

Primary Mathematics Teacher Education Encountering Difficult Terrain: A Personal Perspective

Stephen Norton
Griffith University

The editors will enter dates here. Received: / Accepted:

Abstract. This paper is a case study in preparing prospective primary teachers to teach mathematics at one Australian university. The paper describes the content knowledge of prospective teachers, pre and post a final mathematics curriculum subject. The teaching approach which attempts to account for both content and pedagogical content knowledge is described. The constraints with respect to time and delivery mode are documented as well as the strategies used to maximise learning opportunities. The findings suggest that current mechanisms of student entry enable students with low levels of personal mathematics to enter and indeed pass teacher education courses. The data suggest there is merit in considering specialist or part-specialist primary teachers of mathematics or in extending teacher preparation time in pre-service teaching programs and refocusing their intent to better account for prospective teachers' knowledge of mathematics which is widely considered essential for effective teaching of the discipline.

(The key words below) are used in database searches, so include any important search terms.

Keywords: pre-service teachers, content knowledge, pedagogical content knowledge

Introduction

This paper is set against increasing concern in regard to the standards associated with teaching and learning mathematics and science in Australian schools and universities. Dinham (2013) noted that this has been precipitated in part by participation in the Organisation for Economic Development (OECD) and the United Nations Education, Scientific and Cultural Organisation (UNESCO), increasing the international focus on improving teacher quality. International testing data of school students (e.g., Thomson et al., 2012; Thomson, Wernert, Underwood, & Nicholas, 2007) illustrate that Australian students are losing ground, to Asian students in particular. What has particularly interested commentators is the long tail in the results, with relatively few students achieving at the highest levels. Associated with perceptions of falling mathematics education standards has been increased

scrutiny in respect to the quality of teacher preparation, such as from the Teacher Education Ministerial Advisory Group (TEMAG) (2014). In their summary report *Action Now: Classroom Ready Teachers* they stated:

Not all initial teacher education programs are equipping graduates with the content knowledge, evidence-based teaching strategies and skills they need to respond to different student learning needs.... Initial teacher education providers are not rigorously or consistently addressing the readiness of their pre-service teachers against the Professional Standards. (TEMAG, 2014, p. xi)

Earlier, in the state in which this study was undertaken—Queensland—Masters (2009) undertook a review of literacy and numeracy teaching and learning in response to the state's poor results on National Assessment Program, Literacy and Numeracy (NAPLAN) tests (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA], 2008). Masters reported:

It is important that every generalist primary teacher begins their career with at least threshold levels of knowledge about the teaching of literacy, numeracy and science. This “pedagogical content” knowledge includes knowing how students’ understandings in a subject typically develop, how to engage students and sequence subject matter, the kinds of misconceptions that students commonly develop, and effective ways to teach a subject. (p. ix)

Masters’ (2009) description of what knowledge is needed to teach mathematics is very similar to the knowledge forms described in the recently introduced professional standards for Australian teachers. The Australian Institute for Teaching and School Leadership (AITSL) (2014) released the Australian Professional Standards for Teachers documenting seven key standards. The first five standards relate to the practice of managing classroom discourse and include: (1) understand how students learn, (2) know the content and how to teach it, (3) be able to plan for and implement effective teaching, (4) create and maintain supportive and safe learning environments, and (5) assess and provide feedback and report on student learning. There is a long history of research that supports the argument that effective classroom discourse required by the AITSL standards above is dependent upon teachers’ knowledge of mathematics and the specific pedagogies to teach it. Shulman (1987) identified pedagogical content knowledge (PCK); the innovation here was that knowledge of content needed to be complemented with pedagogy. Ma (1999) studied the subject knowledge of U.S. and Chinese teachers and concluded that teaching conditions in the US, including teacher training, militated against the development of mathematics teaching knowledge and its organisation for teaching. She stated that a *profound* knowledge was necessary for effective teaching and in the main, in the US this was deficient.

Ball, Thames, and Phelps (2008) built on earlier understanding of what knowledge was necessary for effective teaching and identified four distinct domains: (a) common content knowledge, or general mathematical knowledge as distinct from that required for teaching; (b) specialised content knowledge, that content specifically needed for school mathematics teaching; (c) knowledge of content and students or a combination of

knowing about students and mathematics; and (d) knowledge of content and teaching. Zhang and Stephens (2013) extended the model to put a greater emphasis upon “curriculum knowledge,” that is, the importance of official curriculum documents that guide what, how, and when teachers are required to teach particular concepts. Zhang and Stephens used the term *teacher capacity*, tracing the term to the NSW Smarter Schools National Partnership (2010) and Victoria’s Smarter Schools National Partnership (Department of Education and Early Childhood Development, 2009). Teacher capacity related to “Teachers’ ability to understand and act on the reforms that policy makers are seeking to implement” (Zhang & Stephens, 2013, p. 483). The Zhang and Stephens model has four major attributes: (a) knowledge of mathematics that is intended to capture the key mathematics ideas for teaching specific content; (b) interpretation of the intentions of official mathematics curriculum, including a willingness to use them in planning; (c) understanding of students’ mathematical thinking, which includes demonstrating a knowledge of students’ mathematical thinking specific to subject matter and their ability to recognise student errors in mathematical thinking; and (d) design of teaching, that is, to plan a way to progress student understanding, utilising a deep knowledge of the relevant content and understanding of the specific pedagogical complexities to progress student understanding of mathematics. All four attributes interact with each other and in the context of the teachers’ dispositions, attitudes, beliefs, and values. A strength of the Zhang and Stephens model is that mathematical content is a key component and is central to teacher capacity. The model has close alignment with AITSL (2014).

The expectation that teachers of mathematics at all levels know the mathematics at the very least to the level they are expected to teach is almost universal; for example, the U.S. Department of Education (2008, p. xxi) states: “Research on the relationship between teachers’ mathematical knowledge and students’ achievement confirms the importance of teachers’ content knowledge. It is self-evident that teachers cannot teach what they do not know.” In the UK, Burghes (2011, p. 17) echoed a similar recommendation: “A prerequisite to be an effective teacher of mathematics, is that you are confident and competent in mathematics at a level significantly above that which you are teaching.”

In many states in the US (Darling-Hammond, Berry, & Thoreson, 2001) and Europe (Burghes, 2007) trainee teachers are either tested on their mathematics content prior to enrolment or before certification. A major purpose of this is to ensure that registered teachers know the relevant mathematics. In the East, in China, for example, primary school mathematics teachers are content specialists and “Prospective elementary teachers are required to study mathematics systematically and in depth” (Li, Zhao, Huong, & Ma, 2008, p. 422). Ginsburg, Leinwand, Anstrom and Pollock (2005) reported the superior content knowledge of Singapore teachers compared to their U.S. counterparts.

For more specificity as to what content will be taught and when, we can look to mathematics curriculum documents. In Australia, the ACARA (2016) document contains content strands including number and algebra,

measurement and geometry, and probability and statistics. It also has strands for proficiency including understanding, fluency, and problem solving and reasoning. The content of primary mathematics ranges from naming numbers in foundation years to problem solving with whole-number computation, comparing fractions, and adding, subtracting, multiplying and dividing fractions and decimals in Year 6. The minimum standard for Year 7 includes solving ratio problems, creating algebraic expressions, evaluating these via substitution, and solving linear equations. These content requirements are on par with the National Council for Teachers of Mathematics (NCTM) (2000). It is worth noting that fraction, including decimal computation with all four operations, is considered a minimum standard for Year 4 in Singapore and working with ratios is addressed in Year 5 (Ministry of Education, Singapore, 2007). In Singapore, Year 6 students are expected to work with formal abstract algebra. Similarly, the Hong Kong mathematics syllabus has expectations in advance of the Australian expectations (Education Bureau: The Government of Hong Kong Special Administrative Region, 2000).

The Australian Mathematics Syllabus with its emphasis on fluency with basic facts and processes implies pedagogical approaches that have the support of a number of Western curriculum thinkers (e.g., Hattie, 2009; Kirschner, Sweller, & Clark, 2006; Muller, 2000) as well as Eastern educational thinking (e.g., An, 2004; Cai & Cifarelli, 2004; Huang & Leung, 2004; Jensen, 2012; Lafayette De Mente, 2009; Lee, 1996; Li, 2004; Zhang, Li, & Tang, 2004). These authors have a number of common tenets: their view of the discipline is essentially esoteric and hierarchical (Muller, 2000) with fluency with basic facts and processes indispensable for problem solving and reasoning. The Chinese term for this concept is “two basics”: basic skills and concepts followed by basic problem solving and applications. In this regard the *Review of the Australian Curriculum: Final Report* (Australian Government: Department of Education, 2014) echoes this approach: “You cannot teach competencies without content; capabilities must be grounded in content. Critical thinking, in particular, is best embedded in learning areas. Knowledge, competencies and problem solving is the usual formula” (p. 33).

