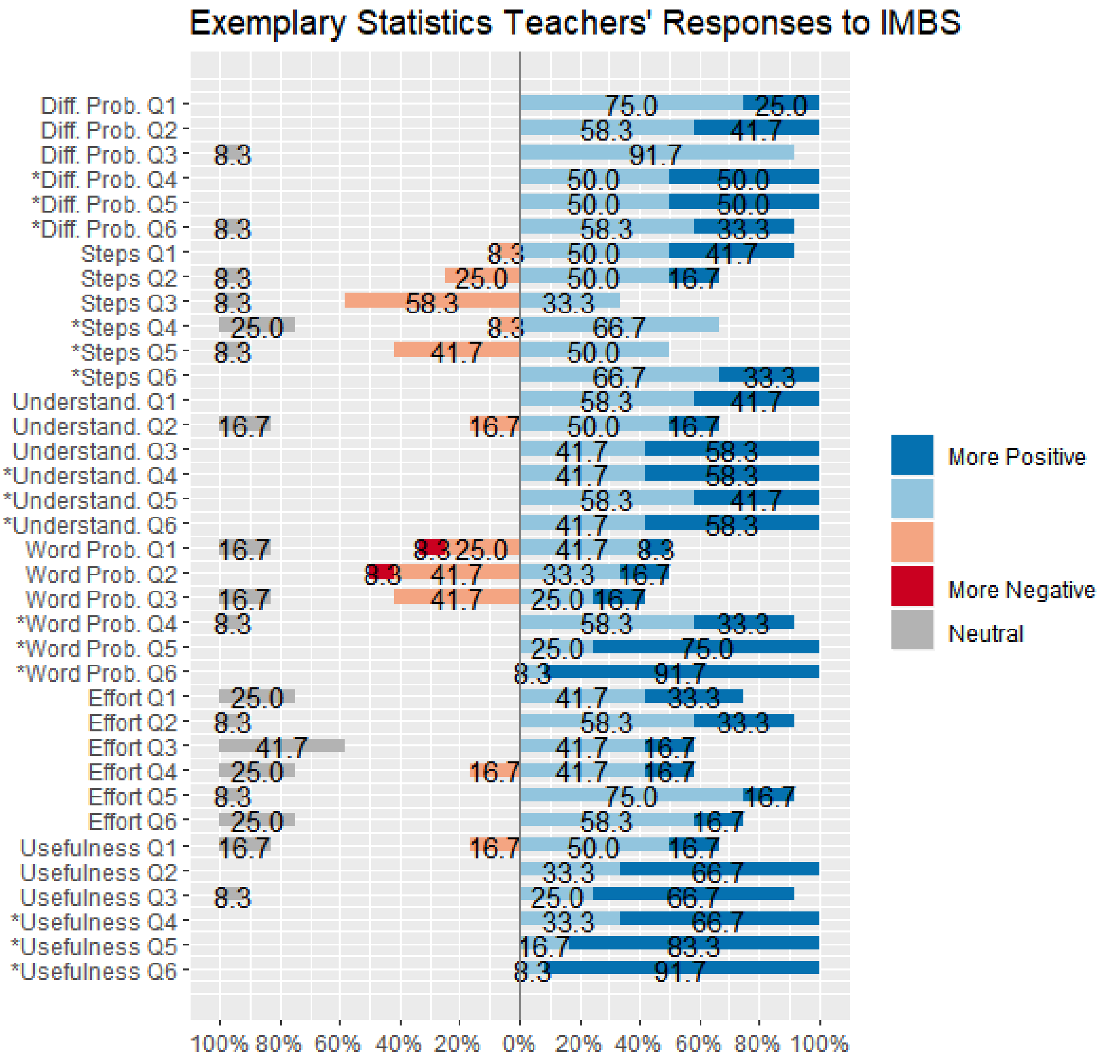

Figure 3. Responses to the IMBS instrument with items grouped by construct (indicated in parentheses). The legend represents the continuum for the constructs rather than the Likert-type response options.

