Group Modeling Competencies of Seventh Grade Mathematics Students

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There exist many studies on modeling competencies and their promotion; however, there is a need to investigate what modeling competencies are and how they vary in lower and higher-achieving groups of students in mathematics within the current modeling literature domain. This study examined how mathematical modeling competencies were displayed in seventh-grade students working in two groups. It was qualitative and took competencies from existing literature into a detailed analysis of the modeling situation. The findings firmly supported the premise that higher-achieving groups exhibit higher levels of modeling competencies than lower-achieving groups. The differences in the levels of groups were more apparent in competencies of a sense of direction, planning and monitoring, understanding, mathematizing, and validating. Recommendations for further studies include studies of heterogeneous groups and further teacher development in modeling.

Society has needed a reform in mathematics teaching where the focus was on meaningful learning and teaching of mathematics and learning mathematics with understanding it (Cobb et al., 1992) since “things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things” (Hiebert et al., 1997, p. 1). To this end, problem-centered teaching and modeling are seen as alternatives to traditional mathematics teaching. Blum (2011) affirms that “by modeling, mathematics becomes more meaningful for learners” (p. 19). Modeling has a vital contribution to what students can learn and how students should learn in mathematics education.

In Turkey, since 2005, there has been an increasing emphasis on mathematical modeling in the mathematics curriculum at the middle school and high school levels. The 2005 curriculum is labeled as a reform-based attempt to achieve the contemporary educational changes in the world (Umay et al., 2006), and it is emphasized in the curriculum that concepts related to mathematics are handled by using concrete and finite life models (Ministry of National Education [MoNE], 2009). In the current mathematics curriculum at the middle school level, mathematical modeling is considered one of the basic mathematical process skills (MoNE, 2017).

Modeling is described as a process in which a non-mathematical problem is solved by applying mathematics (Kaiser & Maaß, 2007). It includes “simplifying a complex real situation, creating a model, working mathematically with it, and interpreting the result in that real situation” (Biccard, 2010, p. 1). This complex real situation is described as a ‘model-eliciting task’ (English & Lesh, 2003; Lesh et al., 2000, p. 608) designed for groups of students to solve. Lesh and Doerr’s (2000) research with model-eliciting problem situations led them to four key aspects of mathematical modeling. First, model-eliciting tasks emphasize representational abilities. If the tasks involve a much broader range of representational competencies, then a broader range of capable students emerge. Second, students often invent constructs that they recognize the need for it (Lesh et al., 1993, as cited in Lesh & Doerr, 2000). Third, when students develop a model, they often use a rich array of interacting representations involving personal systems that they introduced themselves.

Fourth, student experiences in model-eliciting activities serve as prototypes for thinking about other similar situations (Schank, 1990; Doerr, 1994, as cited in Lesh & Doerr, 2000). The following part introduces one of the most frequently used representations of modeling processes in the literature, namely, that of the *modeling cycle*, following the development of modeling competencies from the past until today. Group modeling competencies and modeling competencies in lower and higher achievers in mathematics are also discussed. It concludes with a presentation of the theoretical framework of the study.

Modeling Cycles and Modeling Competencies

The entire modeling process is often represented as a cycle (Greefrath & Vorhölter, 2016). In the literature on mathematical modeling, there is a variety of modeling cycles. These modeling cycles represent the general processes involved in the modeling evolution. There are some variations of processes in the modeling cycles.

In 1985, Blum proposed a visualization of the 'modeling process' (Greefrath & Vorhölter, 2016, p. 6) based on the common concept of models for mathematical application (Greefrath & Vorhölter, 2016). The modeling cycle by Blum is presented in Figure 1.

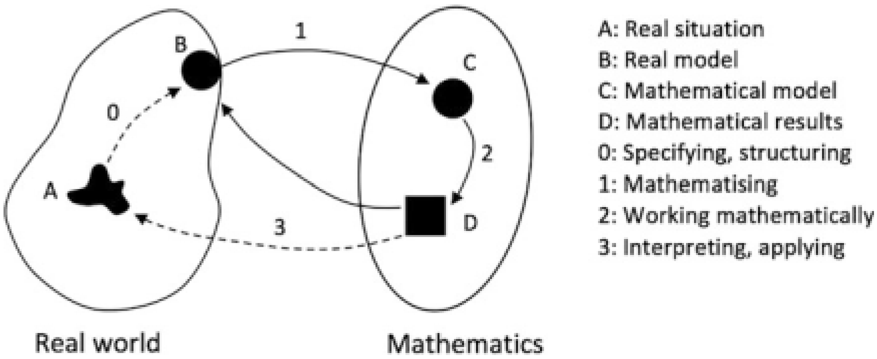

Figure 1. Modeling cycle by Blum (Greefrath & Vorhölter, 2016, p. 7)

In 1997, Blum and Kaiser specified the term 'modeling competencies' by giving a detailed listing of sub-competencies developed along the various phases of the modeling cycle. They are competencies to understand the real problem and to set up a model based on reality (including sub-competencies of making assumptions, recognizing variables, constructing relations between variables, and distinguishing between relevant and irrelevant information), competencies to set up a mathematical model from the real model (including sub-competencies of mathematizing, simplifying, and making suitable representation of the situation), competencies to solve mathematical questions within this mathematical model (including sub-competencies of using heuristics and using mathematical knowledge), competencies to interpret mathematical results in a real situation (including sub-competencies of generalizing solutions and viewing solutions using appropriate mathematical language and/or communicating the solutions), and competencies to validate the solution (including sub-competencies of critically checking and reflecting their solutions, reviewing, reflecting other possible solutions, and generally questioning the model).

The modeling cycle of Blum and Leiβ (2005) is an extension of Blum's original model from 1985 (Greefrath & Vorhölter, 2016), and it often appears in the literature on mathematical modeling (Doerr et al., 2017). They integrated the situation model as a new phase in their modeling cycle (Borromeo Ferri, 2006; Greefrath & Vorhölter, 2016). It showed more detail in considering how a mathematical model is generated (Greefrath & Vorhölter, 2016). The modeling cycle of Blum and Leiβ (2005) is shown in Figure 2.

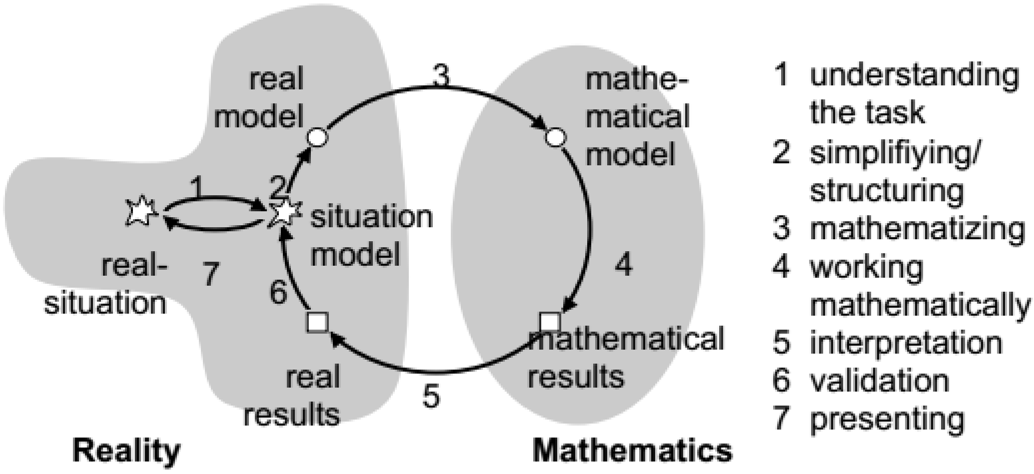

Figure 2. Modeling cycle of Blum and Leiβ (Borromeo Ferri, 2006, p. 87)

Maaß (2006), as well as Kaiser and Stender (2013), added the interpreted solution into Blum's original model as a step between the mathematical solution and reality (Greefrath & Vorhölter, 2016). In her qualitative study, Maaß (2006) aimed to transfer the theoretical frame in Figure 3 into practice. According to the results of her study, she categorized modeling competencies as partial competencies for carrying out the single steps of the modeling process, meta-cognitive modeling competencies, competencies to structure real problems, argumentation competencies, and competencies to judge a solution.

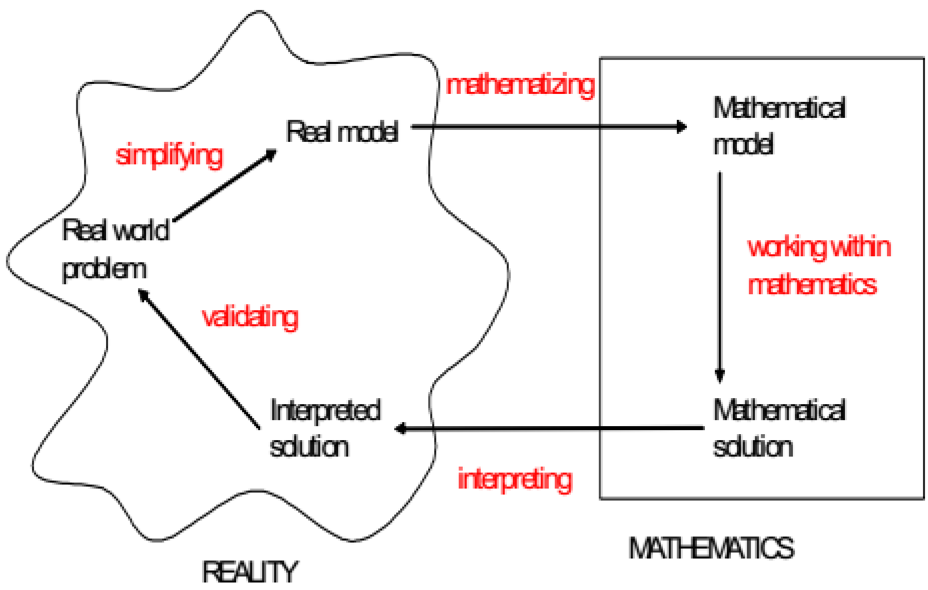

Figure 3. Modeling Cycle of Maaß (2006, p. 115)

More recently, the modeling cycle of Kaiser and Stender (2013) in Figure 4, which is a slight variation of Maaß's (2006) description and further development of earlier proposals, is used in empirical studies (Kaiser & Stender, 2013). It is emphasized that the competencies needed for these kinds of modeling processes are still the topic of current controversial debate; however, metacognitive modeling competencies and social competencies are necessary for the complex modeling examples (Kaiser & Stender, 2013).

