College Developmental Mathematics Students' Understanding of Fraction Operations

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Students enrolled in post-secondary developmental mathematics courses hold mathematical misunderstandings that begin before their college careers. Specifically, these students struggle with conceptual understanding of fractions, ratios, and proportions. The Common Core State Standards in Mathematics outline standards for K-12 mathematics education. Standards for fractions, ratios, and proportions are found in grades 3-7. Through a research instrument, collegiate developmental mathematics students were investigated regarding their understanding of these standards. It was determined that participants showed evidence of limited knowledge of the examined areas involving fractions, ratios, and proportions. Because it seems that this lack of understanding is rooted early in their educational careers and is continued throughout students' academic careers, it is hoped that this work can be a starting point for future work in this field that may apply to investigating ways to bridge these gaps in mathematical understanding

According to the Common Core State Standards in Mathematics (National Governors Association Center for Best Practices [NGA Center], 2010), fractions, ratios, and proportional relationships are studied in grades 3-7. Research reveals that students in those grades begin developing various misconceptions regarding fraction understanding that later manifest in algebra courses (Mou et al., 2016; Fonger, Tran & Elliot, 2015) and that students enrolled in collegiate developmental mathematics courses continue to struggle with these concepts (Steinke, 2017). Because fraction understanding can be linked to success in more advanced mathematics courses, these fundamental concepts must be learned early and thoroughly. Unfortunately, many students progress through elementary, middle, and high school without grasping these mathematical ideas.

This research aims to determine the grades 3-7 Common Core State Standards regarding fractions, ratios, and proportions with which college developmental mathematics students continue to struggle. Although previous research reveals that collegiate developmental mathematics students struggle with fraction concepts, most of this research indicates particular fractional concepts on which they struggled (e.g., Steinke, 2017) rather than connecting these concepts to the mathematical standards they are addressing. This is problematic because several fraction concepts span multiple grade levels. This investigation hopes, in part, to fill partial gaps in the literature in this domain.

Student enrollment in collegiate developmental mathematics courses has adverse financial effects on the institution providing the course and correlates to a strong predictive factor regarding these students eventually dropping out of college (Acosta, 2016; Burley, Butner, Anderson & Siwatu, 2009; Cox, 2015; Fong, Melguizo, & Prather, 2015). Therefore, the results of this study may inform researchers as to grade levels in which intervention should be invested to break this cycle.

Literature Review

As seen below, recent research indicates that fraction understanding on all educational tiers, collegiate developmental mathematics courses, and the persistence of mathematical misconceptions are topics that have been investigated in varying capacities. This study seeks to determine with which mathematical standards regarding fractions, ratios, and proportions collegiate developmental math students continue to struggle. To do so, this study considers literature associated with the dimensions of university developmental math courses, fraction learning and connections, and persistence of misconceptions.

University Developmental Mathematics Courses

A growing proportion of collegiate students are being enrolled in developmental mathematics courses (Acosta, 2016; Fong et al., 2015), "non-credit courses that address the needs of students who are underprepared for college-level courses" (Acosta, 2016, p. 2). These prerequisite developmental mathematics courses typically progress through four topics: arithmetic, pre-algebra, elementary algebra, and intermediate algebra (Fong et al., 2015).

This growing enrollment in college developmental courses is problematic. The annual cost of developmental education at U.S. universities is more than two billion dollars (Fong et al., 2015). The main correlates of university dropout behavior are first-generation, low-income, and minority college students. Because students from these groups are disproportionately represented in developmental education, enrollment in developmental courses has been found a correlate to predict university student dropout (Acosta, 2016; Fong et al., 2015). Specifically, students enrolled in developmental mathematics courses "were less successful in obtaining a degree than students not taking developmental math" (Burley et al., 2009, p. 29). "The three post-secondary courses (across all U.S. Colleges) with the highest rates of failure and non-completion are all developmental math courses" (Cox, 2015, p. 2).

While developmental mathematics courses correlate to predict university student dropout (Acosta, 2016; Burley et al., 2009; Fong et al., 2015), it is dubious if these courses possess actual causal effect upon such apart, possibly, from elongating the student's academic plans. Although these courses negatively impact university costs (Cox, 2015), albeit somewhat mitigated by the lower costs of adjuncts typically teaching these courses, such courses provide otherwise unavailable opportunities for students with college potential to attain college-necessary careers. Although developmental mathematics courses may be hardships to both students and universities, they may be necessary for many students to progress academically and toward their desired careers.

