Development of Prospective Secondary Mathematics Teachers' Noticing Skills in School Experience Course

Berna Tataroğlu Taşdan
Dokuz Eylul University
berna.tataroglu@deu.edu.tr

This study investigates the development of prospective secondary mathematics teachers' (PSMTs') noticing skills in a course conducted at the last year of their teacher education program. PSMTs watched and analysed different instructional videos, including teaching videos of an unknown teacher, their mentor teacher, and their own. The data were collected from five PSMTs through reflection papers and group discussions about what and how they noticed in the videos. The results showed that the PSMTs had different levels of noticing at the beginning of the course and developed their noticing skills over time through the research. It was found that PSMTs' noticing skills differed across different video types. This study revealed the importance of experience in real classroom environment for the development of PSMTs' noticing skills. Suggestions were made based on the study findings.

Introduction

Research in mathematics education has long investigated methods of understanding and supporting the development of prospective teachers' practice. Teacher educators are aware of the challenge of directing prospective teachers' reflections toward different features of mathematics education (Castro, Pino-Fan, & Velasquez-Echavarria, 2018). Current visions of high-quality instruction involve teachers' abilities to implement in-the-moment, pedagogical decision making that is responsive to and builds on students' thinking (Diamond, Kalinec-Craig, & Shih, 2018). Therefore, development of prospective teachers' ability to understand and interpret students' thinking and to decide how to use this knowledge in the process of instruction, referred to as teachers' noticing, is a key part of teacher education programs. Efforts to help teachers and prospective teachers learn to notice are important, because this expertise can support them not only in teaching more effectively, but also in more effectively using their experiences to guide their future actions (Mason, 2002).

Noticing is also considered as an essential ability in mathematics education reforms, as it promotes student-centred instruction and foster mathematical proficiency (National Council of Teachers of Mathematics [NCTM], 2000, 2014). NCTM (2014) stated, "effective teaching of mathematics uses evidence of student thinking to assess progress towards mathematical understanding and to adjust instruction continually in ways that support and extend learning" (p. 53). In other words, when teachers attend closely to what students think, how their work represents about their understanding, and how their mathematical understanding is developing, they create discourse-rich environments where students share, discuss, and reason together about mathematical ideas (van Es, Cashen, Barnhart, & Auger, 2017).

In this study, I investigated the development of prospective secondary mathematics teachers' (PSMTs') noticing skills in a course conducted during the first semester at the last year of their teacher education program. Previous research has mostly examined prospective teachers' noticing by using their own videos, other teachers' or prospective teachers' videos, or both (e.g., Lee, 2019; Mitchell & Marin, 2015; Roller, 2016; Roth McDuffie et al, 2014; Sherin & van Es, 2005; Star & Strickland, 2008; Stockero, Rupnow, & Pascoe, 2017; van

Es, 2011). Most of these studies addressed the potential of videos or the impact of the course on improving teachers' and prospective teachers' noticing skills and discussed the development of noticing skill connecting with participants' experiences. They focused on the analysis of others or own videos without comparing the effects of the teacher who was the main actor in the video. However, this study is initiated on the assumption that the PSMTs' noticing may be affected by whether they know the person in the video or have experience with her/his. To understand this effect, this study focused on the development of PSMTs' noticing skills when they watched and analysed different instructional videos, including videos of an unknown teacher, their mentor teachers, and their own teaching, and on how PSMTs' noticing skills differed across analysing these video types. The study offers different results because it compares the effects of the different types of videos on noticing. The purpose of the study was to investigate the development of PSMTs' noticing skills. The following research questions guided the study:

1. What and how do PSMTs notice when they analyse an unknown teacher's, their mentor teachers, and their own teaching videos?
2. How do PSMTs' noticing skills develop over a course when they analyse different instructional videos?

Literature Review

Teacher Noticing

Although researchers have conceptualized noticing differently, most of them agree on two components: (1) attending to classroom events like student thinking, pedagogical actions, interactions, reasoning, decisions and (2) making sense of these events. While some researchers have focused exclusively on the attending component (Jacobs, Philipp, & Sherin, 2018), some have conceived of noticing as involving identifying, interpreting the important aspects in a mathematics classroom, and decision-making about responding. For example, Star and Strickland (2008) examined what the teachers attend to and what they miss. They considered attending as a prerequisite step for noticing, because if the teachers cannot identify important events, they would have difficulty in making connections between what they notice and broader principles of teaching and learning (Star & Strickland, 2008). Van Es and Sherin (2002; p. 573) have proposed that the skill of noticing for teaching consists of three main aspects: (a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning. While van Es and Sherin (2002) conceptualized noticing by focusing events on a classroom environment totally, Jacobs, Lamb and Philipp (2010) have focused more specifically on student mathematical thinking and have defined professional noticing of children's mathematical thinking. They proposed three interconnected skills of noticing: a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond on the basis of children's understanding. Attending involves teachers identifying the mathematical details in children's strategies. Interpreting is using these strategies as evidence to interpret students' understanding and development. The last skill is based on prior skills and includes teachers' reasoning on deciding how to respond students.

Van Es (2011) focused on what and how teachers notice to capture the nature of the teachers noticing in a video club study by developing a framework for learning to notice student thinking. The first dimension of the framework indicates both whom (whole class,

students as a group, particular students, the teacher) the teachers notice in the video clip and the topic (pedagogical strategies, mathematical thinking, the classroom climate) of their analysis. The second dimension of the framework refers to how teachers analyse what they notice, including both their analytic stances and levels of depth. Analytic stance refers to whether the teachers describe, evaluate or interpret what they observe. The depth of analysis refers to whether the teachers explain their thinking by providing few details or ground their comments in evidence and elaborate on their analyses. The researcher also created a developmental trajectory from Level 1 to Level 4 (Baseline, Mixed, Focused, Extended) to capture the development of teachers' noticing through the mentioned components (van Es, 2011).

Developing Teacher Noticing

One of the factors that affect noticing skill development is teaching experience. Many studies indicate that more experience provides better noticing skills (Jacobs et al., 2010; Roller, 2016; Santagata, Zannoni, & Stigler 2007; Sherin & van Es, 2005; Star & Strickland, 2008). Less experienced teachers have difficulty focusing on students' actions, as they tend to view lectures as chronological but disconnected series of events, and they are not particularly mindful of content issues (Star & Strickland, 2008). Thus, prospective teachers' noticing skills are lower than those of more experienced teachers (Jacobs et al., 2010; Sherin & van Es, 2005; Star & Strickland, 2008; van Es & Sherin, 2002). Barnhart and van Es (2015) stated that without structured support, prospective teachers' analyses tend to focus on the teacher's actions and behaviours rather than student thinking, learning, and sense-making, and they tend to be judgmental and lack evidential support and coherence. Mitchell and Marin (2015) found that prospective teachers could develop the ability to notice across a teacher education course, if their learning were structured in such a way that it allowed for and supported opportunities to notice.