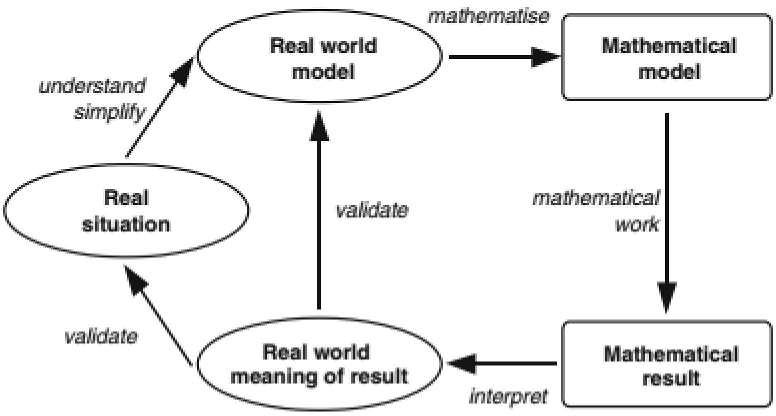

Figure 4. Modeling Cycle of Kaiser and Stender (2013, p. 279)

These various categorizations of modeling competencies led to the inclusion of broad areas of competence and aspects that are not common in traditional mathematics classrooms, such as meta-cognitive and social competencies in this study. To sum up, modeling competencies for this study refer to Maaß's (2006) definition of modeling competencies which is "Modelling competencies include skills and abilities to perform modeling processes appropriately and goal-oriented as well as the willingness to put these into action" (Maaß, 2006, p. 117). Therefore, this study decided to categorize modeling competencies into three areas: cognitive, meta-cognitive, and affective.

Although there are some variations, various authors consistently describe cognitive competencies. In this study, the competencies of Blum and Kaiser (1997, as cited in Maaß, 2006) were used since they were based on theoretical considerations. They covered a detailed listing of the sub-competencies that can assess student progress through the modeling cycle. The cognitive competencies in their view were: understanding the real problem (including sub-competencies of making assumptions, recognizing variables, constructing relations between variables, and distinguishing between relevant and irrelevant information); setting up the mathematical model (including sub-competencies of mathematizing, simplifying, and making a suitable representation of the situation); solving the mathematical questions within the model (including sub-competencies of using heuristics and mathematical knowledge); interpreting the results in the real situation (including sub-competencies of generalizing solutions and viewing solutions using appropriate mathematical language and communicating the solutions); validating the solution (including sub-competencies of critically checking and reflecting their solutions, reviewing, reflecting other possible solutions and generally questioning the model. Arguing was identified by Maaß (2006) as another area of competence necessary for modeling and was therefore included as a cognitive competency. Thus, the seven competencies were identified as cognitive competencies and used in this study: understanding, simplifying, mathematizing, working mathematically, interpreting, validating, and arguing.

Although cognitive abilities are needed to solve a modeling task, they do not work apart from affective and meta-cognitive competencies. Maaß (2006) indicated that meta-cognitive issues, beliefs, and 'sense of direction' (p. 119) from Treilibs' (1979, as cited in Maaß, 2006) work are important for the development of modeling competencies. Masui and De Corte (1999) identified many activities or processes in research on metacognition for their research project. The categories of meta-cognition were taken from Masui and De Corte (1999) to identify meta-cognitive modeling competencies for this study. Mousoulides et al. (2007) found student use of informal knowledge to be a significant factor in modeling. This is student knowledge used that is not specifically from a mathematical realm. Planning and monitoring are meta-cognitive abilities (Masui & De Corte, 1999), indicating general task organizational abilities. Treilibs (1979, as cited in Maaß, 2006) uses the term 'sense of direction' as an attribution to good modelers. Since they appeared to be more critical, this study uses three meta-cognitive aspects (using informal knowledge, planning and monitoring, and a sense of direction) as competencies.

Blomhoj and Jensen (2007) described the key characteristic of the initial modeling process as 'learning to cope with the feeling of perplexity due to too many roads to take and no compass given' (p. 49). It relates to student affective competencies: their beliefs about mathematics, the nature of tasks, how problems are solved, and the value of mathematics in solving real problems. Maaß (2006) found a close connection between positive attitudes toward modeling or mathematics and corresponding student performance. Lingefjard and Holmquist (2005) found clear evidence that attitude has a significant impact on students' modeling competencies over time. It was decided that student beliefs would be a central affective competency investigated in this study. Therefore, the modeling competencies selected for the focus of the study were: understanding, simplifying, mathematizing, working mathematically, verifying, interpreting, and arguing as being the cognitive competencies; using informal knowledge, planning and monitoring, and a sense of direction comprised the meta-cognitive competencies; and beliefs as part of the affective competencies for this study.

Group Modeling Competencies

Students work in groups exclusively when solving a modeling task. Lave (1991) and others (as cited in Derry et al., 1998, p. 27) describe a process by which this group work is characterized as "negotiation." They elaborate that negotiation occurs in groups since each member brings their cognitive background to the group. These unique perspectives enable members to understand and interpret the problems in significantly different ways (Derry et al., 1998). These authors also suggest that ideas shared by one individual and attended by others bring the group members into "greater conceptual alignment with one another" (Derry et al., 1998, p. 30). Furthermore, Zawojewski et al. (2003) approve, based on their experience, that when a small group of students is given an appropriate modeling activity, the group has the potential to bring more mathematical power to a task than any of the individuals involved in the group. From the above, it was decided that the unit of analysis in this study is the group, and the focus is on group modeling competencies.

Modeling Competencies in Lower and Higher Achievers in Mathematics

Modeling, which requires students' conceptions and representations to explain and express a complex system, is often considered too difficult for average or lower achievers. However, modeling research also shows that average-ability students and those in remedial mathematics programs can develop powerful models for describing complex systems (Lesh

& English, 2005; Lesh & Harel, 2003). Maaß (2005) proposed that modeling allows lower achievers to develop long-term affective and cognitive access to mathematics. She also added that the tasks enable lower achievers to perceive mathematics as more useful, interesting, and comprehensible, resulting in remarkable success for lower achievers. In a study by Peter-Koop (2004), primary pupils' real-world problem solving and modeling strategies were investigated in a four-year study. Two out of the four groups were so-called 'low achievers.' It was found that groups, including low achievers, were highly successful in finding an appropriate solution to one of the Fermi-type problems used for a modeling activity. Zubi et al. (2019) conducted a study focusing on nine low-performing students and observing changes in their modeling competencies and mathematical knowledge. It was found that seven of the nine students had developed all competencies and showed progress in their mathematical knowledge. There is also some support for the potential benefits of group work for these students (e.g., Steedly et al., 2008).

On the other hand, Iversen and Larson (2006) also found that high achievers in mathematics students forced more advanced mathematics into their model development while low achievers used simple mathematics in sophisticated ways. Another study by Kaiser and Maaß (2007) showed that high achievers chose more challenging models while lower achievers preferred simpler ways to achieve their final solution. From the studies that obtained surprising and contradicting results with both groups of students, the present study aimed to investigate the modeling competencies of students considered 'lower' and 'higher' achievers in traditional mathematics classrooms. This study was intended to contribute meaningfully to this discussion and advance mathematics learning for all students.

Method

Research Design

A qualitative research design was used in this study since the spectrum of techniques includes observation, interviewing, and documentary analysis (Schurink et al., 2011). Furthermore, it was deemed suitable for answering the research question and documenting the display of the competencies used. This study can be classified as a case study in many of its elements. According to Stake (1995), in case studies, the researcher explores a program, an event, an activity, a process, or one or more individuals in depth. The case(s) are restricted by time and activity, and researchers collect detailed information using a variety of data collection procedures over a sustained period (Stake, 1995). The study set out to examine students' modeling competencies while solving model-eliciting tasks in lower and higher-achieving groups.

Participants

A pilot study was done to select students as per the criteria stated in the aim of the study in terms of 'lower' and 'higher' achievers in a traditional setting. The pilot study took place with a class of 24 seventh-grade students. The class was split into six groups of student's own choice. Each group in the class worked on the same model-eliciting task at each time. They solved three model-eliciting tasks (Big Foot Problem (Lesh & Doerr, 2003), O'Hare Airport (Zawojewski et al., 2003), and Who's for the Long Jump? (Burkhardt, 2007)) for sessions of two hours over three weeks. The lessons took place during normal mathematics class time, and video recordings were made of certain events during these sessions.

Based on the pilot study, it was decided to select four students who performed well at mathematics and enjoy mathematics and four who performed at an average to below-average level and who did not have any learning problems or social problems. Four boys and four girls were selected, and each group consisted of two boys and two girls, thereby not widening the scope of the study to include gender issues. This purposive sampling method was used in the study.

Tasks

The tasks were taken from existing studies to ensure validity and reliability. They were selected with the criterion that they were open to the opportunity to model real-life situations in various ways. It was also necessary that they were suited in difficulty level to the students. The three tasks chosen for the main study were: The Sears Catalogue Problem, The Weather Problem: Where to Live Next? and the Summer Jobs problem. The Sears Catalogue Problem was taken from Lesh et al. (2000, p. 635). Their study dealt with developing thought-revealing activities. This task was adjusted to reflect the Turkish context. Here, groups had to compare prices from ten years ago with current prices and come up with a model to find the value of money (today) worth the same amount ten years ago.