Fraction Learning and Connections

Student understanding of fraction magnitudes and fraction arithmetic skills are both essential in comprehending one another (Bailey, Hansen, & Jordan, 2016). According to Pantziara and Philippou (2011), there exist five sub-constructs regarding the learning of fractions: part-whole, ratio, quotient, measure, and operator. They report that: fourth and fifth-grade students who understand both fractional concepts and procedures outperform those who only understand one of the two; students appear to receive an excessive amount of instruction on the part-whole sub-construct, at the expense of others, specifically measurement; and fraction problems involving variables appear to be the most challenging tasks for students.

Lewis, Gibbons, Kazemi, and Lind (2015), report that fourth-grade students struggle with the symbolic form of fractions and fraction terminology. The same students struggled when asked to portion whole amounts into fractional amounts (e.g., $3 \div \frac{2}{3}$) because they only wanted to perform calculations with whole numbers. Many times, the students would divide a whole into parts without necessarily being concerned with the size of the parts (Lewis, Gibbons, Kazemi, & Lind, 2015). Hansen, Jordan, and Rodrigues (2017) report that "errors on fraction computation problems may reflect misapplication of whole number principles to fractions" (p. 46).

When Fonger, Tran, and Elliot (2015) consider children's creations, interpretations, and connections within and between multiple fraction representations, interviews with children from grades 2-6 reveal that they "often express inconsistencies or mismatches when reasoning across representation types, sometimes posing a hindrance to their problem solving process" (p. 13). They opine that children generally better understand fraction problems that provide them with accessible contexts related to their prior experiences. Several children demonstrated a specific misconception regarding cross-multiplication of fractions when such a calculation was unnecessary. Students also had varying understandings of the meaning of fractions in the context of comparing measurements. The authors believe that students interpreted the measurement task as an operator, the part-whole meaning, and the measurement meaning individually rather than the three having the same innate meaning.

A recent study finds that seventh- and eighth-grade students who struggled with placing fractions on a number line also struggled with algebra skills in ninth grade (Mou et al., 2016). The authors of the study state that providing a stronger focus on fraction magnitudes in early grades would improve performance in later mathematics classes. Understanding fractions is fundamental to expanding number understanding, which is the foundation of advanced mathematics courses. The authors conclude that educators should identify instructional targets—such as fraction magnitude and numerator/denominator relationships—for mathematics education in middle school.

Though fractions are primarily taught in elementary and middle school, the same fraction concepts are applied later in algebra when learning about algebraic fractions. Makonye and Khanyile (2015) note that students who exhibited misconceptions regarding fractions in earlier grades demonstrated the same misconceptions persisting into tenth grade. Through interviewing students, constructing interview questions that necessitate conceptual versus instrumental understanding of fractions, and encouraging students to provide a reason for every calculation, Makonye and Khanyile (2015) reveal that students were able to overcome their misconceptions regarding the simplification of algebraic fractions.

Persistence of Misconceptions

Bailey, Hansen, and Jordan (2017) predict that early specified instruction targeting magnitude understanding and arithmetic skills "is more likely to produce more persistent effects on these same specific skills than early support that targets mathematics achievement more broadly will produce on general mathematics achievement measures" (p. 516). They also conclude that transfer between fraction magnitude understanding and arithmetic skills does not occur until later in a student's learning process. Hansen, Jordan, and Rodrigues (2015) believe that "persistent difficulties with fraction procedures throughout elementary/early middle school are associated with low mathematics achievement at the end of sixth grade" (p. 53). Though elementary and early middle school are the grades in which fractions are taught, Hansen, Jordan, and Rodrigues (2015) report that many students showed little progress during this time in understanding fractions. Because misconceptions regarding fractions persist this way, the intervention must be made in the early years of education. Without such intervention, those misconceptions will again manifest when students enroll in higher-level mathematics courses, including college-level courses, leading to long-term concerns for mathematics achievement.

Bentley and Bossé (2018) show that some college students have limited understanding of fraction arithmetic and employ procedures associated with it—even at times correctly—without understanding why said algorithms are valid. Furthermore, college students demonstrated procedural fraction misunderstanding, including whole number bias, incorrect fraction operation strategies, and division of two fractions, and conceptual misunderstandings, including fraction equivalence and sub-constructs such as part-whole, ratio, operator, quotient, and measure. Concerning the operation of fraction multiplication, for instance, while some students recognized that for, $\frac{2}{3} < 1 < \frac{4}{3}$ and $\frac{2}{3} < \left(\frac{2}{3} \cdot \frac{4}{3}\right) < \frac{4}{3}$, other students believed that the operation of fraction multiplication required common denominators. Regarding division, students failed to recognize that, for $\frac{15}{4} \div \frac{2}{3}$, $\frac{2}{3}$ acted as the *contextualized whole* (Tzur (2000) refers to this as the *referent whole*). Regarding the process of rewriting fractions with common denominators to add them, students disassociated this act from multiplying fractions and did not recognize the process of multiplying by 1.