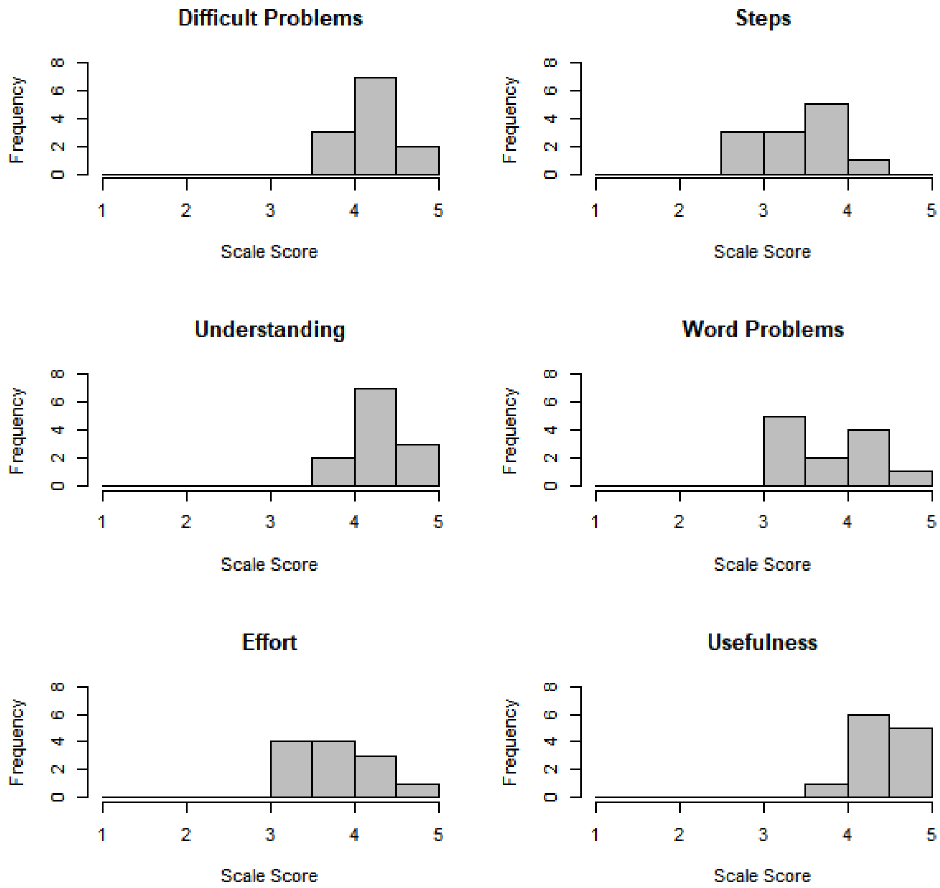

Figure 4. Histograms showing the distribution of scores for each of the IMBS's scales.

While participants generally responded in ways consistent with positive beliefs about mathematics, there were six items notable for the number of negative responses. In the Steps construct (see Figure 3), for three of the items, at least one quarter of the participants responded negatively:

- B2Q2: Word problems can be solved without remembering formulas.
- B2Q3: Memorizing steps is not that useful for learning to solve word problems.
- *B2Q5: Most word problems can be solved by using the correct step-by-step-procedure.

In the Word Problems construct (see Figure 3), for three of the items, about half of the participants responded positively and about half responded negatively.

- B4Q1: A person who can't solve word problems really can't do math.
- B4Q2: Computational skills are of little value if you can't use them to solve word problems.
- B4Q3: Computational skills are useless if you can't apply them to real life situations.

These six items were notable because more than two of the 12 participants responded negatively. Overall, however, the participants individually and as a group responded in ways consistent with positive beliefs about mathematics as measured by the IMBS (Kloosterman & Stage, 1992).