The Weather Problem was taken from Doerr and English (2003, pp. 134–135). In this task, the requirements of the two clients served to guide the group's development of a model for determining which was the best city to locate. As in the Sears Catalogue Problem, some changes were made to fit a Turkish situation. The Summer Jobs problem was taken from Johnson and Lesh (2003, pp. 270–271). In this task, students were given data about a group of workers who sold concessions at an amusement park. This data described the number of hours worked and amount of money earned by each vendor, disaggregated by month and by how busy the park was when the corresponding hours were worked and money was collected. Each group's task was to help the owner hire workers based on hourly wage rates and the number of vendors needed. The tasks selected for inclusion in the study met the six model-eliciting principles (i.e., model construction principle, reality principle, self-assessment principle, construct documentation principle, construct shareability and reusability principle, effective prototype principle) from Lesh et al. (2000). They were used before in modeling research, which increased the validity and reliability of the study. This study extended the use of these tasks to investigate competencies in group modeling. A summary of the tasks is given in the Appendix.

Instruments

The research instruments were sourced from existing literature so that the researcher could revise the instruments ensuring that the reliability and validity of the sourced instruments were well established. The pilot study was done to determine if the instruments were suited to collect the qualitative data. In the pilot study, each group was given the same problem, and they spent some time at the beginning of the session discussing the context of the task as a class. During the first session, the researcher used the Researcher Observation Guide, which focused on certain aspects of group interaction and increased the instrument's reliability. Because of the size of the class, the researcher had difficulty spending quality time with each group. Students in the pilot study used the Group Reporting Sheet.

Several changes to the research instruments were made after the pilot study. This increased the reliability of the instruments. The pilot study was also a valuable orientation for the researcher regarding the practicalities of undertaking the main study. A brief description of each instrument and its contribution to documenting competency display follows.

i) Interview questionnaire

It was designed to document supplementary information regarding student beliefs about mathematics and their experiences regarding school mathematics. It was implemented before and after the modeling program; therefore, two versions were formulated (i.e., pre-intervention and post-intervention questionnaires). The questionnaires included questions on what students believe mathematics is all about, what they enjoy about mathematics, and where they find mathematics relevant in everyday life. Furthermore, there were questions related to their mathematics experience during the program in the post-intervention version of the questionnaire. Their competencies in using informal knowledge were also gauged from their answers to the questionnaire.

ii) The group reporting sheet

This sheet was formulated to enable students to document their working sessions to enable the researcher to identify some modeling competencies. Groups filled out the reporting sheet during the last 15 minutes of the weekly sessions. Students' competencies in planning, understanding, mathematizing, working mathematically, and their sense of direction were gauged from this sheet.

iii) The researcher observation guide and field book

The Researcher Observation Guide was designed to guide the researcher to certain group competencies such as watching and listening and a sense of direction during each contact session. It was used when the transcripts of the recording were analyzed to determine the level of the competencies. Moreover, it allowed the better generation of researcher field notes.

To take down any information that may have transpired to be relevant at a later stage, the Researcher Field Notebook was used during and after the modeling session. After sessions, extensive notes were also taken in a period of researcher reflection.

iv) The written work guide

This was designed for the researcher to assess the Group Reporting Sheets. In conjunction with transcripts, student written work was also viewed. Photographs of written work were taken to support the data collected.

v) The individual response guides

It was designed to supplement information gathered in the interviews. Students completed an individual response sheet and a sheet called 'I think' at the end of each task.

Data Collection

Eight students were selected for the study and met with the researcher weekly for sessions of 1 hour. The sessions took place after the school day had ended. The researcher's role was as the facilitator of the sessions.

Students worked in their groups on three tasks for six weeks. They worked on each task for two sessions over two weeks. Each group's work was video recorded for future transcriptions. All written work was kept for future analysis. At the end of each session, video transcripts, written work, and researcher notes were assessed and combined to determine each competency index. The indexing was done before the next week's session to assure a valid assessment. These were graphed for a visual representation. The narrative description of the competency accompanied the graphical representation.

Groups started with the Catalogue Problem (Task 1), followed by the Weather Problem (Task 2) and the Summer Jobs problem (Task 3). The group reporting sheets were filled in at the end of each session. The researcher completed the researcher observation guide and the written work guide for each session. Once each task was completed, each student spent time completing an individual response guide. Considering validity and reliability, some techniques were employed in the study.

In ensuring internal validity, several strategies were employed in the study while. In this study, the researcher interacted with the participants in different ways to collect additional data about the phenomenon. Other steps considered for establishing internal validity were keeping researcher notes, avoiding generalizations, choosing quotes carefully, and clarifying the bias the researcher can bring to the study. For external validity, the study followed a modeling program that can be adapted to other settings and contexts. The results obtained from the study are similar to other studies done on modeling (Biccard, 2010; Lesh & Harel, 2003). The tasks used are from other research on modeling and have already proven transferability as with the instruments. The data generation was analyzed by using concepts from the existing research. The coding was adapted from the literature study, making them familiar with modeling and transferable to other modeling tasks. For internal reliability, the study was well-planned and conducted consistently. After each session, the researcher made transcriptions used together with the researcher's field notes. Five levels of indexing the competencies (0, 1, 2, 3, and 4) allowed tighter alignment across tasks and groups. In ensuring external reliability, a detailed account of the focus of the study, the researcher's role, the participant's position and basis for selection, and the context from which data was gathered were provided so that another researcher could follow the exact procedures.

Data Analysis

The coding used for the study emerged from the literature study and the pilot study that was run. The competencies selected for this study were beliefs about mathematics, a sense of direction, planning and monitoring, understanding, simplifying, mathematizing, working mathematically, interpreting, validating, arguing, and using informal knowledge. A table was compiled that involved those competencies (except for beliefs about mathematics) and their levels. Based on certain levels of achievement, the competencies were indexed as Level 0, Level 1, Level 2, Level 3, and Level 4. Level 0 indicates that the students did not display the competency at all or displayed it in a wrong way. Therefore, Level 1 to Level 4 indicates levels of achievement for the competency. Level 1 indicates that the competency was observed but in a very fragile or barren state. Level 2 indicates that the competency was observed and contributed to group progress. Level 3 indicates that the competency observed significantly contributed to the group's progress. Lastly, Level 4 indicates that the competency observed enabled the group to make considerable progress in developing a generalizable model. Accordingly, a score for each group was determined from the data. It was not an absolute measurement but an index of competency. Both student written work and transcripts of recordings were coded for each competency. Colour coding was used. Sometimes, several sentences were coded for more than one competency.

Findings

The two groups are denoted as Group 1 and Group 2. Group 1 was considered a mathematically ‘higher-achieving group. It consisted of two girls and two boys labeled M, A, P, and H. Group 2 was considered a mathematically ‘lower-achieving group. It consisted of two girls and two boys labeled C, S, T, and I. Moreover, R stands for Researcher in the parts included in the transcriptions.

Competency 1: Beliefs About Mathematics

Since students were engaged in a task that was not typical of their experience in mathematics classrooms, it was considered that beliefs were an important competency to analyze for this study. It is accepted that they are not measurable in terms of lower or better; therefore, no index was allocated, and no graph was drawn for this competency. The following aspects of this competency are reported.

1. Mathematics in everyday life

Group 1. Student beliefs about mathematics and the real world were obtained from the interview questionnaires. Students’ responses to the question “How do you think we use mathematics in our everyday lives?” before and after the modeling program are shown in Table 1 for Group 1.

Table 1
Changing Beliefs in Mathematics for Group 1

How do you think we use mathematics in our everyday lives? Before the modeling program	How do you think we use mathematics in our everyday lives and What did you learn about mathematics during the program? After the modeling program
“When shopping at the grocery, market, shopping mall, calculating money”	“We use mathematics even though we don’t know. I learnt that it is important to reason in mathematics.”
“While doing job, playing games, calculating score in football”	“When solving real-life problems like bank accounts. I learnt that math is fun, and we will use mathematics in every area until the end of our lives.”
“In the construction of buildings”	“In solving our problems in daily life, we use mathematics. I learnt that my intelligence is suitable for mathematics.”
“When finding the location of Turkey, while operating with the Cartesian coordinate system”	“It teaches us to reason, to use our intelligence.”

Group 1 students’ initial responses showed some routine uses for mathematics in everyday life. Their responses after the program showed subtle changes in their beliefs. They acknowledged that there are many real uses for mathematics outside the classroom except for routine uses, even though they could not give any specific examples.

Group 2. Table 2 illustrates student beliefs in mathematics before and after the modeling program for Group 2.

Table 2
Changing Beliefs in Mathematics for Group 2

How do you think we use mathematics in our everyday lives? Before the modeling program	How do you think we use mathematics in our everyday lives and What did you learn about mathematics during the program? After the modeling program
"Market shopping, business affairs"	"We will buy curtains in the hall of our house, for this, in the measurement calculation. I learnt to solve problems by reasoning."
"In shopping calculations in the market"	"All over life. I learnt to solve questions in a comfortable way."
"In business affairs"	"While solving real problems in daily life. Mathematics seems to be a difficult lesson by everyone, but when doing different."
"In agricultural calculations"	"I learnt that I shouldn't get bored in math lessons and that math is fun, not difficult."

Before the program, Group 2 students' responses indicated limited routine uses for mathematics in everyday life. It was noted that the students who were exposed to those tasks for the first time had views about the use of mathematics in solving complex real-life problems, except for the routine use in simple daily life.

2. The nature of mathematical tasks

Group 1. Students' ideas about what constitutes a mathematical task form part of their beliefs. During Task 1, one of the members in Group 1, M, claimed that they could round off the answer of 'decimal number' to find the pocket money that Ali should get, assuming that decimal numbers are often rounded off in everyday life. Another incident showing that students viewed mathematics as a neat, systematic endeavor was noted during Task 3 while calculating the averages of the number of hours worked by each vendor for each month. They again rounded off the decimal numbers they were working with.