Connecting Fraction Knowledge to Developmental Math

In a study performed by Steinke (2017), 23% of students in developmental mathematics struggled with fractional concepts such as part-whole coexistence. Comprehension of part-whole coexistence is crucial to developing fraction understanding and thus to higher-level mathematics achievement (Steinke, 2017; Bailey et al., 2017). Recognizing that elementary and middle school students struggle to understand various concepts associated with fractions, the question remains as to what extent university developmental mathematics students struggle with fraction understanding and fraction operations. Therefore, this study determines the grades 3-7 Common Core State Standards regarding fractions, ratios, and proportions with which college developmental mathematics students continue to struggle.

Research Methodology

Participants in this study included 75 developmental mathematics students in two developmental mathematics courses at a public university in the Southeastern U.S. The same instructor taught both courses during the Spring 2017 semester, and both courses were demographically and socio-economically similar. Participation was open to all 78 students enrolled in these two classes. All participants had either taken the university mathematics placement exam or did not meet the minimum university SAT or ACT requirements and were placed into developmental mathematics and were freshmen with a nearly even distribution among males and females. Ten of the participants were over 20 years old. All participants had passed their requisite high school mathematics courses, and no participants had taken a mathematics course in a community college prior to this developmental class. All but six participants attended K-12 schooling in the state where the university was located (the particular county for each participant is unknown), and all but 16 of the participants experienced a curriculum purportedly in line with the Common Core State Standards for at least three years.

For this research project, participants completed a test regarding fractions, ratios, and proportions administered during the class's second day of the first week. This timing was selected intentionally. It was deemed valuable for the instructor to develop a positive rapport with the students on the first day of class and engender a level of comfort and trust in the classroom, as it was believed that this would assist student success by lessening students' math phobia. In this first class period, there was no instruction regarding fractions; the class participated only in discussion and trust-gaining activities. Also, students were informed about a voluntary research task in which they could participate during the next class meeting. They were not informed that the research focused on fractions, ratios, and proportions; thus, they could not study these concepts prior to the next class meeting. Participants were informed of the study during the second class meeting and provided participant agreement forms. Participants were given approximately 80 minutes to complete the research task.

This study elected to assess student understanding of fractions based on commonly recognized standards. The Common Core State Standards for Mathematics for grades 3-7 include 51 standards regarding fractions, ratios, and proportions (NGA Center, 2010). The entire list of 3^{rd}-5^{th} grade standards (www.corestandards.org/Math/Content/NF/) and 6^{th}-7^{th} grade standards (www.corestandards.org/Math/Content/RP/) which are addressed in this study are provided in Appendix 1 of this paper.

From these 51 standards, 65 questions were developed by a team of researchers to address the standards. The entire instrument can be found in Appendix 2. The test items were vetted by additional university faculty and K-12 teachers to ensure content reliability regarding the standards. Through different venues (e.g., invitations to investigate and use in professional development settings), over 100 K-12 teachers investigated, vetted, and refined the research instrument prior to its use in this study. This method was used to ensure validity and anticipate reliability with the study population.

While many of the items on the instrument involved performing operations on fractions, all questions were constructed to examine student understanding of the associated standard. While some standards were covered by one question, other more nuanced standards were addressed by two or more questions.

Because the study's goal was to assess student success on grade 3-7 Common Core State Standards regarding fractions and fraction operations and using decimal representations would not demonstrate proficiency in these standards, participants were not allowed to use a calculator on the test. They were asked not to solve fraction problems by rewriting the fractions as decimal values. Participants were also instructed not to guess the answer to a question but rather to skip it if they did not know how to solve the problem. Thus, researchers could identify which problems participants struggled with based on incorrect and incomplete answers, and answers left blank.

Comparing particular test items in the instrument with their associated standard often reveals both similarities and dissimilarities. For instance, test items were constructed based on the standard's mathematical meaning and inferred intention. This necessitated both interpretation of the standards and the creation of an associated test item aligning with the standard. Care was taken to avoid making collegiate students feel demeaned by test items regarding fractions, ratios, and proportions—topics covered in elementary grades mathematics. Thus, at times, numerals were used within test items, making them seem more simplistic, and at other times variables were used, making them seem more in line with algebraic reasoning. The latter was intended to protect participants from feeling insulted by questions on the instrument.

Each participant response on each research item (75 participants × 65 items = 4,875 items) was scored by two of the researchers using the following criteria: Correct, all parts of the student's answer were correct; Incorrect, all parts of the student's answer were incorrect; Partial, at least part of the student's answer was correct; Blank, the student left the question blank; Incomplete, the student started the problem but did not finish (the attempted portion of the problem was incorrect); and Unanticipated, the student provided a correct answer, but it was not anticipated based on the context of the problem. Using code-checking (Miles & Huberman, 1994), all items were scored by varying pairs of the three researchers to ensure inter-rater reliability. All items scored as all three researchers reexamined either Partial, Incomplete, or Unanticipated to ensure consensus. The third researcher, the instructor of the course, remained uninvolved in the scoring of student work to assure the participants that results on the project would not affect class grades.