Noticing has also been described as a component of expert practice that can be learned and improved through support (Jacobs et al., 2010; Jacobs, Philipp & Sherin, 2018). Studies have investigated various strategies for developing teachers' noticing skills through a course (e.g., Diamond et al. 2018; Fisher, Thomas, Schack, Jong & Tassel, 2018; Star & Strickland, 2008; Roller, 2016), professional development programs (e.g., Goldsmith & Seago, 2011; Jacobs et al., 2010), video club interventions (e.g., Barnhart & van Es, 2015; Mitchell & Marin, 2015; Schack et al., 2013; Sherin & van Es, 2005, 2009; Stockero et al., 2017; van Es, 2011; Walkoe, Sherin, & Elby, 2020), lesson studies (e.g., Güner & Akyüz, 2017; Karlsen & Helgevold, 2019; Lee & Choy, 2017) and learning trajectories (e.g., Ivars, Fernández, & Llinares, 2020; Fernández, Sánchez-Matamoros, Valls, & Callejo, 2018). For example, Star and Strickland (2008) investigated the impact of a one-semester teacher education course on prospective teachers' ability to notice classroom events; their course included specific activities that explicitly focused on improving teachers' ability to notice. They concluded that prospective teachers showed significant improvement in their ability to notice classroom events as a result of the course. Roller (2016) examined PSMTs' noticing during a mathematics methods course, where they viewed videos of their own co-teaching in a microteaching setting with peers. In the study, participants used an observation tool to document their observations while re-watching their video and then identified and ranked their top three observations. The results of the study showed that the participants' ranked observations were more likely to attend to students and had a strong focus on mathematics and student learning. Although PSMTs in the study prioritized important moments in their

teaching, they did not implement these observations in practical ways for improving their teaching practice.

Fisher and her colleagues (2018) examined prospective elementary teachers' professional noticing skills as they participated in a video based professional noticing module. This module, titled *Noticing Numeracy Now*, focused on professional noticing of children's conceptual development in whole number and arithmetic reasoning. Their findings indicated that prospective teachers could develop professional noticing skills (attending, interpreting, deciding) as a result of their participation in a video-based module. They also suggested that the experiences that incorporated these skills provided prospective teachers viable and appropriate pedagogies to help them to learn responsive teaching.

These studies have supported the idea that it is possible and essential for PSMTs to gain and develop noticing skills by through explicit instruction in teacher education programs before the PSMTs begin to teach in real classrooms.

Using Videos

One way to develop noticing skills is using videos in teacher preparation programs (Sherin, 2004; Star & Strickland, 2008). Video allows one to enter the world of the classroom without having to be in the position of teaching in-the-moment (Sherin, 2004, p. 13). Sherin and van Es (2005) explained the advantage of using the videos as providing an access to classroom interactions that is not possible during the act of teaching itself. Since videos offer a permanent record of classroom interaction, teachers do not have to rely solely on their memory of what happened. Instead, they can watch a video multiple times at any time and examine what took place from different perspectives (Sherin & van Es, 2005).

Use of videos help both experienced and prospective teachers improve their abilities to notice classroom events (van Es & Sherin, 2002) and presents an opportunity to focus explicitly on pre-service teachers' ability to notice in teacher education programs (Star & Strickland, 2008). Various studies have investigated the role video plays in guiding teachers or prospective teachers to notice classroom events, especially work of Sherin and her colleagues (e.g., Sherin, 2004; Sherin & Han, 2004; Sherin & van Es, 2005, 2009). These studies examined the potential of video to support teacher learning in the context of video clubs. In a video club, a group of teachers meets to watch and discuss excerpts of videos from each other's classrooms (Sherin & van Es, 2009). Sherin and van Es (2005) focused on four middle-school mathematics teachers' and six prospective teachers' noticing in a video club study that used a video analysis support tool for the analysis of the lessons. They found that all participants showed changes in what they noticed and in how they interpreted the events in the classrooms.

As with Sherin and colleagues, other research on video use in teacher education has been carried out by asking participants to watch videos of their own or other teachers or their peers. Borko et al. (2008) pointed out that, in contrast to video from unknown teachers' classrooms, video from teachers' own classrooms situates their exploration of teaching and learning in a more familiar, and potentially more motivating environment. On the other hand, Star and Strickland (2008) highlighted the benefits of watching videos of other teachers' classrooms for prospective teachers, especially as it provides field experience. Considering these claims, it is possible to suggest that prospective teachers should be provided opportunities to watch and evaluate different instructional videos of their own and other teachers in teacher education programs.

Methods

An instrumental case study (Creswell, 2007) was employed in this study, as it aimed to investigate the development of PSMTs' noticing skills through a field experience course, using a bounded case of five PSMTs.

Research Context

This study was conducted with five PSMTs at a university in the west of Turkey through a School Experience course that focused on the PSMTs' field experiences. The content of this 14-week course was designed to scaffold PSMTs' noticing. The author was the instructor of the course and facilitated all the activities during the course.

In their four-year teacher education program, PSMTs take many courses focusing on mathematics, including Calculus, Abstract Mathematics, Analytical Geometry, and Linear Algebra; courses on pedagogy such as Developmental Psychology, Learning and Teaching Theories and Approaches, and Program Development and Teaching; and courses on mathematical knowledge for teaching such as Teaching Methods, Instructional Technologies and Material Design, and Mathematics Curriculum. When they enter the first semester of the last year, they experience a real classroom environment through the School Experience course. They then practice teaching in schools through the Teaching Practice course in the last semester of their teacher education program. In the context of the School Experience course, PSMTs go to secondary schools to observe their mentor teacher's lessons for four lesson hours (each 40 minutes) per week. The School Experience course aims to develop PSMTs' recognition of educational activities in classroom and school by observation; and give them opportunities to plan and conduct in-class teaching practice. Groups of four or five PSMTs are assigned to each mentor teacher, appointed by national education officials. In this study, the PSMTs who were the participants of the research were assigned to one mentor teacher in a group of five people. PSMTs observed their mentor teacher's lessons in groups of two or three people per week throughout the semester. Each PSMT was also expected to teach two hours of lessons during the semester. PSMTs met with the researcher once a week in the faculty. In this time, theoretical and practical lectures were given that scaffolded PSMTs' noticing skills by utilizing videos.

Participants

The participants were five out of twenty PSMTs enrolled in the School Experience course. They were purposefully selected for their situation to video-record their mentor teacher's lesson. Four were females and one was male. Pseudonyms were used for the participants: Elvan, Selen, Gülce, Irem and Yavuz. Since they observed their mentor teacher's lessons in groups of two or three, they sometimes worked as two groups during the research. Elvan and Selen were in the first group; Gülce, Irem, and Yavuz were in the other group. All PSMTs had teaching experience with teaching mathematics individually as private tutors, but they had no teaching experience in a real classroom setting. Therefore, the teaching experience in this course was their first experience in a real classroom setting.