Group 2. Like Group 1, Group 2 still assumed that round numbers were more correct than decimal numbers by the end of Task 1. They had the following discussion that can be related to the nature of mathematical tasks:

R: What have you decided to do, what are you doing now, for instance?

C: Friends have tried to calculate average right now. I have also tried to divide, but it didn't work, it comes out with a comma.

During Task 2, it was noticed that this group had the belief that a mathematics problem is a collection of rules and skills that are unrelated to each other at all since they did not relate the variables given (number of clear days, days below 15 ^{0}C, days above 20 ^{0}C, and average yearly rainfall) for an individual city collectively. By Task 3, they could relate the variables given more easily and deal with the problem as a set of interrelated structures. A change in their beliefs about math problems and the nature of mathematical tasks became apparent. Even when certain beliefs did not change, at least students talked about them.

Competency 2: A Sense of Direction

Group 1. A sense of direction includes the group knowing what they want to do next and how this relates to what they want to achieve at the end of the task. The group early understood the problem and decided on what they had to do quickly. The first thing the group said after reading the task was:

> P: Madam, I think when we find the ratio, the difference between them—in 2017, 10 years before Ali, Ayşe got 30 TL per month as her pocket money which was enough for her expenses. If Ali gets 30 TL now per month, there becomes an imbalance; he doesn't get to buy as much as Ayşe could. If we can find the price ratio between, I mean the imbalance, we will add it to 30 TL and find the pocket money that Ali should get.

During Task 1, it was observed that the competencies of understanding the problem and mathematization were very important for the group's sense of direction. They considered the problem relating to the mathematics (i.e., ratio) that they knew. This gave one insight into their sense of direction in mathematizing the problem. During Task 2, the group tried to find direction by interpreting the variables given in the problem. Although they had some ideas, in the beginning, they did act on some of the better ideas while solving the problem. In Task 3, based on the idea that Levent should rehire the vendor who makes the most money in the shortest period, they started to consider the total amount of money collected and the total number of hours worked by each vendor.

Group 2. As opposed to Group 1, during the first session of Task 1, the group knew what they wanted in the end, but they were not certain about what they must do next. Although they understood the problem, they did not progress further than this for a long time. Again, the competency in mathematizing the task affected the group's sense of direction. They were concerned with 'doing mathematics' with the calculations they had made. During Task 2, this group worked toward understanding the problem and shared each other's opinions. Their sense was directed by their informal everyday knowledge about how hot or cold the cities on the list are. After this, the group read the supporting documentation to find a solution. By comparing the variables given for each city, they determined a direction based on an elimination method. For Task 3, the group understood the task well and mathematized it. Understanding and mathematizing worked together in creating the group's 'sense of direction.' Figure 5 shows this competency for the groups over the three tasks.

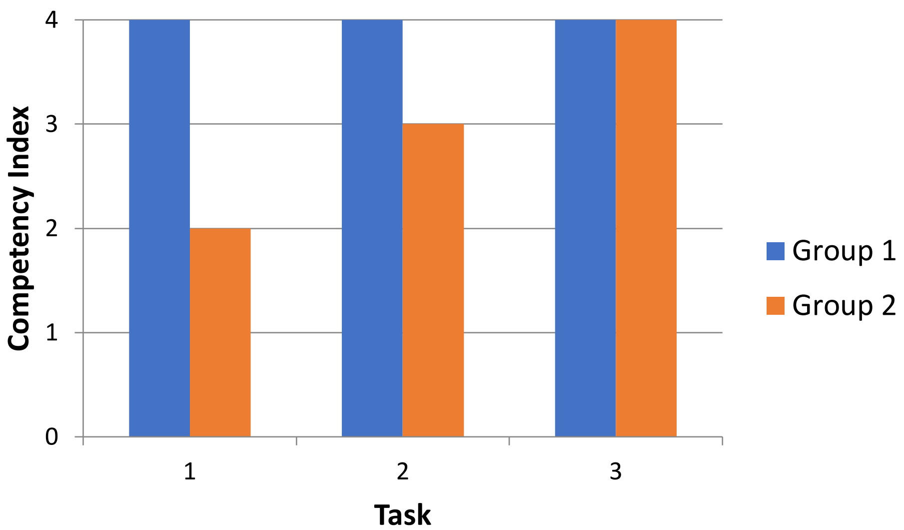

Figure 5. Sense of Direction

Group 1 displayed a greater sense of direction than Group 2. They immersed themselves in the tasks to a better degree. These students tried more approaches, and they also always kept the result of the task in mind. Group 2 was often so immersed in the 'what to do next' or what they were currently doing that they lost the result of the task in mind. Thus, it took more time to decide what they wanted to do as a group.

Competency 3: Planning and Monitoring

Group 1. Planning and monitoring involve managing the problem by organizing aspects of the solution process and keeping track of the solution process. During the first session of Task 1, the group organized the solution process. They managed to keep track of what they were doing by writing things down in a planned manner. During Task 2, students displayed many incidents of planning and monitoring their process. They listened to the suggestions of others and monitored their solution process. Task comments relating to the group's planning and monitoring:

> P: Let's find the median, order the numbers of days from largest to smallest. You can find the most ideal one, then the second ideal or something.

During Task 3, they did plan and monitor their process. There was much discussion, which may be due to the context. In this group, P acted as a planner and monitor. She initiated the ideas and controlled the ideas of the group.

Group 2. Group 2 worked in the most unplanned manner during the first session of Task 1. Students did not spend much time planning and monitoring. The group members did not act actively in managing the problem, and there was minimal discussion about planning and monitoring. During Task 2, students talked to each other, and there was input from all the group members in the discussions. There were incidents where the group started acting as a control. During Task 3, they had autonomy and independence. They planned at the beginning of the first session and reached some consensus on that plan as a group. They continually questioned and challenged their ideas. Figure 6 shows a visual representation of the groups' competency across the tasks.

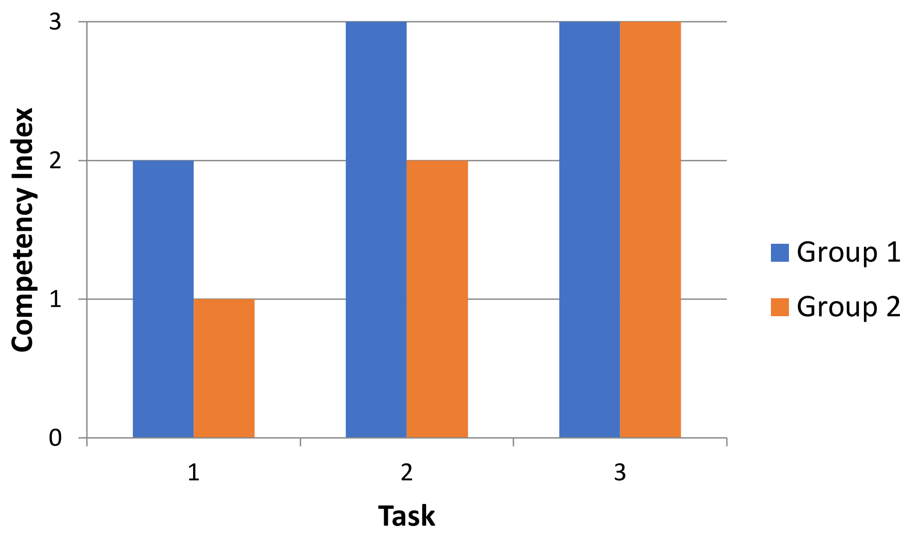

Figure 6. Planning and Monitoring

Group 1 maintained a higher level of planning and monitoring. On the other hand, these competencies were low in Group 2 during the beginning sessions of the program. The groups' planning and monitoring were affected by their autonomy and independence in solving tasks. Besides, teacher guidance and intervention were necessary for demonstrating the competency.

Competency 4: Understanding

Group 1. Understanding is considered as comprehending the written task. The group understood the task during the very first session of Task 1. They carefully examined the supporting documentation to understand the problem and started quickly performing specific calculations. For Task 2, the researcher initially prompted the group to explain their thinking. They checked their understanding with each other.

R: What exactly are you planning to do? What did you understand from the question?

M: We will categorize the cities according to the clients' preferences. For example, we will classify the best in group 1, the middle in group 2, and the worst in group 3.

H: Having the most average yearly rainfall.

A: Mugla has the highest average yearly rainfall.

H: Rize has the highest. Rize…

During Task 2, the group still wanted to do mathematics very quickly, although they understood the problem well. However, they did this to a lesser degree than in Task 1. They spent more time discussing the problem. During Task 3, the group worked more toward understanding the problem than they did in Task 2. They noticed that they needed a deeper understanding of this task than Task 2.

Group 2. Group 2 also understood the task instruction for Task 1; however, this did not guarantee that they would mathematize correctly. They were aware of their lack of progress, and they returned to reading the task many times. During the first session of Task 2, students seemed to understand the problem, but in their method section, it was observed that they involved only one city in each group, and they did not put all cities into the groups. This showed that they did not understand the problem very well. During the initial stages of Task 3, they needed help reading the instruction sheet and the supporting documentation. The group showed a better understanding of this task than the second task. Still, when examining their written work, they noticed that they ignored different times of park attendance (busy, steady, and slow) in their modeling cycle. Figure 7 shows a visual representation of this competency.