Once all the items were scored, the scores were entered into a spreadsheet to directly compare results. The data analysis employed the tabulation of numbers, averages, and percentages. Because this study had no interest in disaggregating data by participant age, gender, or traditional Freshman versus nontraditional students, no effort was made to use statistical methods to search for correlations between these factors and student understanding of fraction concepts. This study intended to investigate the population of participants as a whole.

Since the participants involved in this study were college students, the standards considered were in respect to grades 3-7, and all participants had passed all grades subsequent to grades 3-7 in order to be enrolled in developmental math, it could be assumed that the participants should do well on the research instrument. The following results demonstrate otherwise.

Results

Each standard investigated in this study is identified in the Common Core State Standards for Mathematics (NGA Center, 2010) by an alphanumeric coding structure (e.g., 3.NF.A.1). The first numeral represents the grade level in which the standard should be taught. The next set of letters is known as the "domain." The domain links groups of standards together that revolve around a common concept. NF denotes the domain "Numbers and Operations – Fractions." The next digit represents that the standard listed is one in a cluster of related standards. For example, 3.NF.A.1 indicates that this standard is taught in third grade, under the domain of Numbers and Operations – Fractions, and that it is the first standard (1) in the cluster (A). Notably, in the standards for grades 6 and 7, the NF designator is replaced by RP, denoting Ratios and Proportions

The results from the pretest are separated by their associated grade level and standard. Tables 1, 2, 3, 4, 5, and 6 present the number of student responses per standard that were deemed correct, incomplete, blank, partially correct, unanticipated, and incorrect.

Tables 1-6 align these standards with question numbers on the research instrument (e.g., 3.NF.A.1 aligns with question 1 and 3.NF.A.3.A aligns with questions 5, 6, and 7). Additionally, these tables provide counts of the number of student responses (N = 75) that were in each category. Results are later investigated in the discussion section.

Table 1
Grade 3 standards results

	Standards A1-A3A						
Grade 3 Standards: 3.NF	A1	A2A	A3	A2B		A3A	
Question	1	2	3	4	5	6	7
Correct	15	19	4	6	54	40	13
Incomplete	3	1	0	3	0	0	0
Blank	43	23	2	5	12	16	14
Partial	0	15	68	48	0	0	0
Unanticipated	0	0	0	0	0	0	0
Incorrect	14	17	1	13	9	19	48
Total Not Fully Correct	60	56	71	69	21	35	62

	Standards AB3-A3D						
Grade 3 Standards: 3.NF	A3B	A3C			A3D		
Question	8	9	10	11	12	13	14
Correct	8	24	17	49	50	39	19
Incomplete	8	5	3	0	0	0	0
Blank	42	21	49	12	9	16	18
Partial	7	21	4	0	0	0	0
Unanticipated	0	1	0	0	0	0	0
Incorrect	10	3	2	14	16	20	38
Total Not Fully Correct	67	51	58	26	25	36	56

Table 2

Grade 4 standards results

Grade 4 Standards: 4.NF	Standards A1, A.2 and B.3					
	A.1			A.2		B.3
Question	15	16	17	18	19	20
Correct	11	26	4	55	24	0
Incomplete	0	0	4	0	0	0
Blank	14	27	51	10	9	52
Partial	0	0	6	0	0	0
Unanticipated	0	0	0	0	0	0
Incorrect	50	22	10	10	42	23
Total Not Fully Correct	64	49	71	20	51	75

Grade 4 Standards: 4.NF	Standards B.3.A, B.3.B, and B.3.C				
	B.3.A		B.3.B	B.3.C	
Question	21	22	23	24	25
Correct	2	7	11	25	27
Incomplete	0	1	0	1	0
Blank	42	40	51	31	23
Partial	6	0	0	0	0
Unanticipated	2	0	0	0	0
Incorrect	23	27	13	18	25
Total Not Fully Correct	73	68	64	50	48

Table 3
Grade 4 standards results

Grade 4 Standards: 4.NF	B.4.A	B.4.B		C.5		C.6		C.7
Question	26	27	28	29	30	31	32	33
Correct	2	12	5	22	3	17	17	2
Incomplete	4	1	0	2	0	0	1	4
Blank	60	42	50	29	53	43	42	58
Partial	5	0	0	0	0	1	0	3
Unanticipated	0	0	0	0	0	5	1	4
Incorrect	4	20	20	22	19	9	14	4
Total Not Fully Correct	73	63	70	53	72	58	58	73