Cognitive load theorists (e.g., (Kirschner et al., 2006; Owen & Sweller, 1989) explain that the critical importance of basic facts and processes, in terms of the brain’s capability, is to use long-term memory as a reservoir that frees up short-term memory to solve problems. Good problem solvers, or in this case good teachers, have an extensive range of mathematical facts and techniques stored in long-term memory. This information can be accessed with little effort to be applied to new problems. As a result, short-term memory is freed up to focus on the peculiarities of the new problem or, in the case of teaching, to enable adaptive provision of scaffolding. In contrast, students lacking such reservoirs of key facts, processes, and structures in long-term memory are forced to rely on short-term working memory. Without fluency with basic facts and processes students are likely to be overwhelmed in attempting the multiple steps

required to problem solve. In the case of teachers, without fluency with the content they are unlikely to be able to effectively scaffold student learning in a dynamic classroom environment. With regard to pedagogical approach, the above authors share the view that the teacher is central in controlling the nature of classroom discourse such that the underlying mathematical structures are made explicit. This is essentially the view of the role of the teacher taken by Hattie (2009) who uses the term “visible teaching and learning” (p. 22) to describe teaching within the context of explicit goals, appropriate challenge, deliberate practice aimed at mastery, targeted feedback, and active engagement in learning.

In Australia, as in most of the West, primary or elementary teachers are generalist teachers and are expected to teach mathematics along with English and the full range of humanities and technology and science orientated school subjects. This means that teacher preparation programs attempt to prepare teachers to be classroom ready across a range of disciplines including arts, history, geography, English, physical education and supporting subjects such as lifelong learning, assessment subjects, and ICT based subjects. Program structures of these forms are common across Australia and are certified by the state registering body the Queensland College of Teachers (QCT) and graduates are recognised as being able to teach in all Australian states and are broadly accepted internationally.

A possible consequence of the preparation of generalist teachers is a dilution of focus on any specific discipline domain, with less time to become deeply knowledgeable in a particular discipline. Interestingly, East Asian nations including China have specialist mathematics teachers in primary school (Li et al., 2008; Ma, 1999; Author, 2013). Ma (1999) reports that there are several advantages in having specialist teachers of primary mathematics, including that it concentrates the teacher preparation in the hands of a smaller number of expert mathematics education specialists, raises the incentives for mathematically inclined people to become mathematics teachers, and enables teachers to focus their energies on understanding how to teach mathematics effectively.

Further factors attracting critique in regard to teacher preparation in the West include the application of misguided or misapplied theories of learning (e.g., Farkota; 2005; Hattie, 2009; Kirschner et al., 2006; Meyers, 2012; Muller, 2000). These authors argue that the mathematics education movement that has come to dominate mathematics education discourse in the West in recent decades has emphasised social cultural theory at the expense of the discipline of specific subject domains (e.g., knowledge of the mathematics). In effect, the authors were critical of over-focusing on processes of learning and theories of learning at the expense of learning the subject domain and how to teach it explicitly. Meyers derogatorily used the term “educationalist” to describe academics whom he claimed have come to dominate tertiary institutions, stating they have “cheated [students] of knowledge and skill” (p. 71).

Dinham (2013) attributed part of the trouble with respect to primary teaching standards to the intake into the profession of students who struggle with understanding the mathematics they are likely to teach, reporting that across Australia, acceptance into teaching is characterised by a declining

Australian Tertiary Admissions Rank (ATAR), falling to the 45% percentile and lower. In the study state of Queensland, an Overall Position (OP) score is used rather than an ATAR score. At the time of enrolment for the students reported on here, the official OP cut off for undergraduate students was 13. This is equivalent to the nationally agreed common index ATAR score of about 68 (the highest score being 99.5 and the lowest score being 0). A student with a score of 68 has performed better than 32% of OP eligible students. The approximation “about” is used because the conversion rates vary a little from year to year. From an international perspective, an OP of 13 is equivalent to an International Baccalaureate (IB) score of about 24, with the maximum IB score being 45 and the lowest score 24; this means the student must pass six secondary high school subjects.

The U.S. Department of Education (2008) reported that there is limited data to help form a detailed picture of the nature of primary school teacher education capacity with respect to teaching mathematics, a claim repeated by TEMAG (2014). Those international studies that exist indicate diverging practices and varied expectations and outcomes (Burghes, 2007). Burghes (2011) reported that nations such as Japan, Singapore, Russia, and China had trainee teachers who outperformed English, Irish, Finnish, and Czech trainee teachers in areas of mathematical skills and content knowledge. With this background in mind, this paper considers the advice of TEMAG (2014, p. xii):

Evidence must underpin all elements of initial teacher education, from the design and delivery of programs to the teaching practices taught within programs.... There should be greater transparency across all elements of initial teacher education, from entrance selection to program outcomes.

In regard to this study, acceptance of the critical nature or fundamental concepts and fluency with basic skills justifies the use of tests such as those used by Burghes (2007) that are dominated by questions based on a range of fundamental number processes. The critical nature of number as a foundation to mathematics study is supported by U.S. Department of Education (2008, p. xvii):

Proficiency with whole numbers is a necessary precursor for the study of fractions....A major goal for K-8 mathematics should be proficiency with fractions (including decimals, percentage and negative fractions), for such proficiency is foundation for algebra and, at the present time, seems to be severely underdeveloped.

Thus, the aims of this paper are to document trainee teachers' mathematical backgrounds as a way of accounting for their starting content knowledge, to describe an intervention, and then to assess their growth in content knowledge. Finally, trainee teachers' content knowledge is related to their ability to describe how they would teach key primary mathematics concepts. The totality of data enables the reader to reflect on the effectiveness of preparing trainee teachers to be classroom ready under the constraints of existing programs in at least one teacher preparation institution.

Method

The method is a case study consistent with interpretative inquiry that attempts to gain insight as to what lies behind the phenomenon (Denzin & Lincoln, 1994). It draws on both quantitative and qualitative data to create a narrative of the processes of primary teacher education enacted by the author. The investigative nature of the data exploration and the emergence of themes from the data give the methodology an element of grounded theory methodology (Creswell, 2005; Strauss & Corbin, 1990). While most of the cited data come from the past 3 years, the challenges have remained consistent over the past half-decade that the academic has been testing students, evaluating the courses and conducting surveys. This gives the entry data a longitudinal dimension. Credibility or trustworthiness is attained by reporting the data and analysis in sufficient detail for the reader to judge if the findings are educationally significant (Denzin & Lincoln, 1994).

Study Background Information

What is the mathematics education learning opportunity?

In Queensland, and in the study institution, teacher training pathways include programs which incorporate learning how to teach as well as practical experience in schools. The two dominant pathways are via a 4-year undergraduate degree or a graduate entry pathway with 1 year of teacher preparation. In the undergraduate pathway there are two compulsory specialist mathematics curriculum courses (Mathematics 1 and Mathematics 2) out of a total of 32 courses. As of 2015 this was extended to include a third mathematics curriculum course, Mathematics 3. There are typically at least four school-based practical subjects in a program and the remaining courses are designed to help teachers meet all of the discipline needs; as noted in the literature review these include broad cognitive demands (e.g., science, history, arts, technology, geography, ethics, Indigenous studies, English, music, health and physical education, and professional practice subjects). The usual pattern in Queensland universities is to split the mathematics curriculum subjects according to strands, with Mathematics 1 or its equivalent covering the teaching of number and algebraic thinking and Mathematics 2 the teaching of the strands of chance and data, and space and measurement. Mathematics 3 at the study university was designed to build on the prior mathematics curriculum subjects and to develop skills in student diagnosis and teaching more advanced problem solving.

In some institutions teacher education study can be completed online without any face-to-face lectures and workshops, but usual practice is a mix of lectures, workshops, and online learning opportunities. In the study institution the historical pattern was for two face-to-face curriculum courses of about 40 hours spread over 13 weeks: Mathematics 1 in second year and Mathematics 2 in third year. Recent years have seen this contact time changed to two blended learning courses with some limited face-to-face

learning and some online learning amounting to about 26 hours of face-to-face contact for Mathematics 1 and Mathematics 2. Students could access 32 hours of face-to-face tuition for Mathematics 3 from Semester 1, 2015 and in 2016 this course is scheduled for 24 hours of lectures and workshops.