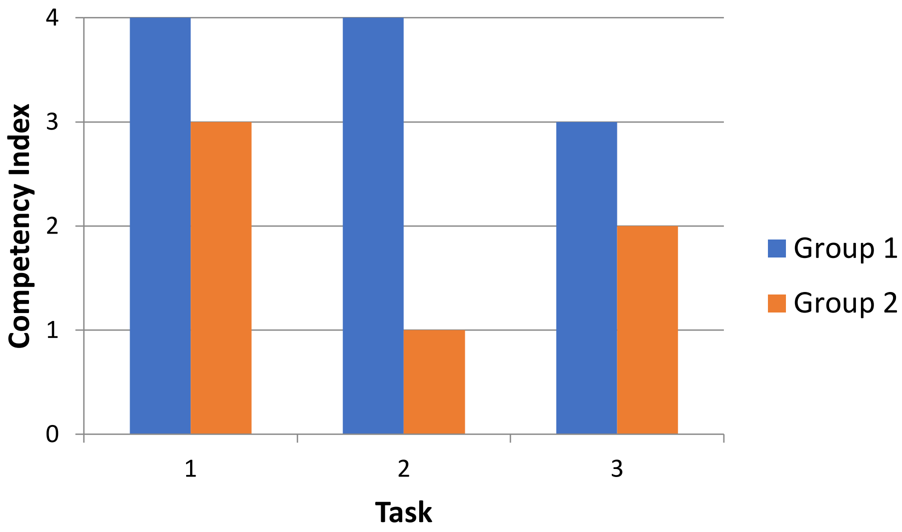

Figure 7. Understanding

Group 1 was mathematically high, and they had a high understanding of the problem during the first task. However, the link between their understanding of the problem and mathematizing was lacking during the first session of Task 1. They maintained higher levels of understanding consistently for the following two tasks. Group 2 was slower in understanding all three tasks than Group 1. They could not bring themselves into the solution process as Group 1 had.

Competency 5: Simplifying

Group 1. Simplifying means extracting the essential features of the problem. Initially, Group 1 started using an average of price differences, so they used all prices on the lists. However, not much later, they decided to use just a few items. They first picked the items assuming they could be bought once a year, and they tried to estimate whether 30 TL is enough for Ali's monthly expenses. For Task 2, it was observed that the group omitted the variable, clear days while choosing the best cities for the first client to live in (one not caring much rain). The decision by the group members to remove clear days was a significant step in simplifying the problem. During Task 3, this group tried to identify several assumptions such that they decided to use the number of hours first. What is noticed is that simplification is part of the students' mathematization abilities. They had to make progress in simplifying the problem to further their solution to the task. Although it is a separate competency, it is integral in mathematics.

Group 2. Group 2 also simplified the catalog items in Task 1. They filtered the lists by subtracting items having prices of 0.99, 1.98, 2.99 TL, etc. They had the idea that if Ali wanted to buy an item with a price of 99 or 98 kr, then the shopkeeper may not give him 1 or 2 kr back. Thus, they grouped items according to their prices to be easier to work with, but it was not a meaningful simplification of the problem. During Task 2, this group eliminated the cities having more days below 15^0 C according to the number of clear days based on the preferences of the first client (a warm and sunny place). In this way, they achieved to determine relevant and irrelevant information. For Task 3, this group used this competency differently than Group 1 did. They decided to calculate the total number of hours worked and the total amount of money collected from each vendor last summer. They used all information in the table, yet they simplified it differently. The visual representation of competency is given in Figure 8.

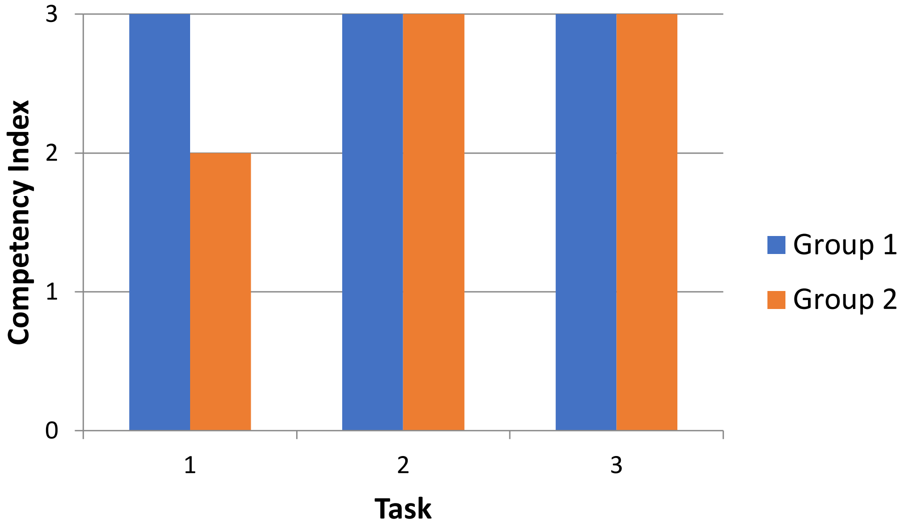

Figure 8. Simplifying

During the first session of Task 1, Group 1 seemed unsure how to deal with the task that had too many numbers. Then, they simplified the vast amount of information by giving meaningful reasons. Group 2 also simplified the data given in Task 1, but their reasons were not as meaningful as Group 1. In Task 2, both groups used a significant sample of the data provided by the task. During Task 3, Group 2 used mathematics by calculating totals, while Group 1 only used sorting of data.

Competency 6: Mathematizing

Group 1. Mathematizing involves translating from the real world to the mathematical world. During Task 1, the group initially planned to take the difference between the sums of the item prices of 2017 and 2007 and then add this difference into 30 TL to find a solution. Later, they decided that an average price difference was what they wanted, so they divided the difference by 12, a ‘monthly’ average. By the end of Task 1, this group had created a third model in which they calculated the averages of the item prices of 2007 and 2017, respectively, and then they took the difference of the averages.

During Task 1, Group 1 decided that calculating an average was relevant, but it was not really. For Task 2, the group’s initial plan was to determine which variables should be involved and which variables should not be in their model. They never needed to use mathematics to solve the problem. The task’s context could be a factor, but a better understanding of the task contributed to their recognizing the structure of the problem more successfully. For Task 3, the group initially planned to find each vendor’s total hours worked and money collected for busy, steady, and slow times respectively, last summer. However, at the end of their calculations, they realized that they could not select the full-time and the half-time vendors, so they decided to create another model by calculating the averages of hours and money of each vendor for each month (regardless of busy, steady, and slow times of park attendance). They tried to build their model based on the assumption that the vendors who collected more money in less time should be rehired.

Group 2. For Task 1, the group first decided to use the averages of the item prices in the lists. By the end of the first session of Task 1, this group had decided to subtract the prices of each item in 2007 and 2017 and then add those differences. After a short discussion, they decided that they should combine their initial model with the final one, so they could find the money that Ali should get. The group was uncertain about their decision. It was seen that although the group understood the problem, they did not mathematize it. During Task 2, like Group 1, this group almost resisted any use of mathematics during the first session. Their initial plan was to compare nine cities based on the clients' preferences (using their personal knowledge or experience). However, they could not reach a conclusion after some discussion. By the end of the session, the group decided to quantify the qualitative information by calculating the averages of the variables given. However, at the end of the first session, the group was still not certain about their decision. They returned to a creative descriptive analysis of the task. They decided to search for the cities according to the number of days below 15^0 C and the number of days above 20^0 C, considering the preference of the second client that of warm and sunny places. During Task 3, it was observed that this group increased their mathematization as the task developed. They created a model based on the assumption that the vendors collected more money in less time. Figure 9 shows the groups' competency in mathematizing the tasks.

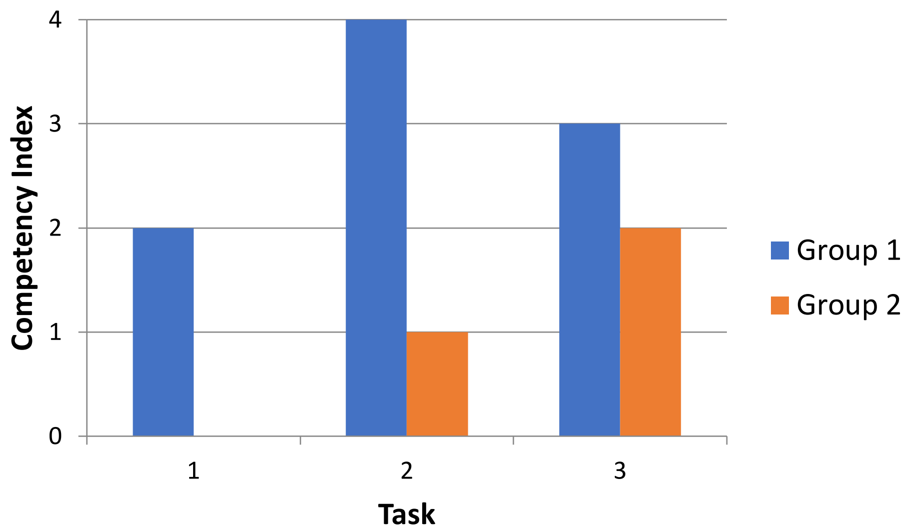

Figure 9. Mathematizing

Throughout the tasks, Group 1 was better at extracting the mathematical concept from the situation than Group 2. This was also related to the group's understanding of the task requirements. Group 2 did not mathematize the problem during Task 1, and they were slow during Task 2; however, in Task 3, they were very much able to mathematize the problem but not fully. They gained from their experiences in modeling. Indeed, proportional reasoning was low in both groups.

Competency 7: Working Mathematically

Group 1. Indexing group working mathematically involves looking for rather than the accuracy of traditional computing but instances of their understanding of the underlying features of the mathematics that they had chosen to apply. During Task 1, sometimes, students used mathematical concepts in ways that were different from their use of the concept in mathematics classes. They wanted to match the sums of item prices in the 2007 and 2017 lists with the pocket money of Ayşe and Ali (should get), respectively.

> A: Can't we find it with ratio and proportion? If Ayşe could buy all the things (making 306,41 TL) with her pocket money of 30 TL monthly 10 years ago then, with how much pocket money of monthly Ali can buy the same things (making 515,07 TL) now?