Table 4
Grade 5 standards results

	Grade 5 Standards A.1, B.3, and B.4.A					
Grade 5 Standards: 5.NF	A.1	B.3				B.4.A
Question	34	35	36	37	38	39
Correct	4	8	1	0	0	12
Incomplete	4	0	1	0	0	0
Blank	53	42	54	52	57	35
Partial	3	2	0	0	0	0
Unanticipated	2	10	4	0	0	0
Incorrect	9	13	15	23	18	28
Total Not Fully Correct	71	67	74	75	75	63

	Grade 5 Standards B.4.B, B.5.A, and B.5.B				
Grade 5 Standards: 5.NF	B.4.B	B.5.A		B.5.B	
Question	40	41	42	43	44
Correct	2	9	0	8	8
Incomplete	5	0	0	0	0
Blank	50	48	54	42	51
Partial	0	0	0	0	0
Unanticipated	0	0	0	19	5
Incorrect	18	18	21	6	11
Total Not Fully Correct	73	66	74	67	67

Table 5
Grade 5 standards results

Grade 5 Standards: 5.NF	B.7.A			B.7.B			
Question	45	46	47	48	49	50	51
Correct	0	14	3	1	14	7	0
Incomplete	0	0	0	2	0	0	3
Blank	65	55	54	57	45	43	52
Partial	0	0	0	2	0	0	14
Unanticipated	0	0	0	0	0	1	0
Incorrect	10	6	18	13	16	24	6
Total Not Fully Correct	75	61	72	74	61	68	75

Table 6
Grades 6 and 7 standards results

	6.RP A.1 & A.2						
Grade 6 and 7 Standards	A.1				A.2		
Question	52	53	54	55	56	57	58
Correct	12	14	1	13	56	36	0
Incomplete	0	0	0	0	1	2	2
Blank	28	34	67	31	12	22	43
Partial	25	0	0	10	0	9	0
Unanticipated	0	0	0	0	0	0	0
Incorrect	10	27	7	21	6	6	30
Total Not Fully Correct	63	61	74	62	19	39	75

	6.RP A.3.C & A.3.D					7.RP A.1 & A.2.A	
Grade 6 and 7 Standards	A.3.C			A.3.D		A.1	A.2.A
Question	59	60	61	62	63	64	65
Correct	1	0	5	0	0	30	12
Incomplete	1	3	2	3	1	0	2
Blank	42	52	58	52	58	34	41
Partial	27	0	0	0	0	0	16
Unanticipated	0	0	0	0	0	0	0
Incorrect	4	20	10	20	16	11	4

Total Not Fully Correct	74	75	70	75	75	45	63

Tables 7, 8 and 9 aggregates all the result for the standards in each grade. This allows for each grade to be seen as a whole.

Table 7

Global results for grades 3, 4, and 5 standards

	Total Results by Grades 3, 4, and 5					
	Grade 3 Standards		Grade 4 Standards		Grade 5 Standards	
	Questions (N)	Percent	Questions (N)	Percent	Questions (N)	Percent
Correct	357	34.00%	272	19.09%	91	6.74%
Unanticipated	1	0.10%	12	0.84%	41	3.04%
Partial	163	15.52%	21	1.47%	21	1.56%
Incorrect	224	21.33%	375	26.32%	273	20.22%
Incomplete	23	2.19%	18	1.26%	15	1.11%
Blank	282	26.86%	727	51.02%	909	67.33%
Total % Not Fully Correct		66.00%		80.91%		93.26%

Table 8

Global results for grades 6 and 7 standards

	Total Results by Grades 6 and 7			
	Grade 6 Standards		Grade 7 Standards	
	Questions (N)	Percent	Questions (N)	Percent
Correct	138	15.33%	42	28.00%
Unanticipated	0	0.00%	0	0.00%
Partial	71	7.89%	16	10.67%
Incorrect	177	19.67%	15	10.00%
Incomplete	15	1.67%	2	1.33%
Blank	499	55.44%	75	50.00%
Total % Not Fully Correct		84.67%		72.00%

Table 9
Total results across all grades of standards

	Total Across Grades	
	Questions (N)	Percent
Correct	900	18.46%
Unanticipated	54	1.11%
Partial	292	5.99%
Incorrect	1064	21.83%
Incomplete	73	1.50%
Blank	2492	51.12%
Total % Not Fully Correct		81.54%

In this study, no corresponding questions were created for standards 4.NF.C.7, 5.NF.B.5, 6RP.A.3.D, 7.RP.A.2.B, 7.RP.A.2.D, and 7.RP.A.3, as they were determined to be outside of the scope of this study.