Data Collection

The data collection process was conducted in three stages; the participants analysed an unknown teacher's, their mentor teacher's and their own lessons, respectively (see Figure 1). Each stage is described in detail below.

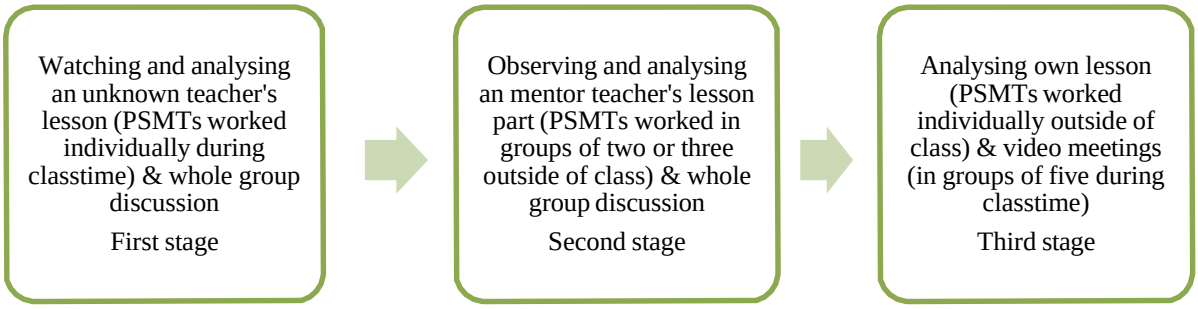

Figure 1. Stages of data collection process.

First stage. At the beginning of the course, in order to determine the pre-existing noticing skills of PSMTs, they watched a 40-minutes lesson video taught by an unknown mathematics teacher teaching the concept of function and were asked to write the events they noticed in the video. Their papers were collected, then the events in the video were discussed as a whole class. During the discussion, I used general prompts such as, "What did you notice?" to stimulate the discussion and to accustom the PSMTs to notice noteworthy events. This group discussion was recorded with a video camera. Although some researchers (e.g., Star & Strickland, 2008; Ulusoy, 2020) suggest use of edited or rearranged videos to provide more focus on teaching and learning, Stockero and colleagues (2017) argue that noticing skill can be improved by using unedited videos of a full class. Since the goal was to examine PSMTs' noticing skills without any guidance, as Stockero and colleagues (2017) suggested, the 40-minutes lesson video included typical classroom events was watched completely without any edit. The video was selected from the researcher's archive by permission of the teacher in the video, meeting ethical considerations. Another factor considered in the selection of the video was the concept taught. The function concept is a suitable mathematical concept, as it reveals different student thinking and misconceptions, and is a basic concept for advanced subjects like limits, derivation, and trigonometry in the Turkish mathematics curriculum.

Second stage. During the course, PSMTs were asked to observe a part of their mentor teachers' lesson and record it via a camera. They worked as groups and recorded roughly 15-minute segments of their mentor teacher's lesson. After the observations, they transcribed and then analysed the lesson segment together in their groups outside of class time. They described what they noticed in their reflection papers and submitted them to the instructor. Then, in the whole group discussion, each group guided their friends through these videos with the transcripts, analysed the selected lesson segment, and discussed noteworthy events in the video. Their peers in the class interpreted and discussed the events in the video. With the instructor's guidance, the group members refrained from commenting unless there was a point to explain so as to not affect their friends' ideas. They were asked to share their comments at the end of the discussion. Other PSMTs who were not the research participants also took observation notes during their mentor teachers' lessons. These PSMTs also shared their observations in the course with illustrations from the dialogues. After the presentations, all the PSMTs explained what they noticed in observed lesson segments. This group discussion was recorded with a video camera.

Third stage. Thirdly, PSMTs began to prepare their two-hour lesson plans for their teaching practice. They had experience in lesson plan preparation from previous courses in teacher education program, so they could prepare their plans as they pleased. They revised

their plans, considering the instructor's feedback on each PSMT's lesson plan. This feedback was mainly based on guiding them toward more student-centred instruction. Next, the PSMTs performed their lessons in line with their plans. These lessons were recorded by a video camera. Due to the large number of the PSMTs in the course and shortage of time, although they each taught for two hours, the PSMTs were asked to evaluate only one hour of their lessons and were able to choose which hour to evaluate. Later, each PSMT watched her/his own teaching video at home and selected one of their two lessons to analyse. They analysed the selected lesson and wrote a report about what they noticed in their teaching experience. After each PSMT submitted a reflection paper, they met with the instructor and determined critical minutes in the video that were worth analysing together in video meetings. Following these informal interviews, two video meetings were organized and conducted. At each video meeting, two or three of the selected segments were watched and analysed. In these meetings, when the PSMTs requested, the video was stopped and the events they noticed were discussed. Sometimes, I also requested to stop the video and ask them questions. Video meetings were also recorded with a video camera.

Since the course aimed to scaffold PSMTs' noticing skills, I purposefully directed both the second group discussion and two video meetings with general prompts (e.g., "What did you notice?") to identify the issues PSMTs noticed, and with specific prompts (e.g., "What did the student say? What do you think about Kaan's thinking?") to foster the PSMTs to think more about students' mathematical thinking. So, I played the role of facilitator to stimulate discussions and nudged the PSMTs to elaborate on specific events about students' mathematical thinking.

To summarize, the data consisted of the PSMTs' reflection papers that they wrote at each stage, video records of three group discussions, and video records of two video meetings. Sometimes, PSMTs' statements were similar across the reflection papers, the group discussions, and the video meetings. To prevent duplication in the data, the reflection papers were used as primary data, while the data from group discussions and video meetings were secondary data. Lesson plans, videos of mentor teachers' lessons, and videos of PSMTs' lessons were also used during data analysis to better understand the context.

Data Analysis

Since the purpose of this study was to examine the development of PSMTs' noticing skills throughout a course, it was useful to level these skills to clearly reveal the development. I adopted the van Es (2011) framework for analysis of the data (see Table 1). Data analysis was carried out in consecutive steps depending on the types of the data, first the reflection papers, and then the group discussions and the video meetings. To start data analysis, I read the reflections line-by-line and reviewed them for clues to the PSMTs' noticing. Each different classroom event that the PSMT mentioned was separated into a noticing statement. I coded each statement in terms of what and how the PSMTs noticed based on van Es (2011) framework. To understand the nature of PSMTs' noticing, I analysed the data by using four components; the first two refer to what PSMTs notice and the last two refer to how they notice: actor, topic, stance and depth of analysis. Actor included teacher, student or students. The focus person of PSMT's statement determined the actor. Thus, a statement that involved the word "student(s)" did not always indicate that the actor was a student. One statement could contain more than one code, i.e., both teacher and students.