There were also some instances where students demonstrated some sophisticated uses of averages. For instance, to find how much more money Ali should get now, they said that they calculated the 'monthly' average of the difference of the sums of item prices in 2007 and 2017 lists by dividing it by 12 in their modeling cycle. In another modeling cycle, they calculated the difference of price averages of items in 2007 and 2017 lists to find the amount of money that Ali should get now. This group created their own meaning for words. They used terms such as 'difference' for ratio in their own way.

> P: You say that the difference is the ratio of 2017 to 2007, don't you?

For Task 2 and Task 3, students decided to use an average again (based on their experiences). During Task 2, they calculated the average number of clear days, days below 15^0 C and above 20^0 C. The amount of average yearly rainfall of nine cities and during Task 3, they calculated the averages of the numbers of hours worked and the amount of money collected for each vendor last summer. Still, they omitted different times of park attendance into their calculations.

Group 2. For Task 1, Group 2 followed a similar solution path to Group 1. While calculating the sum of the differences between item prices in 2007 and 2017 lists of each item, they realized that some of the item prices picked up and some of them dropped. They indicated this successfully in their calculations. They had the following discussion about the subtractions that they were doing:

> R: What did you think about those minuses?
>
> C: In fact, they were pluses; it became minus since we subtracted the larger from the smaller.

Shortly after this, they decided to use an average. However, they tried to find the average item prices in the 2007 and 2017 lists by dividing the totals by 30 (the amount of Ayşe's pocket money ten years ago). For Task 1, both groups had some difficulty working with averages. During Task 2, the group used the inverse operation (subtraction) to find the number of days above 15^0 C and days below 20^0 C for each city. They subtracted the number of days below 15^0 C (given in the climatic information table) from 364. They did a similar calculation to find the number of days below 20^0 C for each city. Here, they accepted one year as 364 days. For Task 3, this group followed a similar solution path to Group 1. They calculated the total number of hours worked and the total amount of money collected for each vendor last summer regardless of different times of park attendance. What was noticed was that, in their calculations, rather than adding all hours and the money of a vendor in three months at once, they first calculated the total hours and the money in each month (adding the hours and the money in busy, steady, and slow times in each month) and then they added those totals to find the total hours and the money of a vendor in last summer. It seemed that they were comfortable doing their calculations, but they used a complex way of calculating totals. During their discussion, one group member said they could use a ratio by dividing the total number of hours worked by the total amount of money for each vendor. Still, the group did not consider it in their modeling cycle. Figure 10 presents a full view of this competency for the groups.

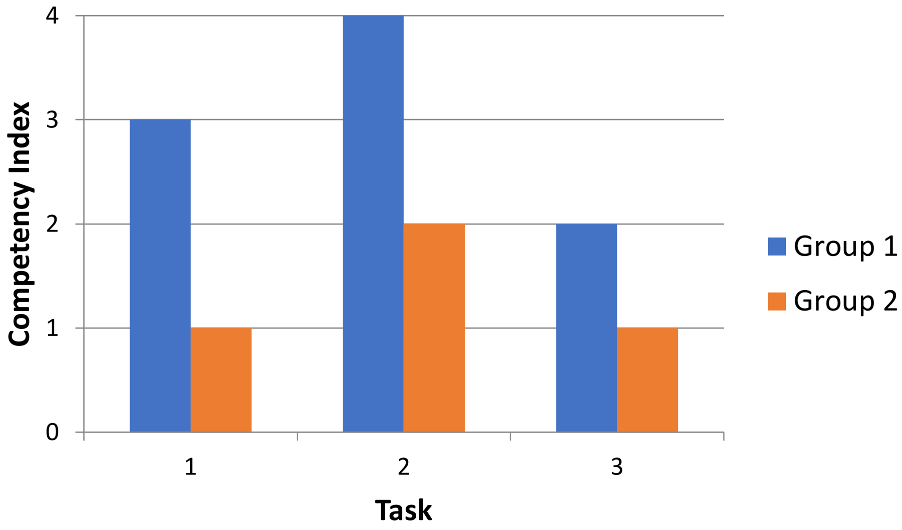

Figure 10. Working Mathematically

Both groups used the four basic operations fluently. They were largely unable to use the mathematics they knew (i.e., average, ratio) in situations different from their mathematics classes. Towards the end of the program, students felt that they were using mathematics rather than doing it.

During the program, it was observable that both groups calculated manually on paper even though the calculator was provided from the start. They preferred manual calculations for some aspects of their work throughout. They stated that they used the full decimal number to be more accurate. However, working with decimal numbers posed problems to Group 2. Group 1 preferred to round off while reaching a conclusion. Figures 11 and 12 show examples of manual calculations of Group 1 and Group 2, respectively.

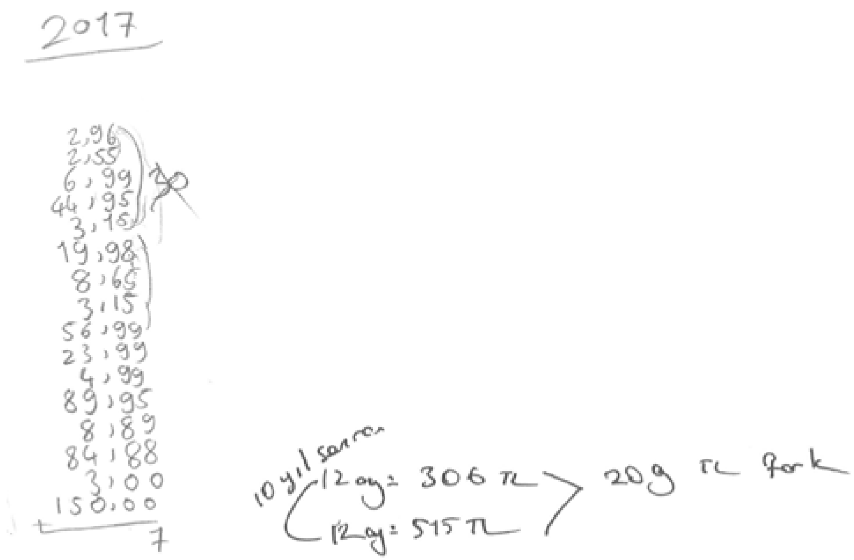

Figure 11. Group 1 Task 1

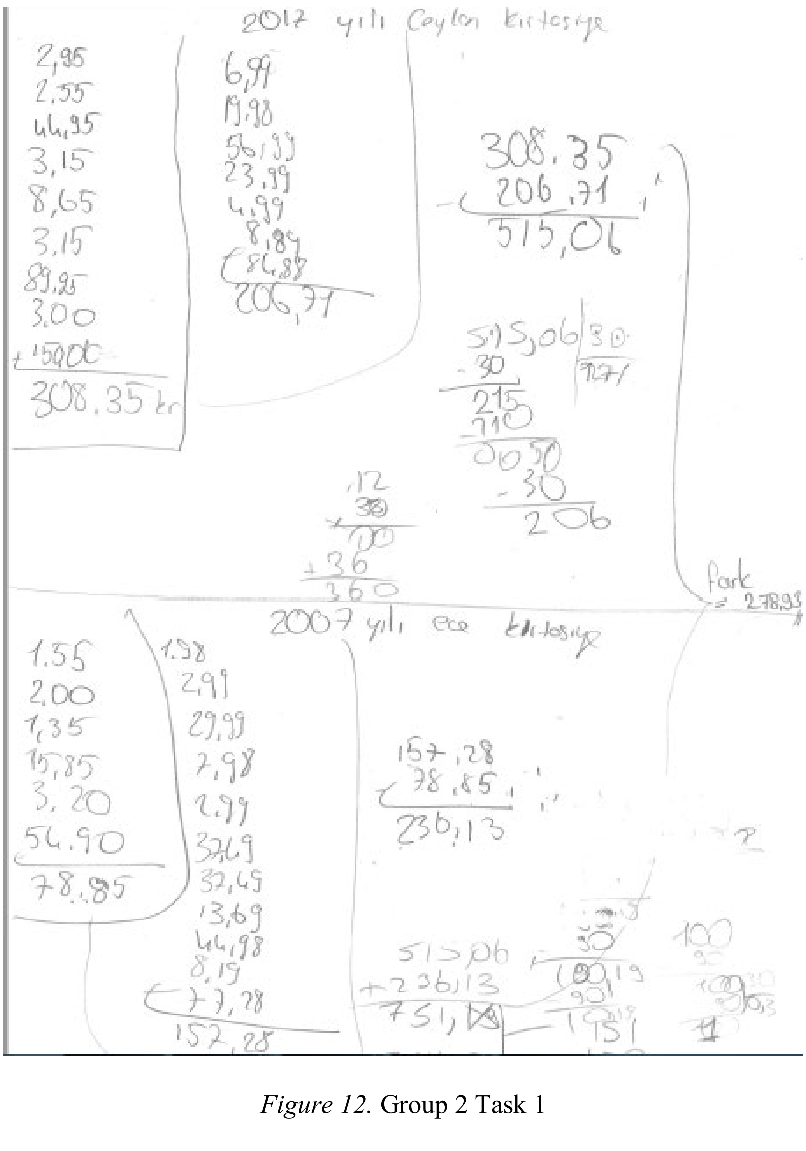

Figure 12. Group 2 Task 1

Competency 8: Interpreting

Group 1. Interpreting was coded according to the group's interpretation of their mathematical solution within the context of the real problem given. Group 1 continually checked that their mathematical solution provided a reasonable answer for Task 1.

> P: But now prices have picked up in 2017 (they found the difference of totals of item prices in 2017 and 2007 as 208,66 TL); if Ali needs the same items, then his pocket money should be more than 30 TL.

For Task 2, this group interpreted their answers by revisiting the task to check whether their recommendation for each client was acceptable. They considered part of the preferences of clients while interpreting their results. During Task 3, this group found the amount of money collected per hour from each vendor and decided on full-time and half-time workers accordingly. They felt so confident in their calculations that they thought there was no need to interpret them.