The second teacher education pathway is via graduate level. Students who have completed a degree which has been judged to contain a solid basis upon which to build teaching expertise enter for a 1-year program. This base includes a passing grade in senior mathematics unless there were special enrolment circumstances such as mature age entry. The level of mathematics studied was not stipulated until recently. As of the time of writing, the current advice is a minimum of a pass in Mathematics A, the least abstract of the formal senior mathematics subjects, and to have passed one science-related course that qualifies towards an OP. The graduate diploma pathway has since been phased out in favour of a 2-year master's program with two mathematics curriculum units. In line with competing institutions, these courses are offered as mixed mode, that is, essentially online with some 8 hours of face-to-face contact over 8 weeks for domestic students. In the study institution as in other institutions providing teacher training, the course structures are in flux and reflect a diversity of practice both in terms of what is taught and how it is delivered and assessed. Institutions attempt to balance quality, costs, and perceptions of what they consider students will find attractive in what is perceived as a competitive market for domestic and, potentially, international students.

The students under study are from one of two of the largest teacher education institutions in Queensland. Entry to either the undergraduate or the graduate pathway has minimal requirements of personal numeracy. It has been the case that almost any Queensland student who gains an OP or who has already obtained a degree can enter primary teacher training either from the accumulation of bonus points or from special considerations including mature age entry.

The bulk of student-related data reported in this study are derived from the 2013, 2014, and 2015 cohort entry into a Graduate Diploma of Primary Teacher Education and the 4th year undergraduate Bachelor of Primary Education in a metropolitan area of Queensland. Post-test data are confined to 2014 and 2015 cohorts. Thus the sample of students was as follows:

A - Graduate Diploma in Primary Education 2013 intake (n=86)

B - Graduate Diploma in Primary Education 2014 intake (n=74)

C - Undergraduate 4th year mathematics curriculum Mathematics 3 2015 (n=179).

The data represent at least 85% of each cohort, thus there is no reason to suspect the sample is not representative of the population of the cohorts. The approximately 15% attrition was related to ad hoc absences or lack of consent to use the data.

What was the mathematics educational history of students?

This question is concerned about what forms of mathematics study the students had undertaken prior to enrolment in my courses. Most of the data used to inform the answer to this question were obtained from a survey at the course commencement. A summary is presented in Table 1.

Table 1. *Extent of High School Mathematics Studied by Percentage of Cohort*

	A (n=86)	B (n=74)	C (n=179)
No senior mathematics	13%	15%	9%
Mathematics A	47%	37%	57%
Mathematics B or equivalent	40%	38%	12%
Missing	None	10%	22%

Key:

A: Graduate diploma pathway 2013

B: Graduate diploma pathway 2014

C: Undergraduate pathway 2015

Of the undergraduate students (Cohort C), 12% reported studying abstract senior high school mathematics that included calculus. For the graduate pathway this proportion was about 40%. Relatively small portions had studied no senior mathematics. In the 2015 undergraduate intake, all students had passed both Mathematics 1 and Mathematics 2, reporting that 66% gained grades higher than a credit for Mathematics 1 and 81% gaining grades higher than a credit for Mathematics 2. Later, the implications of these relatively high proportions of credits and higher grades is discussed in terms of curriculum focus.

What mathematics curriculum teaching approaches were employed?

The section following helps to add transparency to the mathematics teacher education process and provide a context for the reader to evaluate the success or otherwise of the interventions. The author's teaching philosophy was informed by several decades as a mathematics and science teacher at secondary level and two decades teaching mathematics teacher education at primary, middle school, and senior levels, that is, from counting to calculus.

The course had the following overarching aims:

1. To build the content knowledge of trainee teachers as much as possible while teaching the specifics of how to teach the content and processes to children; and
2. To teach trainee teachers how to identify student errors in mathematical thinking and how to plan to remediate these.

In implementing the first objective, the author was attempting to meet "Know content and how to teach it" (Standard 2), and Standard 3 – "Plan and implement effective teaching and learning." The second objective

attempts to meet Standard 5, “Assess, provide feedback and report on student learning” (AITSL, 2014). Some assumptions were made in the structuring of the course.

First, it was clear that there was not sufficient time to teach the content and specific pedagogy of all the strands (number and algebra, space and measurement, chance and data) equally. It was considered it would be better to teach the key pedagogy well rather than attempt to teach a broad spread of specific pedagogy poorly. This approach was informed by ongoing analysis of NAPLAN (MYCEETYA, 2008-2014) data on student error patterns: for example, students Australia wide struggle with basic algorithms (30% of Year 5 students could not carry out a subtraction with renaming problems [MCEETYA, 2014]) and fractions and proportional reasoning were especially problematic for Year 5 and 7 students.

Knowledge of the personal numeracy level of students from prior intakes was central to planning with a focus on number. With this in mind the content focus was on teaching the specifics of teaching whole-number numeration and computation, followed by fraction numeration and computation and then proportional reasoning across the strands.

The course content was divided into *know how to teach* and *know how to do*. There were 110 key concepts that covered most of the critical content of primary mathematics and ranged from naming single-digit numbers to early algebra and included key concepts in all strands. *Know how to teach* indicated that the students could be expected to explain explicitly how to teach the concept, in particular what materials would be used (e.g., ten frames, counters, place value charts, bundle sticks, base ten materials, diagrammatic models), and what specific mathematical language would be used to connect the material representations to symbolic representations. For whole-number numeration and computation the pedagogy was informed by the work of Booker et al. (2010) while the fraction and proportional reasoning teaching was informed by a variety of sources (e.g., Booker, Bond, Sparrow, & Swan, 2010; Brown & Quinn, 2006; Author, 2009; Van de Walle, Karp, & Bay-Williams, 2010). Prior testing informed the decision to treat the 4th year graduate and undergraduate students similarly.

The primary student resource was watching 16 hours of DVD snippets showing explicit modelling of teaching key concepts. The extent to which students used the DVDs is unknown, but lecture and workshop attendance was typically 90% for both undergraduate and postgraduate students until 2015 when undergraduate lecture attendance fell to about 20% towards the end of the 8-week teaching phase. Students cited access to lecture capture, the demands of full-time and part-time jobs, and assessment demands in other subjects as reasons not to attend lectures.

The entry test of content knowledge was conducted in the first workshop before any teaching had occurred. In the second week of the semester, marked scripts were returned and one hour was spent explaining the solutions to the test of content the students had completed in Week 1. Thereafter the test was not mentioned and the test and solutions were not made available to students, nor was there a particular focus on the content in the test, unless it was core to the course content and listed in the documented list of outcomes. It is considered highly unlikely that students

might have found the test online and studied for it, since there was no directive to do so. The students were told that they were to be retested with a similar test upon conclusion of the course and this would account for 10% of their exit mark.

Table 2 sets out the aims and data sources for this study.

Table 2. *Study Aims and Primary Data Sources*

Research aims	Primary data source
1 What starting knowledge (content) of mathematics do students bring?	Primary Mathematics Audit Part A ICSMTT (Burghes, 2007)
2 What is the gain in mathematical (content) knowledge?	Modified Primary Mathematics Audit Part A ICSMTT (Burghes, 2007)
3 What pedagogical content knowledge was demonstrated at the end of the course?	Testing of pedagogical content knowledge as assessed in final examination of course (Author designed).

Assessment of content knowledge

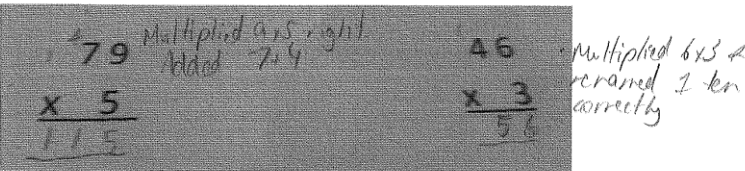
To assess content knowledge, Part A of ICSMTT (Burghes, 2007) was used for the pre-test. The post-test was of the same structure with changes to the numbers and minor changes in form. For example, the pre-test asked students to convert $\frac{3}{8}$ to a decimal and percentage while the post-test asked students to convert $\frac{5}{9}$ to a decimal and a percentage. The post-test data were collected as part of the final assessment; no calculators were permitted and as noted above the score out of 40 was scaled to 10% of the exit mark. Responses were scored consistently with Burghes’ (2007) marking scheme. The test has been used widely internationally and there is no reason to question its validity as a measure of basic mathematical content knowledge.