Groupings

To answer the third research question, responses to individual items were considered. With a sample size of only 12 exemplary statistics teachers, sophisticated techniques such as latent class modelling were unavailable. The degree to which the exemplary statistics teachers responded to items in the same direction was not anticipated, and only three items from the EBI and six items from the IMBS produced responses with appreciable numbers of high and low responses. Because of this, we turned to visual inspection of responses to these items; this would enable us to see only the most obvious types of grouped patterns (e.g., a group of teachers who answered all items low and another group of teachers who answered all items high). Responses to the three aforementioned EBI items are shown in Figure 5, and responses to the six aforementioned IMBS items are shown in Figure 6. With 8 high-low pattern possibilities for the EBI and 64 for the IMBS, there are no apparent patterns to the participants' responses that would enable the creation of meaningful groups.

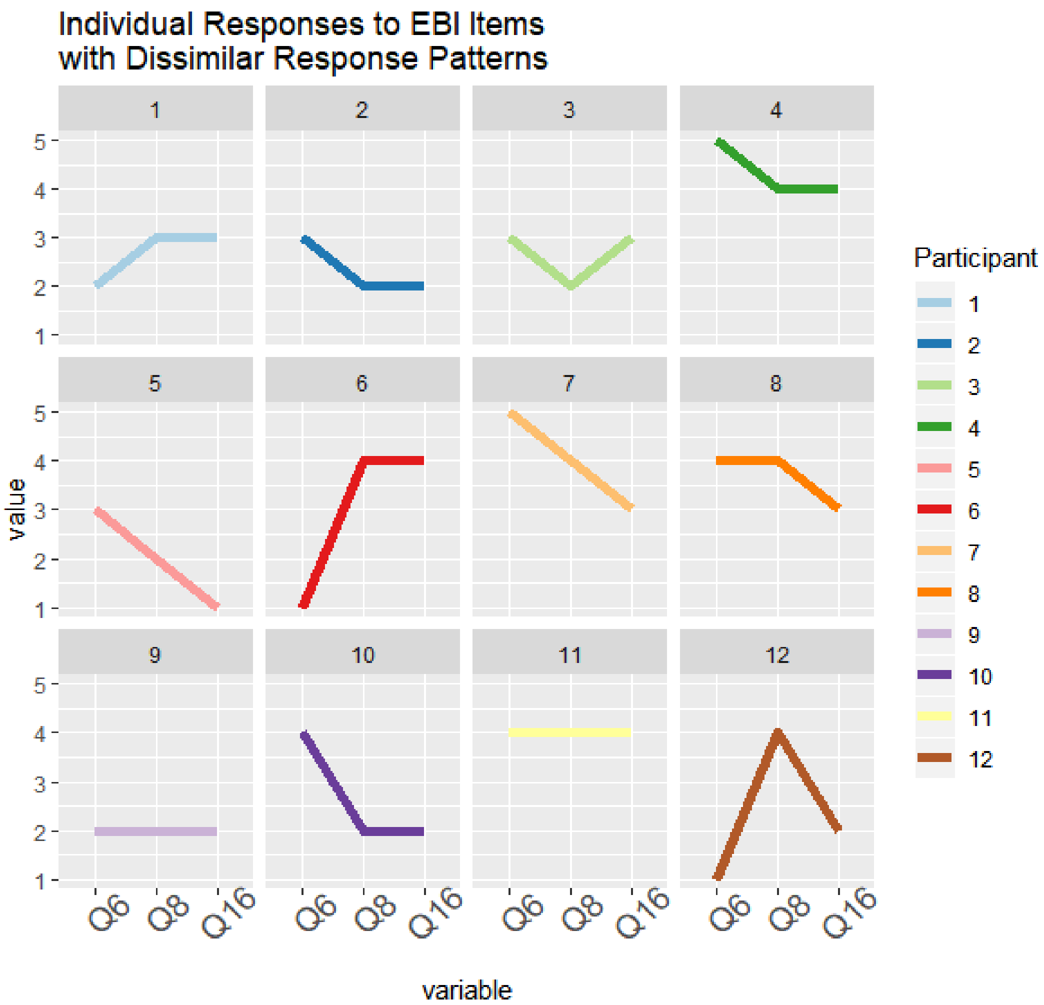

Figure 5. Individual responses to the EBI items that exhibited dissimilar responses.