Table 1
A Framework for Learning to Notice Student Thinking (van Es, 2011, p. 139)

	Level 1 baseline	Level 2 mixed	Level 3 focused	Level 4 extended
What teachers notice	Attend to whole class environment, behavior, and learning and to teacher pedagogy.	Primarily attend to teacher pedagogy. Begin to attend to particular students' mathematical thinking and behaviors.	Attend to particular students' mathematical thinking.	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking.
How teachers notice	Form general impressions of what occurred. Provide descriptive and evaluative comments. Provide little or no evidence to support analysis	Form general impressions and highlight noteworthy events. Provide primarily evaluative with some interpretive comments. Begin to refer to specific events and interactions as evidence	Highlight noteworthy events. Provide interpretive comments. Refer to specific events and interactions as evidence. Elaborate on events and interactions.	Highlight noteworthy events. Provide interpretive comments. Refer to specific events and interactions as evidence. Elaborate on events and interactions. Make connections between events and principles of teaching and learning. On the basis of interpretations, propose alternative pedagogical solutions.

Topics emerging from the data were teacher's pedagogy, students' thinking and learning, students' behaviours, management, mathematics, and other issues (e.g., curriculum, identity). Stance refers to whether the PSMTs describe, evaluate, or interpret what they notice. Describing involves restating what was noticed. Evaluating refers to judging what was good or bad or should have been done differently. Interpreting includes understanding and reasoning about what was observed. To define depth of PSMTs' analysis, I paid attention to if they mentioned general or specific events, provided evidence to support analysis, made connections between events and teaching and learning principles, and proposed alternative pedagogical solutions. A sample from the data analysis of reflection papers was presented in Table 2.

Table 2

A Sample from Data Analysis of Reflection Papers

Statement	What do PSMTs Notice?			How do PSMTs notice?				
	Actor	Topic		Analytic stance	Depth of analysis		Make connections between events and principles of teaching and learning	Propose alternative solutions
		Sub-Code	Code	Analytic stance	Specificy	Evidence		
Elvan I asked students to write the examples we did in the previous lesson as steps of drawing graphics. I attempted to engage the students to the lesson. I aimed for every student to attend and learn.	Teacher Students	Explaning Engaging students Learning	Pedagogy Student thinking	Des	Gen	No		
Selen When I asked if it ended here, he replied "no". I asked "Is there an inflection point?" The student said that it existed, because the second derivative could be found. He got the second derivative, found the root of the second derivative and said this is the inflection point.	Teacher Student	Asking Question Learning	Pedagogy Mathematics Student thinking	Int	Spe	Yes Quotations		
Gülce When I started to teach the lesson through the examples, I lost a lot of time because there was a problem with the board and I could not show some examples that I wanted to use. The lesson would have been more efficient if I had used abundant and understandable examples instead of solving less examples in order not to waste time.	Teacher	Using examples Time management	Pedagogy Management	Int	Gen	No		Proposes to use abundant and understandable examples.
Irem At 19:20, students were distracted and humming began. I tried to draw attention to me by saying "Are we listening?". Here, I think that there is a more appropriate discourse that include I-language, by saying "Friends, I am distracted in the hum. I cannot convey what I want to you".	Students Teacher	Classrrom management	Student Behaviour Management	Eva	Spe	Yes Minutes from video	Using I-language is an appropriate discourse.	Proposes to use I-language
Yavuz As I spoke a little fast, what I said in some places may not be fully understood.	Teacher	Speaking fastly	Other- Identity	Eva	Gen	No		

Finally, I determined the trajectory of the development of the PSMTs' noticing from Level 1 to Level 4. This data analysis process was carried out for each of the three stages. It is necessary to point out that the data of the second stage, carried out in groups, were considered, and evaluated as separate data for each group member.

The findings obtained from the analysis of reflection papers were organized as in Table 3 to illustrate the shift in PSMTs' noticing skills by using number of statements and average percentage. To demonstrate the shift between the stages, average percentage was preferred to the numbers of statements, because the total number of statements in each stage was different. Average percentage was calculated by dividing the number of related statements by the total number of statements in the component. Only the total number of statements in actor and topic was higher than the number of statements per PSMT. This is because, for example, there were statements at the intersection of teacher and students. The students' column indicates the sum of the statements referring to a particular student and students as whole. Quotations from reflection papers were also used to explain findings in detail.

To start data analysis of group discussions and video meetings, I first repeatedly watched three group discussions and two video meetings. Since the discussions were conducted with all the PSMTs in the context of the course, I needed to extract the parts related to the participant PSMTs. In the first group discussion, only Elvan shared her observations; the other four PSMTs did not identify their observations. This was influenced by the fact that four or five dominant PSMTs not involved in the research, talked most of the time in this twelve-person class. The others preferred to approve or remain silent when these people state their opinions. The second group discussion and the video meetings included more data from the participants. I transcribed the video segments that included their ideas. Then, I analysed the transcript texts using van Es (2011) framework, similar to the data analysis process mentioned for reflection papers. The findings obtained from the analysis of this data were presented qualitatively along with sample data excerpts.

To ensure the reliability of my coding, I asked a mathematics education researcher to independently code a part of the PSMTs' statements. There was an acceptable agreement in our coding of the statements and some discrepancies were discussed. Then, I re-analysed the data and compared the codes. I examined these disagreements and solved them by taking opinions from two other mathematics education researchers.

Findings

In this section, the PSMTs' noticing skills (what and how the PSMTs notice) in three stages will be presented comparatively. Next, developmental shifts in the PSMTs' noticing skills will be examined qualitatively by providing data excerpts.

What and How do PSMTs Notice?

Analysis of the three stages indicated some changes in the PSMTs' noticing skills during the course. In Table 3, to illustrate changes between the stages, numbers and percentages of noticing statements were given. At first stage, when the PSMTs analysed an unknown teacher's lesson, they mostly noticed the teacher (83%) and teacher's pedagogy (37%). They described the teacher's moves, teaching strategies, and use of mathematical language in teaching the function concept. Very few of the PSMTs (17%) noticed the students; when they did so, they noticed the whole class. Other topics the PSMTs attended to were students' thinking, mathematics, management, and students' behaviours, respectively.