Discussion

Numerous percentage values are provided in this discussion. Remembering that this study investigated developmental college students' success rates regarding fractions, ratios, and proportions standards from grades 3-7, some of these percentages may be considered somewhat shocking. To highlight these, when appropriate, bold font is employed. Additionally, when standards are quoted in the discussion, they are italicized to assist the reader in making appropriate connections.

Table Summaries

Of the fourteen test items based on third-grade Common Core State Standards regarding fractions (Table 1): students answered only 34% correctly; 26.86% of the responses were left blank; 21.33% were answered incorrectly; and 15.52% were answered partially correct. The most commonly missed test items based on third-grade standards were items number 7 (referencing 3.NF.A.3.A: *Understand two fractions as equivalent (equal) if they are the same size…*) and number 14 (referencing 3.NF.A.3.D: *Compare two fractions with the same numerator or the same denominator by reasoning about their size… Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a fractional model.*) with 48 and 38 people, respectively, answering the questions incorrectly (64% and 51%). The two test items most frequently answered correctly were numbers 5 (3.NF.A.3.A) and 12 (3.NF.A.3.D), with 54 and 50 people respectively answering the items correctly (or 77% and 67%). Notably, the standards associated with the two items on which the students best performed were the same ones on which the students did the worst. The two most frequently skipped questions were questions number 1 (referencing 3.NF.A.1: *Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b*) and question 10 (referencing 3.NF.A.3.C: *Express whole numbers as fractions, and recognize* *fractions that are equivalent to whole numbers*) with 43 and 49 people respectively skipping the questions (57% and 65%).

Fourteen test items were created based on the fourth-grade Common Core State Standards. Of the fourteen items (Tables 2 and 3): 51.02% of the responses were left blank; 26.32% were answered incorrectly; and 19.09% were answered correctly. The most missed test items from the fourth grade standards were questions number 15 (referencing 4.NF.A.1: *Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b)… Use this principle to recognize and generate equivalent fractions.*) and 19 (referencing 4.NF.A.2: *Compare two fractions with different numerators and different denominators… by comparing to a benchmark fraction… Record the results of comparisons with symbols >, =, or <…*) with 50 and 42 people respectively answering the items incorrectly (or 67% and 56%). Number 15 is the single most missed question on the test. The two test items most frequently answered correctly were numbers 18 (4.NF.A.2) and 25 (referencing 4.NF.B.3.C: *...replacing each mixed number with an equivalent fraction…*) with 55 and 27 people respectively answering the items correctly (or 73% and 36%). While item 19 was one on which students did most poorly, and item 18 was one on which students did best, both items referenced the same standard. The two most frequently skipped questions were questions 26 (referencing 4.NF.B.4.A: *Understand a fraction a/b as a multiple of 1/b*) and question 33 (referencing 4.NF.C.7: *Compare two decimals to hundredths by reasoning about their size… Record the results of comparisons with the symbols >, =, or <…*) with 60 and 58 people respectively skipping the questions (80% and 77%).

The fifth-grade questions included in the test consisted of fifteen items. Of the fifteen test items (Tables 4 and 5): 67.33% of the responses were left blank; 20.22% were incorrect; and 6.74% were correct. The most commonly missed test items in this section were numbers 39 (referencing 5.NF.B.4.A: *Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b… In general, (a/b) × (c/d) = (ac)/(bd).*) and 50 (referencing 5.NF.B.7.B: *Interpret division of a whole number by a unit fraction, and compute such quotients… Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*). Twenty-eight people (37%) missed item number 39 and 24 people (32%) missed item number 50. The two test items most frequently answered correctly were numbers 46 (referencing 5.NF.B.7.A: *Interpret division of a unit fraction by a non-zero whole number, and compute such quotients…. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*) and 49 (5.NF.B.7.B) with only 14 people (19%) answering the items correctly. Item 50, among the items on which students did most poorly, and item 49, among the items on which students did best, were both associated with the same cluster of standards, 5.NF.B.7. The most frequently skipped questions were questions 38 (referencing 5.NF.B.3: *Interpret a fraction as division of the numerator by the denominator…)*, 48 (referencing 5.NF.B.7.B), and 45 (referencing 5.NF.B.7.A) with 57 people skipping numbers 38 and 48, and 65 people skipping question 45 (76% and 87%).