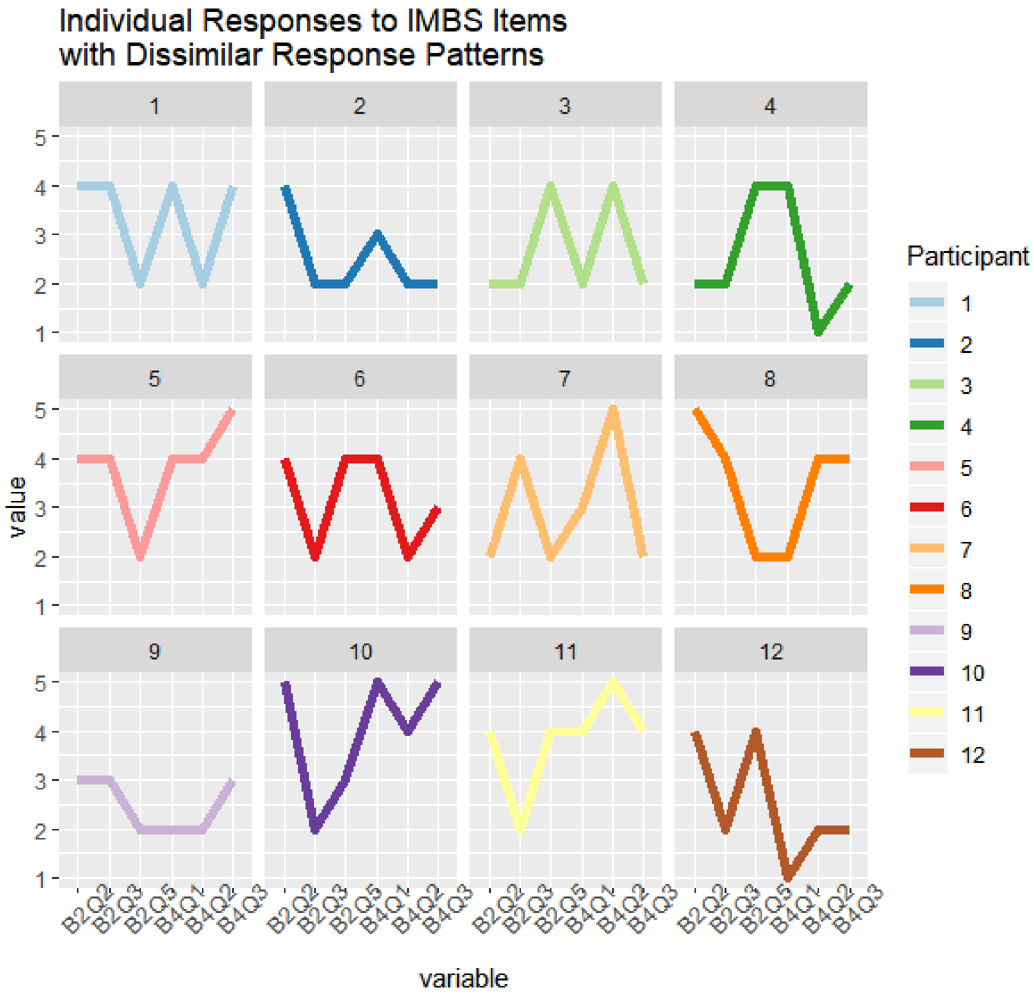

Figure 6. Individual responses to the IMBS items that exhibited dissimilar responses.

Relationship between mathematical and epistemic beliefs

To answer the fourth research question, Spearman rank correlations were calculated using scale scores for each pair of scales and are illustrated in Figure 7. For only four pairs of scales from different instruments did the magnitude of the correlation exceed 0.5, shown in Table 1. Negative correlations between EBI and IMBS scales reflect the scoring of epistemic belief instruments such as the EBI: lower scores tend to be associated with more availing beliefs (disagreement with the characterizing statement), while higher scores are associated with more nonavailing beliefs.