Assessment of pedagogical content knowledge (PCK)

There was no pre-test in pedagogical content knowledge because this was intended to be developed during the course and it would be invalid to test this at the start. The tests of PCK, in effect a measure of teacher capacity, are tests devised by the author to assess trainee teachers’ capacity to identify student errors, explain teaching sequences that would be likely to remediate these misconceptions or errors, and plan to teach specific problems including those of the form that confound Year 3, 5, and 7 students in NAPLAN tests (MCEETYA, 2008-2014). This is consistent with the emerging models of teacher capacity (e.g., AITSL, 2014; Zhang & Stephens, 2013). A sample of questions and curriculum intent is displayed in Figure 1 as are explanations on marking. It is the responsibility of the informed reader to evaluate the validity of such assessment and determine the educational significance of the results.

Question 4

Year 6 student carried out the following computations. Diagnose the errors exhibited by the child. (1 marks)



(a) What is the error in procedure? Why do you think this error was made?
The student clearly knows their multiplication facts and can rename 20 ones as 1 ten. However they revert to the addition algorithm when they get to the tens column and forget to cross multiply, e.g. 4 tens by 3 ones.

(b) Use the space below to show how you would remediate by teaching the vertical Bundling algorithm with appropriate materials. Since the numbers above require renaming to slides 29 hundreds, use smaller numbers to remediate this conceptual error eg., 29 by 3 and use materials appropriate to this level of computation. (5 marks)

Materials	Explanation of connections	Written algorithm
① 29 3 7	1) Start with ones, 3ones multiplied by 9ones is 27 ones.	29 x 3 7
② 29 3 7	2) Record the 7 ones. Rename 20 ones as 2 tens, and place in tens column after bundling with rubber band.	29 x 3 7
③ 29 3 7	3) Now tens. 3ones multiplied by 2tens is 6 tens.	29 x 3 7
④ 29 3 7	4) In the tens column we now have 6 tens, and the 2 renamed tens is 8 tens. Record 8 tens.	29 x 3 7
⑤ 29 3 7	5) Say the computation - 3 by 29 is eighty-seven.	29 x 3 7

Figure 1. Sample of test item assessing capacity to diagnose student errors and scaffold remediation for multiplication.

Part (a) of the question above assesses the pre-service teacher's capacity to assess, provide feedback and report on student learning (Standard 5 of AITSL standards). In this instance the pre-service teacher has correctly identified that the child had fluency with basic multiplication facts, but confounds the addition and multiplication algorithms. Part (b) requires the pre-service teacher to use this information as a basis to describe how this misconception can be remediated. In the sample above they have appropriately used specific meaningful language to connect material

representations to the symbolic recording of the algorithm. Such a response would be awarded full marks.

In the example in Figure 2, error pattern analysis and remediation of fraction addition operation is assessed.

Question 7

A Year 9 student carried out the following computation.

(a) Identify the mistake in thinking (1 mark)

Whole-number thinking.

The student has added straight across the fractions as if they are whole numbers.

(b) Use the models and careful use of language to help remediate this student's thinking.

Be sure to explain the renaming of improper fractions to mixed numbers (5 marks)

* You could also reduce $\frac{6}{10}$ to $\frac{3}{5}$ first and solve. I did it the other way to show how to find lowest common denominator.

Area models	Explanation of teaching
1. $\frac{6}{10} + \frac{3}{5}$ 	1. Can I add these fractions - do they have the same name? Six TENTHS } No they do not. Three FIFTHS }
2. $\frac{6}{10}$ 	2. Rename using the unity rule. By factorisation $\frac{10}{2 \times 5}$ we see that the lowest common denominator is 10. Multiply $\frac{3}{5}$ by one in the form of $\frac{2}{2}$ to make tenths. 3 fifths by $\frac{2}{2}$ is 6 tenths.
3. $\frac{12}{10}$ 	3. Now we have 6 tenths and 6 tenths. I can add these because they have the same name. 6 tenths and 6 tenths is 12 tenths. We can see from the model that 12 tenths can also be written as 1 and 2 tenths.
4. $1\frac{2}{10}$ 	4. We can use fraction strips or our unity rule or look at the model to remove $\frac{12}{10}$ to one and 2 fifths. This is reduced to its simplest form. Therefore 6 tenths and 3 fifths is one and 2 fifths.

Figure 2. Assessment of student thinking for fraction addition and subsequent description of remediation pedagogy.

In Figure 2, the pre-service teacher has identified the student's error in simply adding the denominators, most probably a manifestation of over generalisation of whole-number thinking. The pedagogy associated in remediating this error is considered appropriate in that diagrammatic models are used to convey meaning and this is connected to symbolic recording with specific language in an appropriate sequence. In the example in Figure 3, the pre-service teacher has correctly identified that the student does not understand place value in the context of multiplying decimals. They have been asked to describe the pedagogical basis for correcting this misconception. In doing this, the pre-service teacher has converted the

decimals to common fractions of tenths and represented the product using an area model. Such a response would be awarded full marks.

Question 11

A student was asked to multiply **0.4** by **0.3** and attained the solution 1.2.

(a) What misconception is illustrated? (1 marks)

Misunderstanding of place value and algorithm

(b) Use two different methods to illustrate how the solution can be attained. (5 marks)

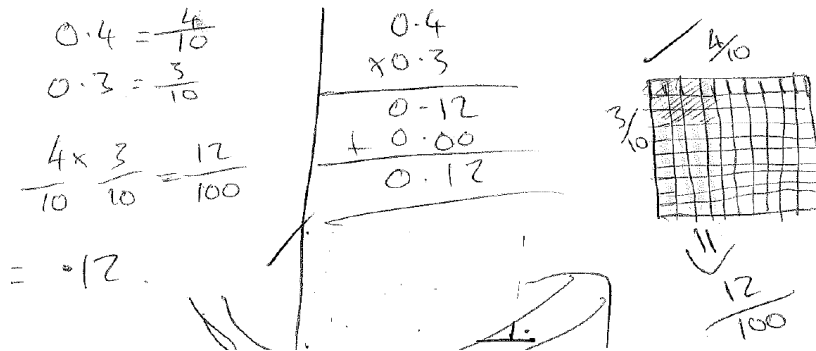


Figure 3. Modelling pedagogy associated with decimal multiplication.

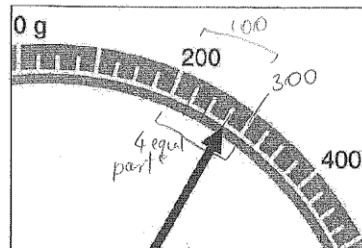
In 2014 the most poorly completed pedagogical content question required pre-service teachers to explain how they would teach a scale question; this question was used in the 2013 NAPLAN (MCEETYA, 2013). Figure 4 illustrates what is considered a coherent response, that is, a response that illustrates the pre-service teacher understands the mathematical concepts involved and how these might be made explicit to students.

Question 8.

The following stimulus was in the Year 7 NAPLN 2013.

Finn put some flour on his kitchen scales.

The picture shows part of the dial.



How many grams of flour did Finn put on the scales?

- (a) What concepts are involved and why might students find such a problem difficult? (1 mark).

• Reading a scale • Subtraction • Addition
 • Fractions → recognising that 3 markings means the total grams between 2 points (e.g. 200 and 300 = 100g) is divided into 4 equal parts.

- (b) Provide a complete explanation of the solution to a Year 6 child. Draw the scale several times below to show each step in the process of explaining the solution (5 Marks)

1. 200 300 Lets look at the quantities first. How many grams are between the points?
$$\begin{array}{r} 300 \\ - 200 \\ \hline 100 \end{array}$$
 100g. ⑥

2. 200 300 There are 3 markers between them, so 4 equal parts, divided into 4 equal parts is 25g, no remainders.
$$\begin{array}{r} 200 \\ + 25 \\ \hline 225 \\ + 25 \\ \hline 250 \\ + 25 \\ \hline 275 \end{array}$$
 We can see that one point above 200g will be 25g MORE. Do the addition and it is 225g. Write it in. Each subsequent point is 25g MORE than the last. Do the addition and write it in.

3. 200 300 The arrow is pointing to the third marker, which corresponds to 275g.
$$\begin{array}{r} 200 \\ + 25 \\ \hline 225 \\ + 25 \\ \hline 250 \\ + 25 \\ \hline 275 \end{array}$$

12

Figure 4. An explanation for the teaching of proportional reasoning in a measurement context (allocated marks 6 out of 6).