Twelve test items were created based on sixth-grade Common Core State Standards. Of the twelve items (Table 6): 55.44% of the responses were left blank; 19.67% of responses were incorrect; 15.33% of answers were correct; and 7.89% of responses were partially correct. The top two most missed items were numbers 58 (referencing 6.RP.A.2: *Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.*) and 52 (referencing 6.RP.A.1: *Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.*), with 30 and 27 people (40% and 36%) missing the items, respectively. The two test items most frequently answered correctly were numbers 56 (referencing 6.RP.A.2) and 57 (based on standard 6.RP.A.2) with 56 and 36 people (75% and 48%) respectively answering the items correctly. Item 58, among the items on which students did most poorly, and items 56 and 57, among the items on which students did best, were all associated with the same standard. The items on which students performed best and on which most students struggled were from the same cluster of standards, 6.RP.A. The most frequently skipped questions were questions 61 (referencing 6.RP.A.3.C: *Find a percent of a quantity as a rate per 100… solve problems involving finding the whole, given a part and the percent*), 63 (referencing 6.RP.A.3.D: *Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities*) and 54 (referencing 6.RP.A.1) with 58 people skipping numbers 61 and 63, and 67 people skipping question 54 (77% and 89%).

There were two test items created based on seventh-grade Common Core State Standards. For these two questions (Table 6): 50% of the responses were left blank; 28% of responses were correct; 10.67% of answers were partially correct; and 10% of responses were incorrect. Eleven people (15%) missed item number 64 (referencing 7.RP.A.1: *Compute unit rates associated with ratios of fractions … in like or different units.*) and 4 (5%) missed item 65 (referencing 7.RP.A.2.A: *Decide whether two quantities are in a proportional relationship … by testing for equivalent ratios in a table…*). Thirty people (40%) correctly answered item 64, while 12 (16%) correctly answered item 65. The most frequently skipped question was question 65 (referencing 7.RP.A.2.A), which 41 people skipped (55%).

By dividing the number of test items that each student answered correctly by the total number of items (65), the average percentage grade for all 75 developmental mathematics students on the test was 18.46%. Adding the number of correct, unanticipated, and partially correct item responses for each student, then dividing by the total number of questions, generates a class average of only 25.56%. Over 51% of all responses on test items were left blank. The test items created based on fifth-grade standards were the most frequently left blank (67.33%,) while the items based on third-grade standards were the most frequently answered correctly (34%). Number 54, regarding ratios, was the singularly least attempted test item, with 64 students (85%) leaving it blank. Number 43 was the test item with the most students answering unanticipatedly (N = 19 or 25%). Most students who answered the item with an unanticipated response used zero or a negative number as a multiplicand to make a fraction smaller than another fraction.

Polarized Results and Items Involving Variables

Interestingly and unexpectedly, different test items for some standards led to problems on which students did the best and the worst. These polarized results occurred with standards:

- 3.NF.A.3.A: *Understand two fractions as equivalent (equal) if they are the same size…* (questions 5 and 7, both including variables);
- 3.NF.A.3.D: *Compare two fractions with the same numerator or the same denominator by reasoning about their size… Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a fractional model.* (question 12, not including variables, and question 14, including variables);
- 4.NF.A.2: *Compare two fractions with different numerators and different denominators… by comparing to a benchmark fraction… Record the results of*

comparisons with symbols >, =, or <...(questions 18 and 19, neither including variables);

- 5.NF.B.7.B: *Interpret division of a whole number by a unit fraction, and compute such quotients... Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* (questions 49 and 50, both including variables); and
- 6.RP.A.2: *Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.* (questions 56, not including variables, and 58, including variables).

At first blush, with some polarized results coming from pairs of problems both including variables, one including variables and the other not, and neither including variables, this may seem to be an indicator that participants were not strongly affected by problems that included the use of variables. More analysis (see Table 10) provides additional details regarding participant performance regarding problems including variables and those which did not. While the percentage of problems answered correctly on standards for grades 3, 5 and 6 demonstrated similar percentages, grade 4 standards showed that more problems not including variables were correct. Both of these findings are interesting. It might have been guessed that problems including variables would be perceived as more difficult. However, apart from grade 4 standards, this did not seem to be the case. For grade 4 standards, students performed better on problems that did not include variables, as may have been guessed. Nevertheless, these findings may be partially in conflict.

Table 10
Comparing problems with variables (V) with those with no variables (NV).

	Problems with Variables with Those Without							
	Standards Grade 3		Standards Grade 4		Standards Grade 5		Standards Grade 6	
	V	NV	V	NV	V	NV	V	NV
Percent Correct	32.00%	35.47%	9.48%	27.73%	7.20%	5.33%	16.33%	14.83%
Percent Blank	34.52%	7.47%	59.26%	25.07%	74.30%	66.29%	53.33%	56.50%

Table 10 reveals larger differences in the percent of problems left blank in respect to standards for grades 3 and 4 and smaller difference for standards for grades 5 and 6. It is hypothesized that the higher percentage of blank responses for problems including variables associated with standards for grades 3 and 4 is due to much of the mathematics instruction and learning in these grades being completed without using variables; thus, problems using variables in respect to standards in these grades may have looked out of place, if not confusing. Commensurate with curricula's increasing employment of variables through the elementary and middle grades, the balancing out of the percentages of problems left blank for grades 5 and 6 standards in respect to using variables, may suggest that students were more accustomed to using variables when looking at topics associated with these standards. Further research is needed to answer many underlying questions.