The three negative correlations noted in Table 1 indicate that as exemplary statistics teachers have more positive mathematical beliefs about Understanding, Word Problems, and Effort, that they tend to have more availing beliefs about Simple Knowledge, Certain Knowledge, and Quick Learning, respectively. The positive correlation in Table 1 indicate that as exemplary statistics have more positive beliefs about Difficult Problems, they tend to have more nonavailing beliefs about Simple Knowledge.

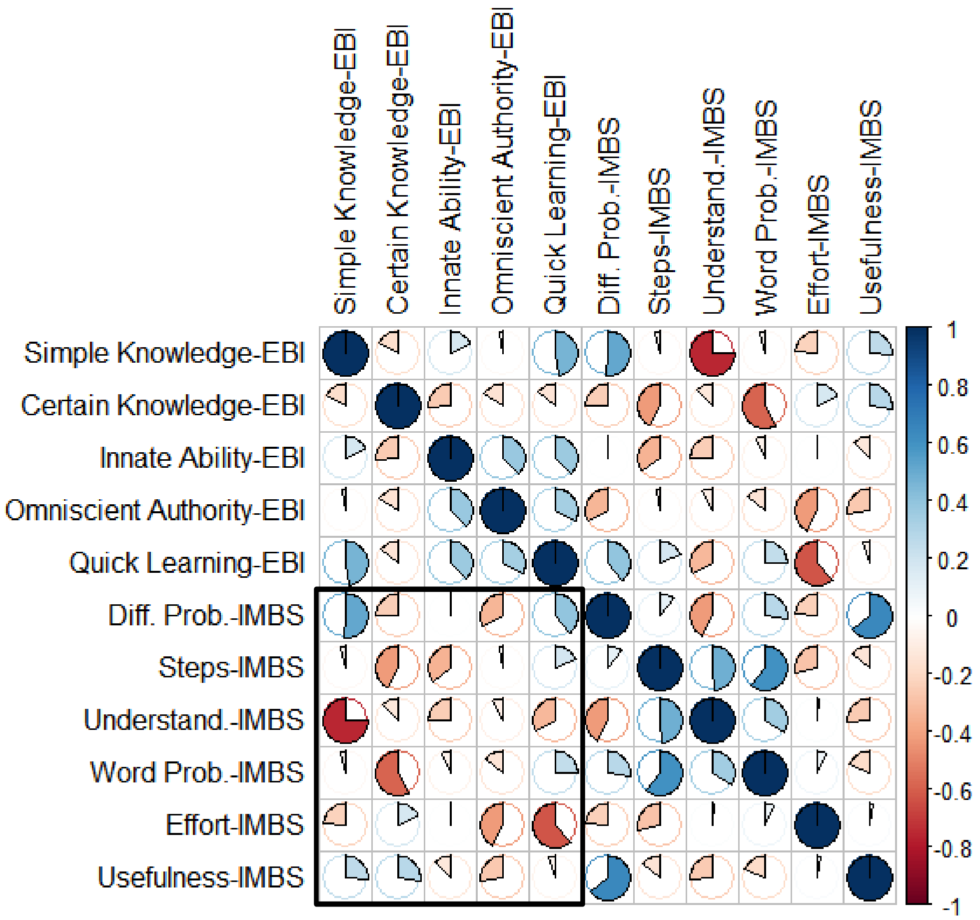

Figure 7. Spearman rank correlations calculated using scale scores between each pair of scales. Within each cell, the proportion of the pie chart filled in illustrates the correlation. Positive correlations are in shades of blue and the area is filled clockwise; negative correlations are red and the area is filled counter-clockwise. Correlations between scales of different instruments are indicated with a box in the lower triangle.