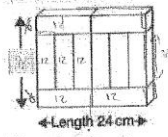
The question illustrated in Figure 5 was given to the 2015 undergraduate cohort. The stimulus material was from the 2014 Year 5 NAPLAN test (ACARA, 2016). The question demands knowledge of proportional reasoning problem solving in the context of measurement. In the response below the pre-service teacher has correctly identified the length

of each block as 12 cm, but failed to see that since three blocks fit into each length, the square ends have side lengths of 4 cm, thus the height of the model is $4+12+4$ is 20 cm. Being unable to undertake this problem solving, the pre-service teacher is unable to describe how he/she might go about teaching the sequence of problem solving. They were awarded $\frac{1}{2}$ mark for identifying measurement as a key concept involved and no marks for PCK.

Question 8

The Year 5 National success rate was 21% on the question below.

35 Meg has blocks with two square faces. She makes this model.



What is the height of the model in centimetres?

centimetres

(a) What are the key concepts involved in this question? (1 marks)

Measurement,

(b) In the space below briefly explain how you would model the correct solution. (5 marks)

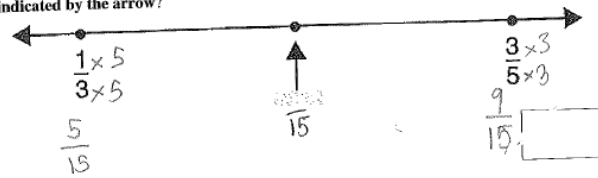
Figure 5. Pre-service teacher response to items assessing pedagogy associated with proportional reasoning.

In the PCK example shown in Figure 6, the pre-service teacher is asked to diagnose student thinking on a fraction naming and renaming question. The content is Year 6 (ACARA, 2016), and the stimulus material was taken from a 2014 NAPALN test. We can see that the children's success rate was on par with guessing probably (eg., 1 in 4) in a multiple choice format. If children had responded with 2 fourths, it is probably an indication of the over generalisation of whole number thinking, that is, they found half way between the numerators (2) and half way between the denominators (3+5) divided by 2 is 4. The pre-service teacher response partially recognises this and was thus awarded a half mark. In explaining how this problem structure might be taught, it was anticipated that the pre-service teacher would teach renaming to a common denominator, and this has been accomplished. We can see that $\frac{1}{3}$ has been renamed to $\frac{5}{15}$ and similarly $\frac{3}{5}$ has been renamed to $\frac{9}{15}$. In fact the pre-service teacher does this via computation and then in part (b) shows how this renaming can be achieved using fractions strips. However, they have not been able to order the fractions, for example, $\frac{5}{15}, \frac{6}{15}, \frac{7}{15}, \frac{8}{15}, \frac{9}{15}$, and subsequently identify $\frac{7}{15}$ as the middle fraction. The pre-service teacher was allocated $2\frac{1}{2}$ out of 5 marks for the partial solution.

Question 13

The question below was asked in the Year 7 NAPLAN test of 2014. The Queensland success rate was 19%. The most common response was $\frac{2}{4}$.

The arrow is half way between the two fractions. What is the value of the fraction indicated by the arrow?



(a) What thinking does this response indicate? ($\frac{2}{4}$) (1 mark)

The student does not see the 3 fractions as parts of whole and has gone from a pattern with numerators and denominators.

(b) Explain how you would teach the solution to this problem and remediate the misconceptions. (5 marks)

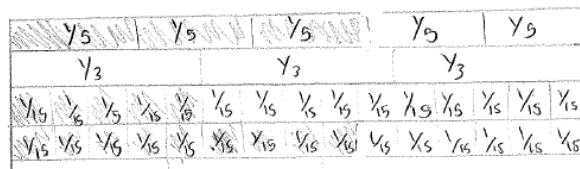


Figure 6. Pre-service teacher's explanation of how to teach finding the midpoint between two fractions.

In 2014 there were 12 questions of this form and in 2015 there were 14. The totalling of marks allocated for each question serves as a guide to the pre-service teacher's PCK.

Results

Starting and ending content knowledge

Students' entry knowledge was assessed using Part A of the Trainee Teacher: Primary Mathematics Audit: Part A (Burghes, 2007). As noted in the Method section, a modified version of this test was used in the post-test of content knowledge. The pre- and post-test summary of scores is presented in Table 3.

Table 3. Pre and Post-test Summaries of Content Knowledge (Total possible score 40)

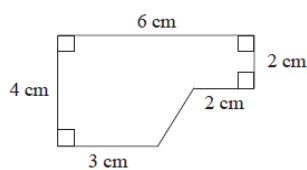
Cohort	Pre-test		Post-test	
	Mean	Std	Mean	Std
Grad dip 2013 (n= 86)	19.752	5.601	na	na
Grad dip 2014 (n=74)	19.675	6.627	25.845	6.557
Undergraduate 2015 (n=179)	19.613	5.078	23.113	6.172

A one-way ANOVA of the pre-test data reported indicates there is no difference between the means of the students in different cohorts ($F = 1.567$, Sig 0.211). It can be seen that in each cohort about half of the students attained less than half marks and about 70% fell between 14 and 26 marks out of 40. By way of reference, Burghes (2011) found that Japanese trainee teachers had a mean of 36.9/40, Russian trainee teachers 30.5/40, Chinese 30.1/40, Hungarian 26.7/40, Finnish, 26.2/40, English 25.5/40, Irish 25.1/40, and Czech Republic 21.5/40. Burghes sampled about 1400 trainee teachers, but did not document the stage of their teacher training.

A closer examination of the pre-test data is revealing. The five questions documented below contain key concepts and processes associated with upper primary mathematics; the success rates are summed in Table 4.

Table 4. *Pre-test: Sample of Items on Trainee Teacher Primary Mathematics Audit*

Sample question	2013 A	2014 B	2015 C
4a) Write each number as a fraction, decimal and percentage.	24.4%	27.1%	16.7%
a) $\frac{3}{8} = \square = \square\%$			
6) What is the value of 5^5 ? $5^5 = \square$	39.6%	34.7%	22.2%
9) If the cost of 3 tickets is \$5.64, how much is the cost of 10 tickets?	49.2%	47.9%	45.6%
10) If \$45 is divided among three friends, Tim, Christine and Andrew in the ratio 3:2:4, how much money will each of them receive?	71.6%	66.6%	60.8%
18) Calculate the area of this shape (not to scale).	27.6%	36.1%	33.3%



Key

A: Graduate diploma pathway 2013

B: Graduate diploma pathway 2014

C: Undergraduate pathway 2015

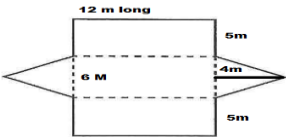
While there is some variation between the cohorts, overall it can be seen that about a fourth could do the division process and convert this decimal into a percentage (minimum standard Year 7 [ACARA, 2016]), about a third understood power notation and could multiply 5 by itself 5 times (the most common error was recording 25--minimum standard Year 7 [ACARA, 2016]), about half could divide \$5.64 by three and multiply this by 10, about two thirds could do a simple ratio problem (minimum standard

Year 7 [ACARA, 2016]), and about a third could find the area of a composite shape.

At the pre-test, overall, less than a third could carry out a relatively simple algebraic substitution. Success on a substitution in the equation to convert degrees C to degrees F that involved fractions and a negative integer ranged from 11% to 23% (minimum standard Year 8 [ACARA, 2016]). Table 5 illustrates the relative improvement on almost identical items that tested basic number concepts and geometric problem solving.