Unfortunately, this study was not formed to answer the question as to why some standards would produce such polarized results in student responses. This certainly warrants future investigation.

Initial Summary

It must be continually recognized that this study's mathematical content under investigation are standards from grades 3-7, and the students involved are college students in developmental mathematics courses. One could argue that the results of this study are poor at best, if not abysmal. In summary: many students left many test items blank; most items were answered correctly by only a small percentage of students; and most students did very poorly overall. This leads to some grave concerns to be addressed as implications of these findings.

Implications

Although students needing more understanding of concepts regarding fractions, ratios, and proportions have been studied previously, there is a gap in the literature regarding when this lack of knowledge may originate. The researchers hypothesized that a lack of mathematical understanding among collegiate developmental mathematics students regarding concepts associated with fractions, ratios, and proportions could be evidenced by investigating their success in responding to questions from standards from grades 3-7. However, while the research participants demonstrated insufficient knowledge of these standards as demonstrated through their work on the research instrument, it is still being determined whether their lack of understanding begins in grades 3-7 or whether these concepts were satisfactorily learned at the time and subsequently lost. Nevertheless, it is seen, that among these research participants, understanding of grades 3-7 mathematical standards regarding fractions still needs to be improved.

According to the results above, the highest percentage of questions answered correctly were those related to third-grade standards (38%). However, the fact that only 38% of questions based on *third-grad*e standards were answered correctly by *college* students supports the hypothesis that developmental mathematics students lack understanding regarding fractions from an early age. This study may imply that, for these research participants, the lack of understanding regarding fractions taught in third through sixth grades persists through all of grade school and on into their college careers.

Specifically, the questions answered correctly the most often were regarding using unit rates in the context of a ratio relationship (question 56), comparing fractions with different numerators and denominators (question 18), and understanding two fractions as equal if they are the same size or point on a number line (question 5). On the other hand, the questions answered incorrectly most often were regarding multiplying by a fraction equivalent to one (question 15), understanding fractions as equal if they are the same size (when the fractions involve variables as well as integers) (question 7), and comparing two fractions with the same numerator or the same denominator by comparing their size (question 14). This may imply that more learning time or different learning experiences may be needed regarding specific concepts in early grades to help students avoid becoming future collegiate developmental math students.

Returning to the finding that some standards led to both best and worst student responses indicates that there remains much opportunity for research in this field. Why did some standards lead to these polarizing results while others did not? Was this due to the nature of the test items or the standards themselves? What in students' backgrounds led to this effect? Countless other questions remain unanswered.

One must ask how many difficulties students exhibited with the test items were due to some items including variables rather than only numeric results. Further investigation should follow to determine how much variables affect developmental mathematics students' skills in solving problems involving fractions, ratios, and proportions. There remains much opportunity for additional research in this area.

Recalling that Makonye and Khanyile (2015) found success in students gaining conceptual understanding regarding the simplification of algebraic fractions through appropriate questioning techniques, there may be ways to make accommodations for developmental mathematics student learning. However, these methods may be perceived as too labor-intensive and time-consuming for college developmental classes with more than thirty students in a class. Nevertheless, these results may demonstrate that developmental college mathematics students' conceptual learning of fraction ideas is possible, given appropriate instructional techniques. More precise ways to correct these misconceptions are not within the scope of this paper; however, it is clear from the lack of fraction understanding among these research participants that this problem must be addressed.

Questions—which cannot be answered in this brief article—arise regarding the nature of the American educational system in which students can struggle with concepts from grades 3-7 mathematics and yet pass all the necessary grades before college and even be enrolled in college. These students passed through middle grades and high school mathematics courses without comprehending many fraction, ratio, and proportional reasoning concepts even though they were required to learn them in school. This continuous cycle of passing without understanding trains students that they don't need to comprehend mathematics because they will still move on to the next, more advanced, mathematics course.

Limitations of Study

Limitations of the study can be localized to the population and size of the sample. Compared to the rest of the study body, the smaller sample size may skew the results of a larger sample. Also, the study is limited in what standards the sample participants were taught in their prior educational experiences, as standards vary from state to state.

Conclusion

After studying the results of 75 collegiate developmental mathematics students on a test regarding fractions, ratios, and proportional reasoning based on third- through seventh-grade standards, it has been concluded that these students performed quite poorly on these problems. Considering that not a single student answered over 45% of the questions correctly, with the average number of correct answers being 12 out of 65, there are significant gaps in mathematical knowledge regarding fractions, ratios, and proportions. Because these concepts are important to higher-level mathematics courses, students must understand them in the third through seventh grades to prevent enrolling in a developmental course.

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