Table 1. *Correlations for pairs of constructs from EBI and IMBS magnitude exceeds 0.5.*

EBI Construct	IMBS Construct	Spearman Correlation
Knowledge is simple rather than complex (Simple Knowledge)	I can solve time-consuming mathematics problems. (Difficult Problems)	0.513
Knowledge is simple rather than complex (Simple Knowledge)	Understanding concepts is important in mathematics. (Understanding)	-0.757
Knowledge is certain rather than tentative (Certain Knowledge)	Word problems are important in mathematics. (Word Problems)	-0.584
Learning is quick or not at all (Quick Learning)	Effort can increase mathematical ability. (Effort)	-0.620

Discussion

Taken as a whole, the exemplary statistics teachers could be characterized as having mature epistemic beliefs consistent with being sophisticated learners (Schommer-Aikins, 2002) and have strong, positive attitudes about mathematical problem-solving. This is not surprising. What was surprising to the author was the degree to which the exemplary statistics teachers had similar responses. The motivation for this work was, in part, the homogenous nature of the statistics teachers in Peters' (2009, 2011, 2014) study who were described as having a robust understanding of variability. The results of this study were originally intended to be used for categorizing the exemplary statistics teachers and then relating these categories with interview data collected from the same group (Whitaker, 2016b). However, as the results for answering the third research question show, the teachers were rather homogenous in their epistemic and mathematical beliefs, with dissimilar responses to only nine of the 64 items administered.

While no groups of dissimilar responses could be identified, the results from the EBI and IMBS suggest that among even such a subgroup as exemplary statistics teachers that beliefs are not homogenous, though there is a remarkable amount of similarity of responses to many items. This similarity of the exemplary statistics teachers' responses to the EBI was unexpected, particularly for the five EBI items where the similar responses were associated with nonavailing beliefs (Items 2, 4, 11, 13, and 25). These responses are not limited to one construct: beliefs recorded as nonavailing related to innate abilities of people (Item 13), the omniscience of authorities (Items 4 and 25), and the simplicity and certainty of knowledge (Items 11 and 2, respectively).

The in-service statistics teachers tended to have availing epistemic beliefs which are associated with constructivism (Mason et al., 2013). In a recent study of the personal epistemologies of statisticians in academia in the United Kingdom, Diamond and Stylianides (2017) found statistics was associated with constructivist beliefs about knowledge while the mathematical foundation of statistics is regarded with objectivist beliefs. The consonance of in-service statistics teachers' epistemic beliefs with the statisticians' epistemic beliefs is notable, and future work about the epistemic beliefs of mathematics teachers may illuminate challenges encountered by new teachers of statistics.

These responses raise several areas for future research. First, while the 'best' epistemological stance is unknowable (Schommer-Aikins, 2002), consistent nonavailing beliefs among the exemplary statistics teachers in this study may mean that some epistemological beliefs may be more appropriate in the context of teaching statistics than in other contexts. Second is a question of measurement: while the responses to these items are recorded as nonavailing beliefs, might these responses be indicative of availing epistemic beliefs within the context of statistics? Third, are there thresholds on the belief continua that could be identified as 'sufficiently availing' to support becoming an exemplary statistics teacher or are there thresholds indicative of beliefs that are '*too* availing' and would hinder such development?

The most notable item with implications for the teaching and learning of statistics was Item 2: "What is true is a matter of opinion" (Schraw et al., 2002, p. 275). None of the exemplary statistics teachers in this study agreed with this item despite agreement indicating an availing (Muis, 2004) or sophisticated epistemic belief (Schommer, 1990; Schraw et al., 2002). While Schommer-Aikins (2002) cautions that the study of epistemology and epistemic beliefs can be personally challenging as one becomes aware of and processes their own beliefs, the field of statistics education does not seem willing to accept the position that opinions are truth. This is evidenced by renewed calls for a focus on statistical and information literacy amid discussions about entering a post-truth era (e.g., Kafadar, 2019; Ridgway, Nicholson, & Stern, 2017; Wild, 2017). As more discussions about evaluating evidence occur (e.g., this was the theme of the 2019 United States Conference on Teaching Statistics), these should be coupled with research about teachers' and students' epistemic beliefs. What teachers and students believe about the nature of knowledge will be critical for ensuring statistical and informational literacy efforts continue to make gains.