Table 3

Number and Percentage of Noticing Statements for Agent, Topic, Stance and Specify

	Agent		Topic						Stance			Specify	
	Teacher	Student(s)	Teacher pedagogy	Students' thinking	Students' behavior	Management	Mathematics	Other	Descriptive	Evaluative	Interpretive	General	Specific
Analysis of an unknown teacher's teaching (First stage)													
Elvan	17	1	7	6	1	3	1	-	8	8	1	15	2
Selen	8	1	3	2	-	-	5	-	7	1	-	6	2
Gülce	7	4	6	4	-	-	2	-	2	3	2	5	2
Irem	8	2	5	2	2	1	4	-	-	6	4	6	4
Yavuz	3	1	1	2	1	1	1	-	1	2	-	3	-
Total	43	9	22	16	4	5	13	-	18	20	7	35	10
Avg.	83%	17%	37%	26%*	7%	8%	22%	-	40%	44%	16%	78%	22%
Analysis of mentor teacher' s teaching (Second stage)													
Elvan& Selen	15	9+10	13	10	11	3	5	-	8	9	1	12	6
Gülce, Irem& Yavuz	10	2+6	8	6	4	-	3	-	1	6	3	7	3
Total	25	27	21	16	15	3	8	-	9	15	4	19	9
Avg.	48%	21+31=52%	34%	25%	24%	5%	12%*	-	32%	54%	14%	68%	32%
Analysis of own teaching (Third stage)													
Elvan	25	9+19	12	24	8	6	10	3	12	17	1	21	9
Selen	21	9+6	17	16	2	5	17	-	11	4	6	9	12
Gülce	10	3+2	9	5	1	1	4	-	2	4	3	3	6
Irem	41	4+9	25	26	7	7	14	4	21	15	7	26	17
Yavuz	11	1+6	7	8	1	3	5	3	5	6	-	5	6
Total	108	68	70	79	19	22	50	10	61	46	17	64	50
Avg.	61%	15+23=39%	28%	32%	7%*	9%	20%	4%	49%	37%	14%	56%	44%

*Note: Rounded to small decimal number.

At the second stage, when the PSMTs analysed their mentor teacher's lesson, their primary focus changed and became the students (52%). This time, they not only noticed the students as a group, but 21% of their statements were about particular students' thinking or behaviour. They described these students either by name or by mentioning them as "a student". In their statements, they focused on the teacher primarily, but with a lower percentage (48%) than in the first stage. Topics the PSMTs noticed, similar to the first stage, were mostly teacher's pedagogy and students' thinking. In the second stage, they also focused on students' behaviour, with a similar percentage of student thinking. Other noticing statements were about mathematics and management.

At the third stage, the PSMTs mostly noticed the teachers again as in the first stage; but this time it was at a lower percentage (61%) than the first stage, though higher than the second one. Like the second stage, the PSMTs attended to both the whole class and particular students (totally 39%). In other words, the difference between teacher and student percentages was largest in the first stage, relatively smaller in the third stage and smallest in the second stage. While the teacher was still the primary actor in the final stage, the topic that the PSMTs most mentioned changed to students' thinking. The potential of the teacher's actions to reveal and support student thinking was expressed in the last stage. Moreover, the PSMTs addressed the teacher's pedagogy by connecting to students' understanding and learning. From highest percentage to lowest, the PSMTs commented on pedagogy, mathematics, management, and students' behaviour. Unlike the first two stages, some statements in the last stage fell into the "other" category. The PSMTs talked about factors such as speaking style, tone of voice, and excitement as they analysed their own lessons.

Findings about how the PSMTs noticed showed that they exhibited a change from evaluating general events to describing specific events, as they provided evidence and made connections between events and the principles of teaching and learning (see Table 4). They mostly evaluated the classroom events in the first and second stage, while they described more in third stage. Interpretive approach percentages were close at all stages (14-16%). The reason why percentage was preferred to interpret data was that the total number of statements in each stage was different. Maximum number of statements were seen at the last stage, because the PSMTs analysed their own lessons step by step in detail, in a manner similar to storytelling. Therefore, although no percentage change was observed, the increase in the number of interpretive statements (17 statements) was noteworthy and showed the improvement in the PSMTs' interpretive approach in their analysis.

If we look at specificity, in the first stage, the PSMTs commented on the observed events in a general way (82%). They preferred to mention the teacher's pedagogy superficially, as they did not focus the details on the events and did not elaborate on their analysis. The decrease in percentage of general statements and the increase in percentage of specific ones were noteworthy in both the second and third stages. At the third stage, the percentage of general and specific statements were close, as the PSMTs started to go deep into the events.

Consistent with these findings, the PSMTs provided evidence to support their analyses at the last stage, while they almost never did at the first two stages (see Table 4). They began to present excerpts from the teacher or student and identify minutes in the videos, making their explanations and interpretations of the events more detailed. They also made connections between events and the principles of teaching and learning mostly at third stage. While the PSMTs offered six pedagogical solutions on the basis of their analyses at the first stage, this number doubled at the last stage. In the second stage, the fact that they did not make any suggestions on pedagogical solutions is worth discussing.

Table 4
Depth of PSMTs' Analysis

Stage	Providing evidence (n)	Making connections between events and principles of teaching and learning (n)	Proposing alternative pedagogical solutions (n)
First	-	2 (Irem)	6 (2 Gülce, 4 Irem)
Second	1 (Gülce, Irem & Yavuz)	2 (Gülce, Irem & Yavuz)	-
Third	52 (2 Elvan, 3 Selen, 4 Gülce, 40 Irem, 3 Yavuz)	10 (1 Gülce, 9 Irem)	12 (5 Gülce, 7 Irem)

While the data of the five PSMTs has been examined quantitatively, the developmental shifts in the noticing levels of each PSMT is discussed separately and qualitatively below.

Developmental Shifts in PSMTs' Noticing Skill Levels

I begin by describing the PSMTs' noticing levels for each stage (see Table 5) and then illustrate the developmental shift of each PSMT from first to third stages by providing examples from reflection papers and discussions.

When we look at how the PSMTs moved through the noticing levels over the three stages (actually over the course), an overall improvement is seen, although not at every stage. While the PSMTs initially exhibited characteristics of baseline or mixed noticing, they reached more advanced levels of focused or extended noticing at the end of the research process.

Table 5
PSMTs' Levels of Noticing Skills

	Elvan	Selen	Gülce	Irem	Yavuz
First stage	Level 2	Level 1	Level 2	Level 2	Level 1
Second stage	Level 2	Level 2	Level 2	Level 2	Level 2
Third stage	Level 3	Level 3	Level 3	Level 3	Level 2

It is also seen at Table 5 that no PSMT reached extended noticing (Level 4). Although Irem made many connections between events and teaching and learning principles, and provided alternative solutions at the third stage, her comments still focused on teacher actions in general situations. So, Irem was also not evaluated as demonstrating Level 4 noticing skill.

To illustrate the PSMTs' developmental shift over the course, two PSMTs (Selen and Gülce) were selected to be examined in detail, to honour the page limit and to prevent repetition. These two PSMTs were chosen because they attended classes regularly throughout the semester and were mostly active in discussions.