Group 2. For Task 1, the task's context highly depended on the group's ability to mathematize the problem, which was low in both groups. Therefore, Group 2 had a low model and interpretation. For Task 2, this group followed a similar way of mathematizing the problem to Group 1, and they interpreted their results in that way. They also used their informal knowledge about climate change in those cities in their interpretations. During Task 3, the group found a mathematical solution without considering different times of park attendance. They were sure about the mathematics they had used for this task; however, since they did not understand the problem fully and did not work mathematically well, they did not interpret their solution within the task's context. Figure 13 shows the groups' competency in interpreting their solutions.

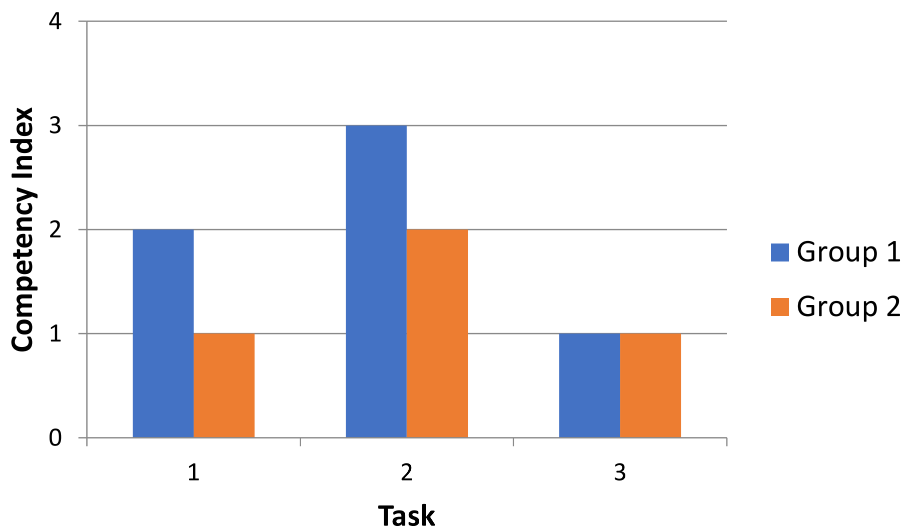

Figure 13. Interpreting

Group 1 interpreted more regularly and steadily than Group 2 for three tasks. Sometimes, both groups did less mathematize the problem; therefore, they could less interpret the results of the problem. Group 2 showed adequate interpreting for Task 2 and struggled during Task 1 and 3. For both groups, there was more interpreting of results for Task 2 than for the other tasks. This could be because of the context of the task.

Competency 9: Validating

Group 1. Validating refers to students verifying that their model is consistent and satisfies the conditions of the real situation context. It also involves checking the solution. For Task 1, initially, Group 1 checked if 30 TL was enough pocket money that Ali could buy all the items in the 2017 catalog list. It was a way for the group verification of the situation presented in the problem in a real-world context, but it was not a true means of the group's validation of the model created. After that, they intended to verify their solution of 42 TL as the pocket money that Ali should get now by developing another model. The validation used in Task 1 was related to their use of extra mathematical knowledge. During Task 2, the group validated that their method would work for both clients by revisiting the problem and ensuring that the preferences of both clients were met. They also used their informal knowledge while validating their solution. In Task 3, the group's sense of correct mathematics superseded the need to validate the solution. In their model, they concentrated on how to select full-time and half-time vendors and tried to validate their model in that realm.

Group 2. Group 2's validation used in Task 1 remained in the realm of intuitive validation. During Task 2, the group showed more instances of validating their work and models than they did in Task 1. Towards the end of the task, one of the group members realized that their model did not work for the other client, that they selected only one city for each group of the best cities, the second-best cities, and the worst cities. Yet, they did not develop a feeling that their model must apply to the other client. In Task 3, this group did not validate their solutions and models. Like interpreting, their ability to validate the model was directly affected by their low understanding of the problem. Figure 14 shows how the validating competency of the groups was over the three tasks.

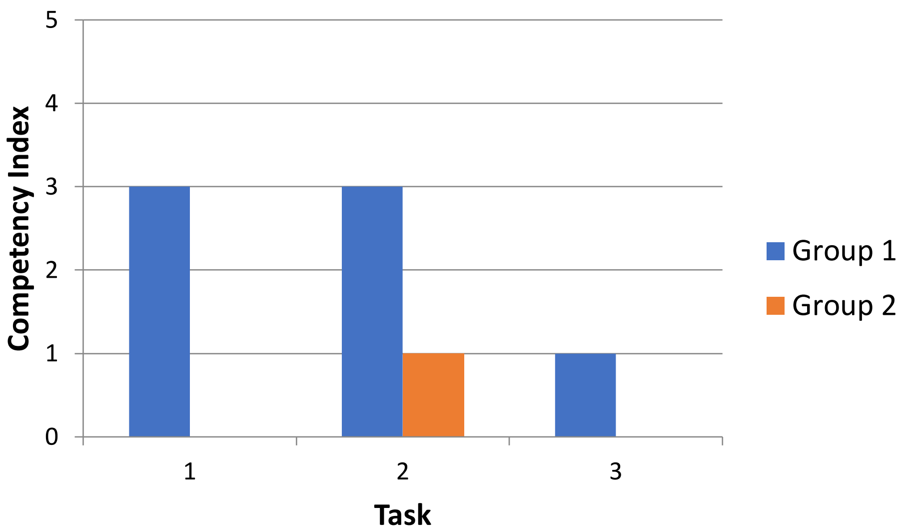

Figure 14. Validating

The task's context seemed to factor in students validating their models in generalizable means. There were more instances of validation in Task 2 than in the other tasks. Group 1 validated all their solutions and models to a greater extent than Group 2 did in all three tasks.

Competency 10: Arguing

Group 1. Arguing refers to reasoning, which entails connecting thoughts to explain something to oneself or others. During Task 1 and 2, Group 1 sustained a more extended period of argumentation on a particular problem feature than Task 3. They displayed several instances of linking their logical thoughts or ideas by providing the next logical thought by a different student than the previous logical thought over the first two tasks. Still, they did not maintain this for Task 3.

Group 2. During Task 1, Group 2 did not follow a line of thought for more than one sentence or two, but they improved this by Task 2. More group members were involved in the process of development of an argument in Task 2. During Task 3, a poor understanding and mathematizing the task influenced their arguing competency. Figure 15 shows the groups' arguing competency across the three tasks.

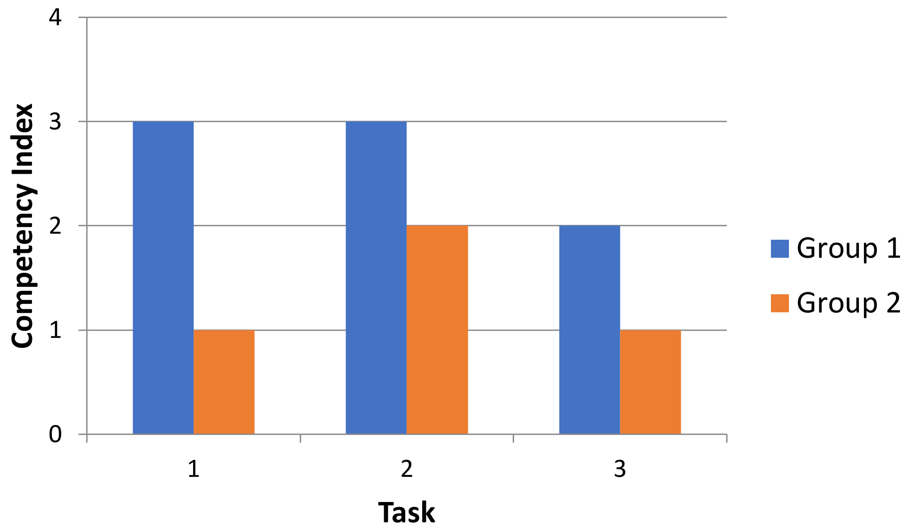

Figure 15. Arguing

Group 1 could reason through many aspects of the tasks more than Group 2. Their language and mathematical ability assisted with this. They maintained a level of arguing over the three tasks since they were in touch with each other and were trying to understand each other more than Group 2. Group 2 generated shorter and less efficient arguments over the three tasks.

Competency 11: Using Informal Knowledge

Group 1. This is related to student knowledge in their real lives about the task. During Task 1, Group 1 decided that they should distinguish the items based on their time of consumption. They made some general comments about the task. In Task 2, this group used their informal knowledge to check the solution that they found. In Task 3, Group 1 did not use their informal knowledge. This was because of the context of the task.

Group 2. During Task 1, Group 2 grouped items in the lists according to their prices based on their daily experience. Even though they used what they knew in their real lives about the task, this was some low use of informal knowledge to approach it. For Task 2, this group also used their informal knowledge to assist with the task. Like Group 1, they did not use any informal knowledge about the task during Task 3. Figure 16 shows a visual representation of this competency index.