While this study addresses the dearth of research about statistics teachers' beliefs at a time of increased focus on evaluating evidence, there are important limitations. First, the sample of exemplary statistics teachers means that these results cannot be applied more broadly to statistics teachers that would be considered typical. Furthermore, because of the critical-case sampling (Patton, 2002) using nominations by experts in the field, this group of exemplary statistics teachers may not be representative of all exemplary statistics teachers. That is, the participants in this study may differ in important ways from other statistics teachers who would be considered exemplary.

The choice of instruments in this study was also limited. Much of the research in this area has focused on the beliefs of students, and instruments have largely been developed with students as the target population. While a clear justification for the appropriateness of the EBI and IMBS instruments was presented, the validity of the score interpretations from these instruments is tenuous and demands further research. For example, the sample in this study was too small to meaningfully investigate the factor structure of either instrument, and future studies could determine if the proposed factor structures are appropriate with teachers. As noted above, there may be other challenges to the use of these instruments – particularly the EBI – with statistics teachers, and other instruments could be developed.

New instruments might assess the constructs in these frameworks or others in a robust way or include epistemic and mathematical beliefs as part of a larger instrument connected with another framework. For example, Expectancy-Value Theory (Eccles, 1983, 2014; Eccles & Wigfield, 2002) has become widely used in statistics education research through the widespread use of the Survey of Attitudes Toward Statistics instrument (Ramirez, Schau, & Emmioğlu, 2012; Schau, 2003). This framework, applicable to both students (e.g., Whitaker, Unfried, & Batakci, 2018) and teachers (e.g., Batakci, Bolon, & Bond, 2018) might accommodate epistemic beliefs within either the existing beliefs construct or another background construct. Similarly, the relationship between beliefs and classroom practices suggests that proposed models for the statistics learning environment (e.g., Bond, Batakci, Bolon, & Whitaker, 2019) may accommodate epistemic beliefs as well.

Conclusion

Little is known about the beliefs of statistics teachers despite calls for more research about affective constructs (e.g., Gal et al., 1997). Teachers' beliefs are important because they may affect their classroom practices and students' beliefs, play a role in informational literacy (Hofer & Bendixen, 2012), and are related to their views on assessment (e.g., Martínez-Sierra et al., 2020). As the amount of statistics in school mathematics curricula increases globally (International Commission on Mathematical Instruction, 2011), an increasingly large group of mathematics teachers is being called on to teach statistics. What these teachers believe about mathematics and knowledge is likely to affect the instructional decisions they made and their students' outcomes. It is possible that different epistemic or mathematical beliefs may be more advantageous for teachers of mathematics than for teachers of statistics. Moreover, understanding the types of beliefs held by exemplary statistics teachers may inform future teacher preparation and professional development supporting mathematics teachers.

Though often included in mathematics curricula, real disciplinary differences between mathematics and statistics have implications for teaching and learning (Cobb & Moore, 1997; Franklin et al., 2007; Garfield & Ben-Zvi, 2008; Moore, 1988) and teacher preparation (Conference Board of the Mathematical Sciences, 2012; Franklin et al., 2015). Some of the differences may be more apparent than others. While the role of uncertainty and the emphasis on data from real-world problems may be an apparent more apparent for statistics, and the differences extend even to the materials provided for teachers and textbooks (Whitaker, 2016). Therefore, teachers' mathematical and epistemic beliefs might influence a myriad of pedagogical decisions, their self-efficacy for teaching statistics, their potential to learn and adapt to the new discipline they are being asked to teach, and ultimately their success in teaching the material. Perhaps teachers' beliefs might one day be considered in allocation of teaching assignments, teaching supports, and professional development may, though more comparative and predictive work would be needed before any recommendations could be made. Moreover, because teachers' beliefs affect students' beliefs and achievement, mathematics teachers tasked with teaching statistics whose beliefs are nonavailing for learning may instill these same nonavailing beliefs in their students.