The Case of Selen

When Selen analysed the unknown teacher's lesson in the first stage, she usually focused on the teacher's pedagogy and the mathematics the teacher taught (e.g., "*He wrote different relations and asked students to examine which ones were functions*"). She almost never mentioned the students. In only one statement she attended to the whole class and noticed

the students' confusion about an image and value set, but she did not interpret the reason behind this confusion: "*Function examples were written and examined. There, students asked what the value set in the question was for which the domain set were given, and they needed to find the image set. The teacher said that it could be written infinitively, and to provide a particular solution, said that one of them is the set of real numbers.*" She stated general impressions and described and evaluated what she observed (e.g., "*The teacher invites students to the board to solve the questions that required finding the domain and image sets*"). She had no interpretive comment in the first stage. She provided neither evidence nor connections between classroom events and teaching and learning principles. Therefore, in the first stage, Selen's noticing skill was considered as baseline (Level 1).

Since Selen did not express an opinion, no findings could be obtained from the first group discussion.

Selen analysed their mentor teacher's lesson with her group friend (Elvan). The mentor teacher was teaching normal lines in graphics in the video-recorded segment. In their analysis, although they primarily attended to mentor teacher's pedagogy, they began to pay attention to particular students' thinking. They presented general impressions, but they also determined noteworthy events. For example, they noticed students' overall behaviour, a student's thinking, and a teacher's approach when they stated: "*There was humming among the students. The student said mx + n with low voice. The teacher did not hear it, continued to solve the question, and asked 'Do you have any questions?'. When there was no sound, he then passed to another question.*" Also, they focused on the students' thinking and made inferences based on the students' response: "*He asked the student what the derivative of the product was. The student said only the derivative of the product in the question, not the general formula. The student had a lack of knowledge at this point, and the teacher reminded him of the derivative of the composite function.*" Unlike the previous stage, by evaluating these and such specific events, they started to examine the teacher's pedagogy in terms of whether he supported students' thinking. They focused on the teacher's actions, such as reminding prior knowledge, questioning, hearing, or not hearing students' thinking. Although they began to refer to specific events or moments that they observed, they were inconsistent in elaborating and providing details to develop their analyses. They still maintained an evaluative and descriptive stance. Therefore, the group's, and namely Selen's, noticing had the properties of mixed noticing (Level 2).

Selen's statements in the second group discussion were similar to what she and Elvan wrote in their reflection paper, with an attempt to understand students' thinking and behaviour. For example, she commented about the students' prior knowledge of derivation. In addition, she questioned the reason why students were passive in the lesson with this statement:

> There were deficiencies in students' previous knowledge. For example, there were those who did not know some of the rules of derivation. Even, the teacher asked a question to the boy who was at the blackboard and he could not answer it. There was a lack of interest in the students because we had already solved these questions with the students in previous lesson. They just solved what we didn't do. They also did extra things; they told the teacher what they did not understand from the questions. In other words, I think that was the reason why students were passive in the lesson. They seemed to have learnt from two different people (from the teacher and us) before. (Second group discussion- analysis of mentor teacher's lesson)

As seen in her statement, Selen attended to the students' lack of knowledge. She also indicated that they (Selen and Elvan) had been able to solve the questions that the students could not successfully solve in the previous lesson. She explained that the students did not pay enough attention to the lesson for this reason. These explanations showed that Selen

attempted to interpret the students' learning and behaviours. Thus, she began to adopt an interpretative approach. These findings from the second group discussion confirmed that her noticing level was mixed.

At the third stage, Selen analysed her own lesson, where she taught graphing a polynomial function. In her analysis, although not as weighted as in the first stage, her statements were more about the teacher (herself). She focused on her pedagogical approach (e.g., questioning, reminding of prior knowledge, making connections between mathematical concepts) and mathematics (e.g., roots of the function, derivative of the function, the increase and decrease of the function, the maximum and minimum points). However, this time, she referred to student's specific mathematical understanding and also tried to find the reason behind a student's understanding/confusion (e.g., "*The student was confused about the double root when examining the plus and minus signs. The reason is that one of the roots of the f function is double. Here, again, I pointed out to the student that we examined the sign of derivative function's roots*."). While making her comments, she was not mainly evaluative as in the first two stages, but either descriptive or interpretive. For the first time, she focused more on specific events and also presented evidence of her analysis. She gave data excerpts of students' responses. Although her noticing has improved compared to previous stages, she still did not connect the events to general teaching and learning principles. Thus, her noticing was evaluated as focused (Level 3).

In the first video meeting that Selen's lesson was watched, at the moment we paused the video and began to talk about it. There, a student had come to the board and tried to solve a problem graphing the polynomial function. While finding the points where the function intersects the axes, he was confused at the points where it intersects the *x* axis. He thought x=0 was a double root, and he should examine signs for the function. Selen reminded the student what they were looking for and when they should examine signs, and that they should do that to examine the derivative function's roots (see Figure 2). In the video meeting, while analysing this case (which Selen also mentioned in her reflection paper), the following dialogue took place:

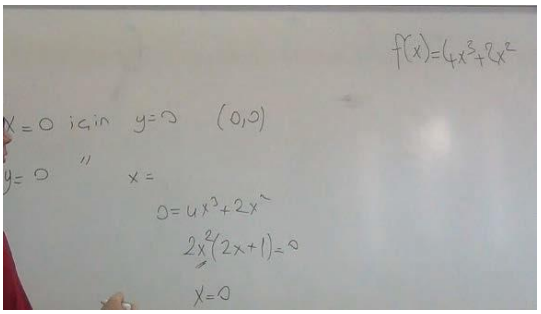

Figure 2. A screenshot from Selen's lesson

Researcher (R): What have you noticed so far? What would you say about the student's solution?
Yavuz: He (the student) said that x=0 is a double root.
Gülce: He took it into parenthesis first. Parenthesis of x^2. Then he said there is a double-root for $x^2=0$. He said -1/2 comes from here.
Selen: That's why I said double root.
R: He was a little confused, what happened? The solution was going right.
Selen: He thinks why she (the teacher) asks. It's all like that, they want me to confirm their accuracy at every step.
Elvan: After that, the student did something, Selen asked what was done. He said immediately 'sign examine,' but he said it with no confidence. He was uncertain. Selen, when you did not answer, the student said 'What are we going to do? Should we examine signs?' Selen said, 'hmm, let's do it.'

In her reflection paper, Selen had mentioned this student's mathematical thinking and interpreted the reason for the student's confusion. In the video meeting, she acknowledged that the students mostly expected approval from teacher for what they did. While making these comments, it was noteworthy that both Selen and other PSMTs used descriptive expressions and could not deepen the events sufficiently, even though they considered the reason for student thought.

In video meetings, Selen also criticized her use of technology and classroom management. She stated that she had prepared a different material in Geogebra (a dynamic mathematics software), but she taught from scratch because Geogebra did not work, and they could not fix it before class.