Table 5. *Sample of Items on Modified Trainee Teacher Primary Mathematics Audit with Post-study Success Rates. 2014 Graduate Diploma Pathway Students (n=74) and Undergraduate Pathway 2015 (n=179)*

Post-study sample question	2014	2015
4a) Write each number as a fraction, <u>decimal</u> and <u>percentage</u> . $\frac{5}{9} =$	74.4%	56.3%
6) What is the value of 3^5 ?	76.4%	61.4%
9) If the cost of 5 tickets is \$7.10, how much is the cost of 8 tickets?	78.4%	60.1%
10) If \$84 is divided among three friends, Sam, Luke and David in the ratio 3:4:5, how much money will each of them receive?	83.3%	62.1%
18) Calculate the area of this shape (not to scale).	48.6%	31.8%



With respect to the post-test data, the differences between the Graduate Diploma average scores and the Undergraduate final average scores were significantly different ($F= 9.888$, Sig 0.002). While the average score on the equivalent content test increased by 31.3 % for the Graduate Diploma students, the equivalent increase for the Undergraduate students was 17.8%. On these basic skills with number fluency and simple problem solving with geometry, both cohorts improved. Most improvement was seen in questions where the underlying basics including place value, key operations, and ratio conventions were explicitly taught during the course. For example, on the pre-test between 27% and 17% could convert $\frac{3}{8}$ to a decimal and percentage while at the end of the course the success rate for carrying out the same process for $\frac{5}{9}$ was between 56% and 74%. Still, the fact that between a quarter and almost half of the students could not carry out the basic skills tasks is a cause for concern since this is fundamental upper primary school content. On the questions that were not taught as part of the course content there was sometimes some improvement, but the scores were

still relatively low. For example, the questions on natural, rational, irrational, and integer numbers had low scores as did the algebra substitution questions, identifying similar and congruent shapes, and the simple area calculation (Q. 18) as well as probability questions.

As noted in the statistic quoted above, the improvement of the graduate entry students was considerably greater. It is not the intent of this paper to explore this difference, but sufficient to note that lecture attendance was much greater among the graduate cohorts. This might be indicative of higher levels of commitment from more mature aged students. What is more educationally significant than a difference between different cohorts is the struggles large portions of both cohorts had in understanding upper primary mathematics.

Pedagogical Content Knowledge (PCK)

PCK was assessed by the author-designed tests at the end of the teaching cycle, the structure and analysis of which is described in the Method section. A summary of the overall scores is presented in Table 6.

Table 6. *Means on Pedagogical Content Knowledge Post-Intervention for 2014 Graduate Diploma Pathway Students (n=74) and Undergraduate Pathway Students 2015 (n=179)*

Test	Min	Max	Mean	Std. Dev
Post-pedagogical content knowledge (2014, n=71)	13/72	72/72	47.853	13.524
Post-pedagogical content knowledge (2015, n=173)	9/72	68/72	42.964	12.773

For the 2014 data, Pearson correlation coefficients between starting content and finishing content indicated a strong positive correlation ($r=0.737$, $n=66$, $p=.000$). The correlation between starting content and PCK was moderate ($r=0.549$, $n=63$, $p=.000$) and the correlation between final content and PCK was relatively strong ($r=0.597$, $n=66$, $p=.000$). A strong relationship between starting knowledge and final grade was reflected in the dominance of students who had studied Mathematics B gaining high total marks. In the top quartile (18 out of 72 grades) 15 of these students (83%) reported having studied Mathematics B. Equivalent figures were found in 2013 and earlier years and reported elsewhere (e.g., Author, 2012). In short, as in prior years, students who studied Mathematics B dominated the top marks while students who studied no senior mathematics, or Mathematics A, filled the lower portion of the grade distribution, taking into account the examination and take-home assignment that did not require high levels of personal numeracy to be demonstrated. With respect to the 2015 undergraduate data, the failure of so many students to fill in their student numbers on the entry survey makes pre-post paired comparison on the basis of high school or tertiary study of mathematics subjects unreliable.

However, it is clear that success on earlier mathematics curriculum courses did not prepare them for success on the tests reported in this paper. There was a statistically significant difference between the two post-test means ($F=7.327$, sign 0.007) with the graduate students performing better on an equivalent test.

In terms of specific PCK, the tests focused on assessing the teaching of whole-number numeration and computation as well as fractions and proportional reasoning. The analysis of the success rates on different content domains helps in the future planning of such courses. The breakdown of 2014 mean marks is informative (see Table 7). Each question was marked out of 6.

Table 7. 2014 Graduate Entry Student Results on Pedagogical Content Knowledge ($n=71$)

Question	Concept	Mark /6	SD
1	Assessing and teaching naming numbers	4.804	1.343
2	Assessing teaching naming numbers	4.819	1.295
3	Assessing subtraction errors and remediation	4.884	1.505
4	Assessing multiplication errors and remediation	4.391	1.364
5	Assessing division errors and remediation	3.659	1.695
6	Teaching fraction naming	4.617	1.689
7	Teaching fraction addition	4.261	1.633
8	Teaching proportional reasoning in measurement context	4.906	2.009
9	Teaching proportional reasoning in scale context	1.964	2.511
10	Teaching proportional reasoning in data context	2.862	2.625
11	Teaching percent problem solving in data context	4.289	2.415
12	Assessing and teaching decimal computation (multiplication)	2.377	2.413

The data indicate that most students were relatively successful in assessing student errors in whole-number contexts and describing teaching to remediate these. They were less successful in describing the teaching of proportional reasoning. The scale question (Q. 9) was described in Figure 4 in the Method section.

In 2015 the test structure was similar and the results similar, as illustrated in Table 8.

Table 8. 2015 Undergraduate Entry Student Results on Pedagogical Content Knowledge (each marked out of 6 except Q14 which was marked out of 8)

Q	Concept	Mark /6	SD
1	Naming numbers and assessment	5.086	1.008
2	Assessing and teaching addition	5.109	1.177
	Assessing and teaching subtraction	5.138	1.345
4	Assessing and teaching multiplication	4.434	1.627
5	Assessing and teaching division errors	3.569	1.971
6	Teaching fraction problem solving Year 5	4.225	2.238
7	Teaching fraction renaming Year 5	3.306	2.411
8	Teaching proportional reasoning in measurement context Year 3 to 5	3.584	2.645
9	Teaching proportional reasoning in measurement context Year 5	2.633	2.541
10	Teaching fraction problem solving	1.662	2.049
11	Assessing and teaching decimal multiplication	2.841	2.319
12	Teaching percentage problem solving	3.636	2.664
13	Assessing and teaching fraction renaming	3.147	2.536
14	Teaching problem solving in early algebra contexts /8	2.942/8	2.641

As with the graduate intake data, the undergraduate data indicate that most students experienced success in diagnosing and then describing the remediation teaching of early number including the four basic computations. Students struggled to do and to describe the teaching of upper primary number associated with fractions and proportional reasoning as indicated by scores less than or about 3/6. The most difficult question in 2015 was to list the concepts then explain the teaching of a Year 7 NAPLAN (MYCEETA, 2014) question. The stimulus was: " $\frac{1}{3}$ of the birds were cockatoos and $\frac{1}{4}$ were pigeons. He saw 6 more cockatoos than pigeons. How many birds did David see?" The Queensland Year 7 student success rate was 9% and the average mark for the 4th year undergraduate students was 1.662/6 or an average less than one-third the possible mark. While almost all students could name the concepts involved (fraction renaming, fraction computation and multiplication) few (about a quarter) could answer the question correctly and were thus unable to explain how questions of this structure might be taught. In addition, Question 13 asked the students to describe the teaching of a fraction half way between $\frac{1}{3}$ and $\frac{3}{5}$ on a displayed number line; 60% of the undergraduate students were unable to carry out

the process of renaming the fractions to a common denominator and placing $\frac{7}{15}$ as half way between $\frac{5}{15}$ and $\frac{9}{15}$. On a more positive note, about half of the 2015 students could calculate 0.3 by 0.4, and show models to teach this process. It should be noted that on the pre-test only 14% of the 2015 intake could calculate the area of a computer chip 0.2 mm by 0.3mm. The structure of these questions, and how to teach them, was explicitly taught in the course.

Overall, the proportion of students in 2014 who did not gain 50% of marks in the PCK section of the paper was 17.8% in 2014 and 31.1% in 2015. In this regard the lesser gain on the PCK section of the assessment mirrors the lesser gain in basic content and the lower basic content mark on intake to the course.

While the teaching of the concepts associated with fractions and proportional reasoning across strands had been covered in lectures and workshops, it is evident that many students in each intake method remained unclear on the concepts themselves and could not explain how to teach them in a reasonably coherent manner. Top candidates from both programs could effectively diagnose common student misconceptions and clearly plan teaching approaches to remediate misconceptions identified, in effect demonstrating reasonable teacher capacity and meeting key discipline-specific AITSL standards with respect to the teaching of mathematics.