By examining exemplary statistics teachers' epistemic and mathematical beliefs, this study contributes to this area and found that the teachers in this study held strongly positive beliefs about mathematical problem-solving and availing epistemic beliefs. However, some notable patterns of similarity of responses to items were present and counter to the dominant trend of positive beliefs. These patterns raise questions about the role epistemic beliefs might play in the teaching and learning of statistical literacy as well as the appropriate measurement of these important constructs.

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Appendix

Two versions of the EBI are used in the literature: a 32-item version and a 28-item version. The 32-item EBI was the instrument originally developed (Bendixen et al., 1998; Schraw et al., 1995), and "revision of the 32-item version into a 28-item version produced the current version of the EBI" (Schraw et al., 2002, p. 264). While the terms *revision* and *current version* suggest that the de rigueur EBI has 28 items, sustained use of the 32-item version while citing the publication for the 28-item version (e.g., Schraw & Olafson, 2002, 2008) has resulted in both versions being adopted by researchers. The conflation of the citations has practical implications for both using the EBI and interpreting results from studies that use the EBI. Clarification of some of these points is briefly outlined below.

Item-Factor Alignment

The full text of the EBI is available for both the 32-item (Schraw et al., 1995) and 28-item (Schraw et al., 2002) versions. However, none of the original papers containing validity evidence (i.e., Bendixen et al., 1998; Schraw et al., 1995, 2002) list the item-factor alignment for every item. The most information available in these original publications is the empirical item-factor alignment for items that had a loading of 0.30 or larger from exploratory factor analyses; the intended factors for all items are not given. Later researchers have published explicit lists of the item-factor alignment, but multiple versions exist with minor discrepancies. For example, Teo (2013) provides the item-factor alignment adopted in the present study, but a different item-factor alignment is implied by Ravindran et al. (2005).

Reverse Coding

Because little information is presented in the original publications about the items that did not have factor loadings above 0.30 (Schraw et al., 1995, 2002), discrepancies have emerged in which items have been reverse coded. Schraw et al. (2002) explicitly list Items 6 and 9 of the 28-item version as being reverse-coded, but others should be reverse-coded, too. Walter (2009) and Gatsos (2015) both use the 28-item EBI and reverse code Items 6, 9, 2, and 28, but both reverse code a different fifth item. Citing personal communication with G. Schraw, Walter reverse codes Item 24; Gatsos reverse codes Item 22. A close reading of the EBI items suggests that the Item 22 of the 28-item version "The more you know about a topic, the more there is to know" should be reverse coded because agreement with it is associated with more availing epistemic beliefs; this is Item 24 of the 32-item version, the possible source of the confusion. Teo (2013) lists several items that either should be reverse coded or have a factor loading in the opposite direction from what is expected and therefore are candidates for reverse coding: Items 2, 6, 14, 19, 20, 24, 30, 31, and 32 of the 32-item EBI. Based on this Teo's findings and suggestions, Items 2, 6, 19, 22, and 28 of the 28-item EBI were reverse coded in the present study.

Scale Scores

The issue of inconsistent reverse coding becomes especially problematic when scale scores are considered. The original papers containing validity evidence all propose constructing scale scores using only items with a factor loading of at least 0.30 (Bendixen et al., 1998; Schraw et al., 1995, 2002), and the 15 items with such factor loadings identified by Schraw et al. (2002) were used in the construction of the scale scores in the present study. However, some of these 15 items are among those with inconsistent recommendations about reverse coding, suggesting that caution is needed when comparing scale scores across studies. Moreover, as more data are collected, researchers may find different subsets of items that have factor loadings of at least 0.30. Using a subset of 27 items from the 32-item EBI, Teo (2013) found that all had factor loadings above 0.30 – though some items did not load with their hypothesized factor. Together with the previous comments about item-factor alignment and reverse coding, researchers using the EBI should explicitly describe which version of the EBI they are using, which items are included on each scale, which items are reverse coded, and how scale scores are computed to support comparisons across studies.