While watching Irem's lesson in second video meeting, Selen identified her ideas on her classroom management by making the following comments. She also gave examples from the practice of the mentor teacher:

> Irem is just like me. I was very busy with Geogebra too. Then, the students said that it would be more comfortable if you used the mouse while zooming in. I guess, I may have been in front of them and limited the field of view. ... Our mentor teacher always does such things by using the mouse. He never gets in front of the board. (Second video meeting- analysis of Irem's teaching)

In conclusion, Selen's comments in the two video meetings showed that Selen focused more on specific events regarding the teacher or students and interpreted them with more detail. These findings from the video meetings support that her noticing was at Level 3 at the final stage.

The Case of Gülce

In the first stage, when Gülce analysed the unknown teacher's lesson, she mostly focused on the teacher and his pedagogy. She used general impressions like "*He used multiple representations (list, sets and the Cartesian coordinate system) to illustrate functions.*" She attended to the mathematical thinking of the whole class and particular students and the details of the mathematical content. For example, when she said, "*The teacher said that the function is a special relation. However, this was a rough definition said verbally. Therefore, it was seen that there were students who could not make sense the function*" she interpreted the teacher's verbal explanation about the concept as inadequate for the students. Although she mostly mentioned general events, she also referred to particular events and interactions in the class. She was dominantly evaluative in her comments. Consequently, Gülce exhibited the characteristics of mixed noticing (Level 2) at the first stage. Like Selen, Gülce did not state an opinion in the first group discussion too. So, no findings could be obtained there.

At the second stage, Gülce showed the same noticing skills as in the first stage, although there were minor differences. Unlike the first stage, she (and her group members) focused more on students, especially on some students' thoughts. In addition to teacher's pedagogy, student thinking and mathematics, they shared their observations on students' behaviour. Although evaluative comments predominated, there was an increase in their interpretive statements and a decrease in the descriptive ones. They began to provide a little evidence from teacher's questions and students' responses to support their analysis. Although they began to refer noteworthy events and individual student thinking and adopt an interpretive stance, they mostly continued to focus on the teacher via general statements. Hence, their noticing, namely Gülce's noticing, still had the features of mixed noticing (Level 2).

In the second group discussion, Gülce and her group members shared the video of their mentor teacher's lesson and then whole class analysed and discussed the noteworthy events. During the lesson, at a time when the teacher dealt with the increase and decrease in sinus

function, a student asked a question: "*Does area mean integral?*" The mentor teacher responded that area was not related to the topic of the class. The PSMTs noticed and began to discuss this interaction. One PSMT criticized the teacher's reaction and proposed that he should have said, "*Yes, it's true, but this is another topic*." Some other PSMTs argued that the teacher's reaction was correct and adequate because it prevented them discussing an unrelated topic. Gülce commented on this event with the following sentences:

> …the area is not always an integral, so if the teacher agreed with this, it would lead to a misconception later. At the moment, we found his response sufficient because he could not explain it in more detail. (Second group discussion- analysis of mentor teacher's lesson)

This statement showed that Gülce was thinking about the effect of teacher action on student thinking, and that she had started to attend to student's thinking and adopt an interpretive approach. In the same meeting, Gülce commented on the mentor teacher's management and use of technology by proposing an alternative solution as below:

> It is good to use Geogebra. But we were stuck in one thing. He (the mentor teacher) drew the graph of x^3 in the class. He followed that by handling the class well, but he could have prepared more in advance, because he has lots of materials when he opens his file. But he did not prepare it before, he did it in the lesson. (Second group discussion- analysis of mentor teacher's lesson)

In this excerpt, Gülce identified her observations on the teacher's use of technology and consequently proposed that he could prepare all materials before the lesson. She also mentioned the events that she (with her group members) noticed about the mentor teacher's pedagogical actions which reveal students' thinking and the students' participation in the lesson:

> The teacher used brainstorming; we liked that. The question related to zero was about learning what was in a student's mind, but there were many answers from the class. Many students participated in the lesson who did not normally do so. (Second group discussion- analysis of mentor teacher's lesson)

It is seen from Gülce's statements in the second group discussion that although she noticed noteworthy events in students' thinking, her main focus was still on the teacher's pedagogy. So, these findings obtained from second group discussion support her noticing to be classified as mixed.

When Gülce analysed her own lesson in the last stage, where she taught asymptotes of functions, she primarily focused on the teacher (herself) and teacher's pedagogy, as in the first stage, but this time she analysed specific events with more depth. She discussed the pedagogical approaches in terms of students' understanding. Instead of student thinking, student behaviour and mathematics, Gülce attended to classroom management for the first time. Again, although she evaluated the most, her interpretive statements reached the highest percentage of the three stage. For example, she stated, "*In the twelfth minute of the lesson, a student asked, 'Is it back because it is a double root?' I said, 'No, it is approaching -1 from both sides.' Having said that, it would have been more memorable if I had zoomed out the graph and showed that it wasn't.*" Here, she evaluated her response to student and interpreted the interaction by proposing an alternative solution on the basis of the evidence from video. By emphasizing the importance of using visual elements to provide meaningful learning, she attempted to connect this specific event to teaching and learning principles. This attempt showed that Gülce's noticing began to move towards an advanced level. When evaluated with the other findings from this stage, her noticing was considered focused (Level 3).

In the sections from the video meetings, where she analysed both other PSMTs' and her own lesson, Gülce identified noteworthy events concerning students' mathematical thinking and understanding, and also commented on mathematical content. She focused on students'

behaviours (e.g., listening and participating in the lesson), students' thinking (e.g., finding asymptotes of a rational function) and the details of the mathematical content (e.g., asymptotes, slope). She grounded her discussion in particular segments of the lesson and used these segments as evidence to support her interpretations. Gülce also offered pedagogical solutions like guiding students to find the mistake themselves, reminding students of the focus of the lesson to use the time productively, and preparing technological materials by predicting what problems may be encountered. All these findings from the video meetings confirm the decision from the data in reflection papers, namely that Gülce's noticing was focused (Level 3).

Discussion and Conclusion

The findings of this study revealed that the PSMTs had different levels of noticing at the beginning of the course and they developed their noticing skills over time through the three stages of the research. The PSMTs' noticing skills were at Level 1 or Level 2 at the beginning, and they reached more advanced levels (Level 2 or Level 3) later on. For example, Selen started at Level 1 and progressed step by step to Level 2 and Level 3. Gülce, on the other hand, moved to a Level 3 at the last stage after demonstrating Level 2 skills in the first two stages. Similarly, other studies have also indicated that PSMTs' noticing skills were at lower levels before involvement in any research process (Güler, Çekmez, & Çelik, 2020; Guner & Akyuz, 2020; Jacobs et al., 2010; Lee, 2019; Roth McDuffie et al., 2014; van Es, 2011) and that noticing skill could be taught and improved with support (Fisher et al., 2018; Schack et al., 2013; Sherin & van Es, 2005; Star & Strickland, 2008). In the present study, the School Experience course was designed to scaffold the PSMTs' noticing skills. During the course, the PSMTs were provided theoretical knowledge and opportunities to analyse different instructional videos individually and collaboratively and to discuss the events in the videos in whole class or small group discussions. They were also guided by the instructor's prompts during the discussions. The content and teaching process of the School Experience course that they participated may have been effective in developing their noticing skills.