Discussion and reflections

In this section the findings are summarised and reflected upon. The first aim was to investigate the pre-service teachers' mathematical background and mathematics content knowledge. The survey data indicate that most students had completed low-level mathematics subjects at high school (e.g., Mathematics A or no senior mathematics) and this was especially the case among the undergraduate intake. The starting content knowledge was assessed with a basic content test (Burghes, 2007). The data suggests that many students in either entry pathway could not do basic primary or lower secondary school mathematics, with over half failing or nearly failing the entry test. The results compare unfavourably with other nations including trainee teachers in China, Russia, Hungary, Finland, England, and Ireland (Burghes, 2011). Inability to carry out accurate division and convert a decimal to a percentage, or to carry out basic whole-number problem solving, prior and post learning, was cause for concern. Upon entry over half of the students found any mathematics associated with fractions, proportional reasoning, and algebra challenging. This finding is consistent with earlier research on teacher knowledge (Norton, 2011; Norton, 2012) and a cause for concern in a number of Western nations studied by Burghes (2011).

In the undergraduate pathway the majority of students reported grades of credit and above but very few of these students could be considered competent in upper primary mathematics after having

completed Mathematics 1 and 2. The concerns raised by authors cited in the literature review with respect to the application of misguided or misapplied theories of learning (e.g., Farkota; 2005; Hattie, 2009; Kirschner et al., 2006; Meyers, 2012; Muller, 2000) may account for this finding. Unfortunately the low mathematics knowledge of students graduating from Mathematics 1 and 2 lends evidence to the Australian Government Department of Education (2014) statement:

Scepticism is building in relation to so called 21st century thinking and skills, a movement which has permeated some of the educational establishment in some countries. Essentially this movement focuses primarily on competencies to the neglect of knowledge, and tries to minimise learning of content based in disciplines, preferring generalised attempts at inter- disciplinary. (p. 34)

The more generic competencies based approach to teaching pre-service teachers how to teach mathematics assumes that some specific pedagogy could be modelled and students could apply the approaches to different mathematical content. For example, multiplicative thinking associated with whole numbers could be taught with bundle sticks, base ten materials, and area models, and pre-service teachers could transfer the pedagogical principles to teaching fractions. Yet the data contradict this assumption. The lack of focus on mathematical understanding of upper primary mathematics may have been compounded by methods of course delivery including limited lecture and workshop time and multimodal delivery outlined in the Method section.

The second aim was to document the intervention and to evaluate its success under the constraints in which it was enacted. The description of the intervention shows attempts to account for students' mathematical content while developing their capacity to analyse student thinking and plan to teach from this basis. Consistent with constructivist inquiry (Denzin & Lincoln, 1994) it is the responsibility of the reader to determine if the model has transferability to their situations.

With respect to the improvement in content knowledge there was greatest gain in content that was a focus of the courses, that is, mathematics that was explicitly and repeatedly taught. Still, it is a cause for concern that very significant portions of students could not carry out upper primary mathematics associated with fractions and proportion. The positive correlation between entry content, exit content, and ability to explain the teaching (PCK), particularly the teaching of upper primary content, is consistent with earlier findings (Author, 2012).

As noted in the literature review there is a broad consensus in the literature that effective teaching of mathematics requires the teacher to know the mathematics irrespective of what pedagogical approach is undertaken (e.g., Ball, Hill, & Bass, 2005; Dinham, 2013; Goulding, Rowland, & Barber, 2002; Greaves, 2014; Hattie, 2003, 2009; Masters, 2009; Shulman, 1987, 1999; Silverman & Thompson, 2008; Zhang & Stephens, 2013). Further, thorough knowledge of the content is anticipated by teaching standards documents

(e.g., AITSL, 2014; QCT, 2012) and recommended by TEMAG (2014). Thus, the findings that even after course completion very significant proportions of each cohort struggled with critical primary mathematics is a cause to reflect upon.

The third aim of the paper was to assess the pre-service teachers' PCK. Following tuition, most students excelled in explaining how to teach whole-number computation and problem solving, but half or more struggled to do and to explain the teaching of upper primary mathematics associated with fractions. It was clear that whole-number pedagogy was not transferable to fraction pedagogy. For example, teaching whole-number multiplication (see Table 7 and 8) was well explained, but most could not explain how to teach decimal or fraction multiplication. The finding supports the U.S. Department of Education (2008, p, xxi) statement: "It is self-evident that teachers cannot teach what they do not know."

Local institutional reform

At the institutional level, the data have prompted a review of the integration of content and pedagogy in the two postgraduate master's mathematics curriculum courses offered from 2015 onwards. At the undergraduate level the data have prompted a review of the preparatory role of earlier curriculum courses and the consideration of offering a basic mathematics content course prior to commencement of mathematics curriculum courses. The plan by the federal government to test basic content knowledge of prospective teachers prior to registration (Maiden, 2015) adds urgency to this internal reform. The challenge will be to teach content in a way that best supports effective pedagogy. To teach content in isolation from pedagogy may lead to the replication of teaching that the students experienced when they were at school, and this is unlikely to be the most effective way to break the cycle of ineffective teaching practices.

Wider teacher preparation reform

While this is a case study, with data drawn from one teacher training institution, the data is likely to have wider implications. The institution is a major teacher training institution and has met all the standards required of the regulating governing body (QCT) in order to have its programs and courses registered. The students who enrol in this institution are under similar constraints to those that exist at other institutions in this state and highly likely more broadly across other Australian states. We do not know just how representative these data are of the broader Australian primary teacher education student population, since as noted by TEMAG (2014), little transparency of practice exists across institutions. This paper helps to provide some illumination on the process of teacher preparation by presenting a model of how one academic attempted to account for content and PCK and how to assess this.

The strong correlation between entry mathematics knowledge and exit teaching knowledge offers one solution. That is, entrance to primary teacher education should be on the proviso of good grades in senior

Mathematics B or its equivalent. Mathematics B is intermediate senior mathematics that contains calculus and statistics to hypothesis testing. Mathematics B is reasonably abstract, as distinct from Mathematics A which contains little algebra or demand for abstractions. Earlier research by the author, along with the data presented here, indicates that while successful completion of Mathematics B was a good predictor of success on primary mathematics tests and a demonstration of primary mathematics pedagogy, the completion of Mathematics A was not. Recent entry requirements to insist that entry to primary teaching be on the basis of passing of Mathematics A or its equivalent and study of one senior science subject are likely to be a low hurdle that will have little impact on overall standards. Unfortunately, only about 20% of the Queensland and Australian cohorts enrol in Mathematics B or equivalent forms. This suggests that demanding Mathematics B or equivalent as a prerequisite will likely reduce intakes into primary teacher education such that the supply of new teachers may not satisfy industry needs as older teachers retire or younger teachers leave the profession.

The exit data on content and PCK of both graduate and undergraduate students suggest that about a third of each cohort graduates are reasonably proficient in upper primary mathematics to the extent that they might be described as classroom ready. The data indicate that it might be time to re-evaluate the wisdom of the generalist teacher education approach and to do as some of the East Asian nations do and have primary content specialists for mathematics. In this way most of the allocated training time is directly associated with learning to teach mathematics. Further, primary mathematics content specialist teachers could focus most of their subsequent professional development on becoming more effective teachers of mathematics, a situation that Li et al. (2008) consider very powerful in reforming Chinese primary mathematics teaching.

It is worth noting that most students could learn to succeed in explaining the teaching of whole-number numeration and computation. This indicates that it is worth exploring the option of primary teacher accreditation from Prep to Year 3, which is largely restricted to whole-number computation and related problems. The fact that about a third of students demonstrated a good knowledge of fractions and proportion and could transfer this to teaching problem solving in unfamiliar contexts at the end of the courses suggests that these pre-service teachers could aspire to a second level of accreditation enabling them to teach primary Years 4 to 6.

If relatively indiscriminate enrolment policies and the generalist primary teacher model persist, there is an argument to extend the number and duration of mathematics-specific courses. Primary teachers are expected to teach mathematics for between one fifth and one quarter of the school day, but in the undergraduate pathway they spend 3 courses out of 32 (9.4%) of their tertiary study learning to teach this subject. In the graduate pathway this has been one eighth of tertiary learning time. It is noteworthy that subjects such as English, history, and geography have a common base in the reading, analysis, and construction of text based on letters and sentences, while mathematics is a distinct, largely unsupported difficult discipline based on numbers, algorithms, algebraic conventions, and geometry

structures. Further consideration needs to be given to the focus of such courses to account for pre-service teachers' content and PCK.

The data contained in this paper are a cause for concern for the study institution and likely more broadly. More positively, the data suggest directions for reform.

Acknowledgements

I would like to thank Dinham (2013) for inspiring the title of this paper.