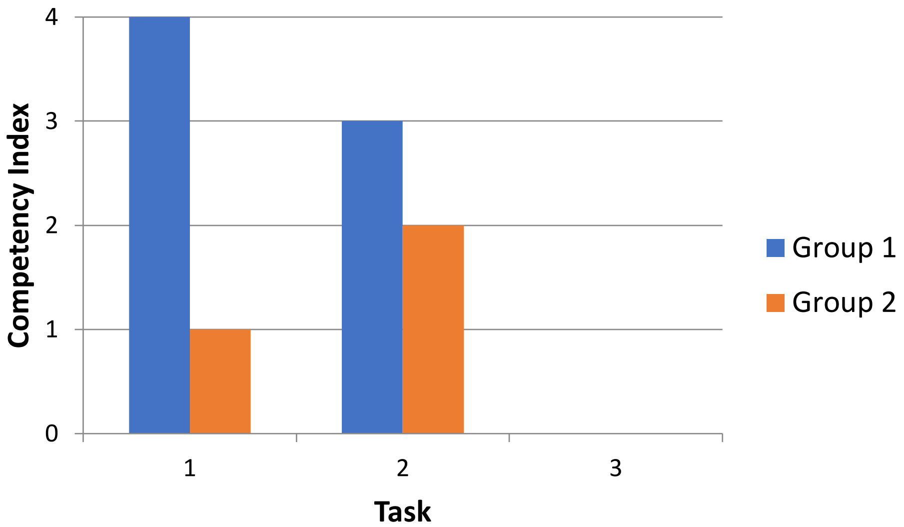

Figure 16. Using Informal Knowledge

It is clear from the graph that Group 1 used their informal knowledge to a greater degree. In both groups, sometimes students' comments during the group interactions about the task had effects on task resolution, but sometimes they did not. It was interesting to note that, during Task 3, there was no incident of using some informal knowledge to assist with the task for both groups. This competency was also dependent on the context of the task.

Discussion and Conclusion

This study investigated the modeling competencies of seventh-grade students stereotyped as lower and higher achievers working in groups. Results revealed that modeling competencies of students in this study were moderate. Just as competencies do not display themselves in a linear way (Biccard, 2010), the groups displayed different levels of competence in different areas of modeling. In terms of particular competency, both lower and higher-achieving groups showed higher levels of competence in simplifying, a sense of direction, planning, and monitoring than that of mathematizing, validating, interpreting, and arguing during modeling tasks. According to Lesh et al. (2000), during their early interpretations of modeling problems, students often only focus on a subset of the information available. It was observed that the groups used simplifying for different reasons and at different times during the task. In the present study, the group's sense of direction was investigated on two levels: knowing what they wanted to do next and what they wanted to achieve at the end of the task. In many of the tasks, both groups knew what they wanted to achieve at the end of the task, but they were not certain about what to do next. In this study, it was observed that the group's planning and monitoring was related to the meta-cognitive ability of each student in the group. During the beginning sessions of the program, the groups' planning and monitoring were low. This might result from highly structured settings that students are exposed to in traditional classrooms. Students displayed a higher level of planning and monitoring when they started to talk to each other to a greater degree. The context of the task also could contribute to that. Indeed, students will reach higher competency levels if they become more involved in modeling situations. Moreover, subtle changes were observed in students' beliefs about mathematics after being exposed to only three tasks over six weeks.

In the present study, it was found that group competencies in mathematizing, interpreting, validating, and arguing were low for both groups throughout the tasks. This might be because these competencies do not often appear in traditional classrooms. Treffers (1993) explained that several factors are necessary for children to mathematize a problem automatically: personal reality, mathematical knowledge, and the context of the problem. Moreover, it can be said that students need more exposure to such real problems. Interestingly, even though the higher-achieving group had a high understanding of the problem and high mathematical skills, they did not see the link during the first task. By the second task, they showed a better mathematizing ability. It was also noticed that both groups did not use sophisticated levels of mathematics in their solution processes. The group competencies in interpreting, validating, and arguing followed much the same way as mathematizing. Fewer students mathematized the problem, the less they could interpret their results in terms of the real problem. They were sure that their mathematics was correct and assumed that their solution was correct, but then they did not check whether their final solution provided a credible answer in terms of the real problem. It was also observed that both groups maintained a level of arguing over the tasks, but this did not assist in usable results of their work. From this study, it can be concluded that mathematizing is the most critical competency during modeling. This is consistent with what Biccard (2010) found in her study that mathematizing filters into, and is comprised of, other competencies. It was observed that simplification was affected by the group's mathematizing ability, and it affected further mathematizing the problem. Moreover, the significant mathematization of the problem was very important in the groups' sense of direction. Their sense of direction was impeded by the struggle to mathematize the problem. The group's ability in one competency affected their ability in many other competencies, and this is in line with the evidence that the competencies are interrelated and interdependent (Biccard, 2010).

In the present study, it is of particular interest to explore the differences between lower and higher-achieving groups regarding the modeling competencies used for the study. As we expected, the higher-achieving group showed higher levels of achievement in almost all competencies during the program. They started the program with greater initial competencies. They could use their 'mathematical toolbox' (Jensen, 2007, p. 144) to a much greater degree than the lower-achieving group throughout the tasks. They displayed a greater sense of direction during the program. This parallels the finding of Treilibs' (1979) that good modelers exhibited a high sense of direction and consequently performed better than other students (as cited in Maaß, 2006). In the study, it was also observed that the reading and reading comprehension of the students in the lower-achieving group were lower than those of the higher-achieving group. This was a significant factor in their low understanding of the tasks. According to Hyde (2007), reading allows students to understand the task instructions, and allows students to connect, visualize, infer and predict, and play a role in meta-cognition. In this study, it was pleasing to observe that the lower achievers engaged with the tasks much more than they did with their schoolwork. This is in line with what Legé (2005) found in a study that some students who had done nothing all year became involved and engaged in the modeling tasks during a mathematics remediation course. Yet, it was observed that the lower-achieving group did not benefit from their grouping much. They needed more assistance from the researcher in progressing through the modeling cycle.

Several recommendations can be made for practical research and teaching from this study. Firstly, it would be better if groups were allowed to complete modeling tasks in one session rather than in weekly sessions. Secondly, groups should be required to present their solution and model to other groups after completing a task. Teachers or researchers can video record presentations and use a rubric to assess them consistently. Then, the period of the program must be adjusted accordingly. Thirdly, students should be exposed to various members of their groups. It may benefit the field if the modeling competencies in heterogeneous groups are investigated and the results are compared. Along with this, students should be exposed to modeling tasks that incorporate different areas of mathematics. Fourthly, effective teacher modeling knowledge is necessary so that students can display competencies even better. Lastly, since it was difficult for some students to attend the sessions after school hours, further studies should occur within the mathematics classrooms.

Since modeling tasks require a vast amount of reading, students' reading and reading comprehension abilities are significant in understanding and completing tasks successfully. Thus, the connection between competency in modeling and reading ability should be considered for future study. Moreover, a full study of each competency should take place. In this study, it appeared that mathematizing is a critical competency in modeling, so its identification and integrity in other competencies is another area that will allow a fruitful study. Finally, the competencies selected for this study were necessary for successfully concluding tasks. They cannot be displayed using traditional instruction and are essential to the display of mathematics as its own competency. However, they are not the only ones evident during typical group modeling activities. Other factors can be identified or selected for studies that contribute to modeling competency display.

The study aims to give important consideration and recommendations to modeling that can be realized in mathematics classrooms as a mathematical activity. This study provides extensive knowledge of group modeling competencies of lower and higher-ability students, which will assist with designing teacher development programs and implementing modeling tasks in the classroom. Since there is a benefit to both lower and higher-ability students, modeling tasks can be implemented in all school types and for all ability students who need to improve mathematics teaching and learning.

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Appendix.

Task 1: Catalogue Problem (Lesh et al., 2000)

Instruction

Hello, my name is Ali and I need some help with a problem. My parents are really unreasonable. My sister, Ayşe, is 10 years older than me. When she was in Grade 7 her pocket money was 30 TL per month. I also get 30 TL per month. With 30 TL I cannot buy as much as she could 10 years ago. To prove this, I collected some information about prices now and 10 years ago. What I need from you:

- Use my price information to determine how much pocket money today would be the same as 30 TL 10 years ago.
- Write a report for me to give to my parents.

  Describe your method and your conclusions.

  Show that you accurately figured out how much money gives me the same spending power as 30 TL did 10 years ago.

  Explain your method so other children in similar situations can use it to figure out what their allowances should be.
- Remember, my parents do not like emotional or illogical arguments.

Thanks

Ali

Supporting Information: Catalogue prices from 2007 and 2017.

Task 2: The Weather Problem (Doerr & English, 2003)

Instruction

The Global Travel Agency is interested in starting a re-location service to help advise people who are moving to a new area. The travel agency needs your help to develop an advising system for choosing places for their clients to live. The clients are primarily interested in the climate: how much rain, how cold it gets, how hot it gets, and if the days are sunny or cloudy. Each of these factors, however, is not of the same importance to every client.

Two potential clients have sent letters to the agency describing their preferences and asking for the agency's advice on the best places for them to live. The agency also has gathered some information on the nine cities. You have been asked to:

1. Develop a rating system for comparing the climates in different places. Be sure your system will really help the agency evaluate places, even those not listed.
2. Write two letters for the travel agency with a recommendation for each of the clients. You should put the cities into three groups: the best cities, the second-best cities, and the worst cities. This way the client will know which cities to consider living in and which cities to avoid.
3. Explain to the travel agency how your rating system works and why it is a good one.

Supporting Information: Letters of two clients, climatic information of nine cities.

Task 3: The "Summer Jobs" Problem (Johnson & Lesh, 2003)

Instruction

Last summer Levent started a concession business at Youth Park. His vendors carry popcorn and drinks around the park, selling wherever they can find customers. Levent needs your help deciding which workers to rehire next summer. Last year Levent had nine vendors. This summer, he can have only six—three full-time and three half-time. He wants to rehire the vendors who will make the most money for him. But he doesn't know how to compare them because they worked different numbers of hours. Also, when they worked makes a big difference. After all, it is easier to sell more on a crowded Friday night than on a rainy afternoon. Levent reviewed his records from last year. For each vendor, he totaled the number of hours worked and the money collected—when business in the park was busy (high attendance), steady, and slow (low attendance). You have wanted to evaluate how well the different vendors did last year for the business and decide which three he should rehire full-time and which three he should rehire half-time.

Write a letter to Levent giving your results. Include in your letter how you evaluated the vendors. Give details so Levent can check your work and give a clear explanation so he can decide whether your method is a good one for him to use.

Supporting Information: For each vendor, hours worked, and money collected last summer.