The PSMTs' noticing development was not linear from the first to the third stage in this study as in the study of van Es (2011). They all improved, but this development varied by individual. Sometimes they improved between the stages, and sometimes they remained at the same level. No PSMT reached Level 4, similar to some other studies (Güler, Çekmez, & Çelik, 2020; Guner & Akyuz, 2020; Lee, 2019). This may be because a one semester course was not sufficient for the PSMTs to reach extended noticing. As previous studies pointed out, these results once again showed that the development of noticing requires time, and noticing is not an easy skill to acquire for the PSMTs (Diamond et al., 2018; Jacobs et al., 2010; Lee, 2019; van Es, 2011).

Two of PSMTs progressed in their noticing between the first and the second stage, and four of them progressed in the transition from second to third stage. In other words, the PSMTs developed more in the transition from the second to the third stage. During this transition, participants had the chance to analyse the lessons of all the mentor teachers that they and their peers observed, either via videos or observation notes, and they discussed the noticed events as a whole class. Thus, the PSMTs had the opportunities to see and evaluate the classroom practices of different teachers. These experiences may be the reason for their more apparent progress from the second to the third stage.

The PSMTs focused more on the teacher in the first and third stages, however, their focus changed to the students in the second stage and began to refer to particular students' thinking

or behaviours. As to topic, teacher pedagogy had the highest percentage in all stages. But in the last stage, the PSMTs identified teacher pedagogy by associating teacher's actions with students' learning and thinking. These findings all showed that the PSMTs tried to understand more the behaviours and thoughts of the students they observed in the second and third stages. In other words, the PSMTs' noticing improved at the stages where they were present in the classroom as an observer or an actor. Similarly, other studies have agreed that PSMTs and teachers focused on teacher pedagogy at early stages of the research content, and that they moved to focusing on students' thinking by the aid of video analysis (Guner & Akyuz, 2020; Roth McDuffie et al., 2014; Mitchell & Marin, 2015; Sherin & van Es, 2005; Stockero et al., 2017; van Es, 2011).

In this study, the PSMTs mostly identified the events dealt with the mathematical content in the first stage, then the third, and the least in the second stage. This can be interpreted as the PSMTs' focusing more on the teacher's pedagogy and the mathematics while analysing the unknown teacher's and their own lessons. Perhaps the PSMTs attended to mathematics less when they primarily noticed the students in their mentor teacher's lessons. In other words, noticing teacher pedagogy and mathematical content were synchronous in this study. For the case of Selen, it was seen that she always considered mathematical content as much as other topics. This finding may give an idea about Selen's content knowledge, but it can't go beyond, as data was not collected regarding the PSMTs' content knowledge.

In contrast to related research that management was noticed in participants' initial analysis (Mitchell & Marin, 2015, Star & Strickland, 2008, van Es, 2011), the PSMTs mostly mentioned management in the last stage of this study. For the case of Gülce, it is seen that she mentioned management for the first time while analysing her own teaching. One explanation might be that the PSMTs focused on and criticized themselves in the last stage, and therefore the classroom management they achieved or failed came to the fore. In addition, Lee (2019) indicated that the PSMTs in her study focused on themselves and started to focus on classroom management as their noticing improved. This supports the idea that the PSMTs' focus on classroom management does not create a contradiction with noticing development. The emergence of the 'other' category at the last stage and the fact that this category mostly consists of identity was also a result of the PSMTs focusing on themselves. In the category of "identity," similar to the PSMTs in Roller's (2016) research, they referred to attributes they were happy about or wanted to change like their voice, speaking style or excitement level.

The PSMTs' noticing moved from evaluating general events to describing specific events by providing evidence and making connections between class events and principles of teaching and learning over the research. Unlike the previous studies where the PSMTs or teachers exhibited a shift from evaluative to interpretive approach (Sherin & van Es, 2005; van Es, 2011), the PSMTs in this study moved from evaluative to descriptive stance. Similar to the probable reason for the previous results, the limited research period may be a reason for this result. Although the percentage of interpretive statements were close to each other at all stages, the increase in the number of interpretive statements was considerable and showed the improvement in the PSMTs' interpretive approach in their analysis. This was due to the difference in the total number of statements at each stage. Particularly, in the last stage, which includes the most statements, the PSMTs sometimes reported chronological sequences of events. This may be because the PSMTs were working with their own teaching experiences for the first time. This finding contrasts with Sherin and van Es' (2005), where participants listed events chronologically at the beginning of the study, and in the second year, organized their essays more around particular events identified as noteworthy.

Although identifying the events as chronological sequences, the PSMTs also began to focus on specific moments in their lessons as the research progressed. There was a significant increase in the percentage of focusing on specific events from the first stage to the third stage. Similar results could be found in prior studies (Goldsmith & Seago, 2011; Sherin & van Es, 2005; van Es & Sherin, 2008). However, even at the last stage, the fact that the PSMTs had still the tendency to focus on general events is a proof that it is challenging for the PSMTs to give up their traditional approach and elaborate on events. Nevertheless, their growing attempts to provide evidence for their analysis, make connections between events and teaching and learning principles, and propose alternative teaching approaches based on their analyses in the last stage also confirmed the conclusion that there was an improvement in their noticing skills. A remarkable finding was that the PSMTs did not make any suggestions while analysing their mentor teacher. This finding may be the result of their evaluation of the mentor teacher as pedagogically sufficient or even a role model.

The findings of this study were limited to five PSMTs' data from their reflection papers and discussions during a one semester course. The findings show that, like many previous studies (e.g., Jacobs et al., 2010; Roller, 2016; Santagata et al., 2007; Sherin & van Es, 2005; Star & Strickland, 2008), support helped the PSMTs to improve their noticing skills. Based on this conclusion, this study also suggests that noticing skills should be supported in the courses of teacher education programs like the School Experience course. While specifically examining the PSMTs' noticing skills during the School Experience course, gaining field experience, observing and following mentor teachers, and living their own teaching experiences were very valuable practices for their noticing skills' development. As Roller (2016) described, studying videos not only of other teachers or PSMTs but also of their own provides the PSMTs with an opportunity to purposefully consider their own teaching practices. In future studies, the development of PSMTs' noticing skills in video club studies can be researched and supported by incorporating more videos of mentor teachers and PSMTs. This study showed that it is not easy for PSMTs to acquire and improve this skill in only one semester. Since the development of noticing skills requires a long process (Jacobs et al., 2010), future studies which will examine PSMTs' noticing skills should also be planned as long-term research.

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