References

- An, S. (2004). Capturing the way of teaching: The learning-questioning and learning-reviewing instructional model. In L. Fan, N-Y Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 462-502). New Jersey: World Scientific.
- Australian Curriculum and Reporting Authority. (2016). Australian Curriculum F-10 Mathematics. Retrieved from <http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1>
- Australian Government Department of Education. (2014). *Review of the Australian Curriculum Final Report*. Retrieved from <https://docs.education.gov.au/documents/review-australian-curriculum-final-report>
- Australian Institute for Teaching and School Leadership. (2014). *Australian Professional Standards for Teachers*. Retrieved from <http://www.aitsl.edu.au/australian-professional-standards-for-teachers/standards/list>
- Ball, D., Hill, H., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, Fall. Retrieved from www.aft.org/pubs-reports/american_educator/fall05
- Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59, 389-407.
- Booker, G., Bond, D., Sparrow, L., & Swan, P. (2010). *Teaching primary Mathematics*. Frenchs Forest: Pearson Hall.
- Brown, G., & Quinn, G. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher*, 62(4), 28-40.
- Burghes, D. (2007). *International comparative study in Mathematics training*. CfBT Education Trust. Retrieved from <http://www.cimt.plymouth.ac.uk/papers/icsmtt.pdf>
- Burghes, D. (2011). *International comparative study in Mathematics training: Recommendations for initial teacher training in England*. CfBT Education Trust. Retrieved from

<https://www.nationalstemcentre.org.uk/res/documents/page/International%20comparative%20study%20in%20mathematics%20teacher%20training.pdf>

- Cai, J., & Cifarelli, V. (2004). Thinking mathematically by Chinese learners: A cross-national comparative perspective. In L. Fan, N-Y Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 71-106). New Jersey: World Scientific.
- Creswell, J. (2005). *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research*. New Jersey, Pearson Merrill Prentice Hall.
- Darling-Hammond, L., Berry, B., & Thoreson, A. (2001). Does teacher certification matter? Evaluating the evidence. *Educational Evaluation and Policy Analysis*, 23(1), 57-77.
- Denzin, N., & Lincoln, Y. (1994). Part V the art of interpretation, evaluation, and presentation. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 479-483). Thousand Oaks: Sage Publications.
- Department of Education and Early Childhood Development (2009). *Key characteristics of effective numeracy teaching P-6*. Melbourne: Author.
- Dinham, S. (2013). The quality teaching movement in Australia encounters difficult terrain: A personal perspective. *Australian Journal of Education*, 57(2), 91-106.
- Education Bureau: The Government of Hong Kong Special Administrative Region. (2000). *Primary Mathematics Curriculum 2000*. Retrieved from <http://www.edb.gov.hk/en/curriculum-development/kla/ma/curr/pri-math-2000.html>
- Farkota, R. (2005). Basic maths: The brutal reality. *Online opinion*. Retrieved from <http://www.onlineopinion.com.au/view.asp?article=3785>
- Ginsburg, A., Leinwand, S., Anstrom, T. & Pollock, E. (2005). *What the United States can learn from Singapore's world-class mathematics system: And what Singapore can learn from the United States*. Washington: American Institutes for Research.
- Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal*, 28(5), 689-704.
- Greaves, A. (2014). *Supply of specialist subject teachers in secondary schools: Report to Parliament 2: 2013-14*. Brisbane: Queensland Audit Office.
- Hattie, J. (2003, October). *Teachers make a difference: What is the research evidence?* Paper presented at ACER Research Conference, Building teacher quality: What does the research tell us? Melbourne.
- Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. London. Routledge.
- Huang, R., & Leung, K. (2004). Cracking the paradox of Chinese learners: Looking into mathematics classrooms in Hong Kong and Shanghai. In L. Fan, N-Y Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 348-381). New Jersey: World Scientific.
- Jensen, B. (2012). *Catching up: Learning from the best school systems in East Asia*. Melbourne: Grattan Institute. Retrieved from

http://grattan.edu.au/static/files/assets/23afab6b/129_report_learning_from_the_best_main.pdf

- Kirschner, P., Sweller, J., & Clark, R. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist*, 41(2), 75-86.
- Lafayette De Mente, B. (2009). *The Chinese mind*. Tokyo: Tuttle.
- Lee, W. O. (1996). The cultural context of Chinese learners: Concepts of learning in the Confucian tradition. In D. A. Watkins & J. B. Briggs (Eds.), *The Chinese learner* (pp. 45-67). Hong Kong: Comparative Education Research Centre.
- Li, J. (2004). Chinese cultural model of learning. In L. Fan, N-Y Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 124-156). New Jersey: World Scientific.
- Li, Y., Zhao, D., Huong, R., & Ma, Y. (2008). Mathematics preparation of elementary teachers in China: Changes and issues. *Journal of Mathematics Teacher Education*, 11, 417-430.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Maiden, S. (2015, 27 June). WA teachers to get new exam on literacy and numeracy. Retrieved from <http://www.perthnow.com.au/news/wa-teachers-to-get-new-exam-on-literacy-and-numeracy/story-fnhocwho-1227417625429>
- Masters, G. (2009). *A shared challenge: Improving literacy, numeracy and science learning in Queensland primary schools*. Camberwell, Vic.: Australian Council for Educational Research. Retrieved from <http://education.qld.gov.au/mastersreview/>
- Meyers, D. (2012). *Australian universities: A portrait of decline*. Retrieved from www.australianuniversities.id.au
- Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA). (2008-2014). *National assessment program, literacy and numeracy: Numeracy*. Carlton, Victoria: Curriculum Corporation.
- Ministry of Education, Singapore. (2007). *Mathematics Syllabus Singapore*. Retrieved from <https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/2007-mathematics-%28primary%29-syllabus.pdf>
- Muller, J. (2000). *Reclaiming knowledge: Social theory, curriculum and education policy*. London and New York: Routledge Falmer.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston VA, NCTM.
- Norton, S. (2012). Prior study of mathematics as a predictor of pre-service teachers' success on mathematics content and pedagogical content knowledge tests. *Mathematics Teacher Education and Development*, 14(1), 2-26.
-

- Norton, S. (2011). How deeply and how well? How ready to teach mathematics after a one year program? *Mathematics Teacher Education and Development*, 12(1), 65-82.
- NSW Smarter Schools National Partnerships. (2010). *Smarter schools national partnerships on improving teacher quality*. Retrieved from <http://www.nationalpartnerships.nsw.edu.au/resources/documents/ITQ-HATDETGuidelines.pdf>
- Owen, E., & Sweller, J. (1989). Should problem solving be used as a learning device in mathematics? *Journal for Research in Mathematics Education*, 20(3), 322-328.
- Queensland College of Teachers (QCT). (2012). *Australian professional standards for teachers*. Retrieved from <http://www.qct.edu.au/Standards/>
- Shulman, L. (1987). Knowledge and teaching: Foundations of the reform. *Harvard Educational Review*, 57(1), 1-22.
- Shulman, L. (1999). Knowledge and teaching: Foundations of the new reform. In J. Leach & B. Moon (Eds), *Learners and pedagogy* (pp. 61-77). London: Sage.
- Silverman, J., & Thompson, P. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, 11, 499-511.
- Strauss, S. & Corbin, J. (1990). *Basics of Qualitative Research: Grounded Theory Procedures and Techniques*. London, Sage.
- Teacher Education Ministerial Advisory Group. (2014). *Action Now! Classroom Ready Teachers*. Retrieved from http://docs.education.gov.au/system/files/doc/other/action_now_classroom_ready_teachers_print.pdf
- Thomson, S., Hillman, K., Wernert, N., Schmid, M., Buckley, S. & Munene, A. (2012). *Highlights from the TIMSS AND PIRLS 2011 from Australia's perspective*. Melbourne: Australian Council for Educational Research.
- Thomson, S., Wernert, N., Underwood, C., & Nicholas, M. (2007). *TIMSS 07: Taking a closer look at mathematics and science in Australia*. Melbourne: ACER. Retrieved from http://www.acer.edu.au/documents/TIMSS_2007-AustraliaFullReport.pdf
- US Department of Education. (2008). *Success: The final report of the National Mathematics Advisory Panel*. Retrieved from <http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>
- Van de Walle, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally*. Boston: Allyn & Bacon.
- Zhang, D., Li, S., & Tang, R. (2004). The "two basics": Mathematics teaching and learning in mainland China. In L. Fan, N-Y Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 189-207). New Jersey: World Scientific.
- Zhang, Q., & Stephens, M. (2013). Utilising a construct of teacher capacity to examine national curriculum reform in mathematics. *Mathematics Education Research Journal*, 25, 481-502.
-

