An Evaluation of Mathematical Tasks Designed by Pre-Service Teachers Within the Framework of Task Design Principles

Özge Gün
Bartin University
ozgegun@bartin.edu.tr

Fatih Taş
Bartin University
fatihtas@bartin.edu.tr

It is important to support pre-service teachers in terms of mathematical task design so as to increase the success of their future teaching. The purpose of this study was to evaluate mathematical tasks designed by pre-service primary and elementary mathematics teachers within the framework of task design principles. For this purpose, a total of 43 tasks corresponding to the learning objectives in primary and elementary school mathematics curriculum were examined through content analysis. Results revealed that pre-service teachers have considered aim of task, classroom organization, students' prior knowledge and multiple start point principles for all or most of their tasks; measurement and evaluation principle for almost half of their task; and instruction for using materials and student's misconceptions and difficulties principles for very few of their tasks. This study can provide teacher educators a guideline for improving their programs to support pre-service teachers in terms of mathematical task design.

There is a revival of interest in task design in mathematics teaching (Watson, 2008). It is proposed that "what students learn is largely defined by the tasks they are given" (Hiebert & Wearne, 1993, p. 395). There are several kinds of impact resulting from engaging teachers in the tasks that they designed (Zaslavsky, 2007). The role and nature of mathematics-related task in enhancing teacher learning is one of the significant aspects of mathematics teacher education and there is an attempt to conceptualize it (Zaslavsky, 2007).

In research studies, it is observed that there are many different uses of tasks in mathematics teacher education. Watson and Mason (2007) listed aspects of task-use in teacher education settings, whether pre-service or in-service, which are common throughout the world. It includes asking teachers to: work on mathematics; then use similar tasks in practice; analyze task structures and observe lessons using them; observe, analyze, compare teaching and learners' work using similar tasks. According to Swan (2006), to involve teachers in the design process is also another way to engage them with tasks. He used this approach in his study which involved all stages of the design process with teachers and re-design.

In task literature, there are two frequent uses of the word 'task' referring to one as problem (Brousseau's (1997) epistemological notion of problem) and the other as task (Doyle's (1983, 1988) academic-task approach) (Herbst, 2008). Doyle (1983) defined academic tasks as "the products that students are expected to produce, the operations that students are expected to use to generate those products, and the resources available to students while they are generating the products" (p. 161). In related literature, the conception of mathematical task is used more similar to Doyle's notion of academic task. More recently, Stein et al. (1996) proposed a conceptual framework based on the construct of a mathematical task, referred to Doyle's notion of academic task, and defined a mathematical task as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea, or skill" (Henningsen & Stein, 1997, p. 528). In this study, task refers to Henningsen and Stein's (1997) definition of mathematical task.

In National Council of Teachers of Mathematics' Mathematics Teaching Today document, 'worthwhile mathematical tasks' is one of the standards for effective mathematics teaching and it states that "the teacher of mathematics should design learning experiences and pose tasks that are based on sound and significant mathematics" (Van de Walle et al., 2010, p. 535). In Turkey, there has been an interest in tasks in mathematics teaching environments since 2005. In 2005, there had been a reform movement in Turkish mathematics curriculum for K-12 schools which focused on students' conceptual learning and a learning developed through personal experience and real-world situations. The 2005 curriculum manual offered a rich and diverse collection of sample activities along with particular learning objectives for all grades of K-12 schools. It is the most detailed one among the curricula of 1948 through 2018. As for the current curricular document (i.e., the 2018 curricula), it does not offer any specific examples of tasks; it only offers learning objectives that students are supposed to know by a certain grade level. It is believed that it is important to mention tasks in curricular documents since it provides possible ways in which the student may engage in activities which may promote learning certain concepts for both in-service and pre-service teachers.

Mathematical Task Design

In task design studies, it is generally observed that researchers have investigated some features of mathematical tasks and the effects of these features on some variables. For example, in their study, Stein et al. (1996) analyzed 144 mathematical tasks in terms of two interrelated dimensions: tasks features and cognitive demands. Designing tasks that encouraged conceptual development, Silver et al. (2009) focused on the task's mathematical and pedagogical features.

There is also a great deal of research focused on types of tasks implemented in mathematics classrooms. An early classification of tasks by Doyle (1983) was four general types: (1) memory tasks, (2) procedural or routine tasks, (3) comprehension or understanding tasks, and (4) opinion tasks. More recently, Swan (2005; 2011) identified different task genres that promote concept development and select a rich of that kind. They were (1) classifying and defining; (2) interpreting and translating between multiple representations; (3) testing and evaluating mathematical statements conjectures; (4) creating and solving variants of mathematical problems; and (5) analysing reasoning and solutions.

There are many principles elicited from the literature review for successful mathematical task design. In her paper, Bell (1993) introduced principles for designing teaching. She stated that tasks proposed to students requires some form of feedback, some type of intervention by teacher or task (flexibility provided by task), a multiple starting, intensity, and reflection and review. In Stein's et al. (1996) mathematical task framework, task features included multiple solution strategies, multiple representations, and mathematical communication. They also identified task conditions (e.g., the extent to which tasks build on students' prior knowledge and the appropriateness of the amount of time that is provided for students to complete tasks) as one of the factors that influence students' implementation. A more comprehensive work of Swan (2007) included many principles of task design elicited from the literature review as:

> the importance of focusing directly on significant conceptual obstacles and general structural features of situations; eliciting, confronting and building on the knowledge students already have; carefully juxtaposing questions and stimuli so as to produce surprise, tension and cognitive conflict that may be resolved through reflection and discussion; using tasks that are accessible, extendable, encourage

decision-making, creativity and higher order questioning; using multiple representations to create bridges between concepts; and using tasks that allow students to shift roles and explain and teach one another (p. 219).

Additionally, Watson and Mason (2007) mentioned other factors including "ethos and atmosphere; established practices and ways of working; students' expectations of themselves and of each other as influenced by the system and their pasts; and learners' sense of self-confidence, agency (mathematically and socially) and identity" (p. 207). In order to describe tasks that encouraged conceptual development, Silver et al. (2009) focused on the task's mathematical and pedagogical features. They stated that while mathematical features of a task include the mathematical topics of the task and the mathematical cognitive demands inherent in the task, the pedagogical features refer to the organization and enactment of the task.

In Turkey, there are few studies which handle mathematical task comprehensively (Olkun & Toluk-Uçar, 2007; Özmantar & Bingölbali, 2009; Uğurel & Bukova-Güzel, 2010). Olkun and Toluk-Uçar (2007) described some basic parts of a constructivist mathematical task in their book about activity-based mathematics teaching for elementary education as: (1) intuitive phase, (2) constructed task, (3) discussion-explanation, (4) comprehension/reach rule, (5) implementation, and (6) evaluation. Moreover, in the 2005 curriculum manual, sample activities included subtitles such as: lesson, grade level, content strand, sub-strand, skills, objectives, materials and tools, teaching and learning process, and measurement and evaluation. In a comprehensive work from Özmantar and Bingölbali (2009), a collection of principles of a mathematical task design was presented as purpose (aim, learning objectives, perceived purpose), use of time, classroom organization, student's prior knowledge, multiple start point, comprehensiveness, required materials, the roles of student, the role of teacher, student's misconceptions and difficulties, measurement and evaluation, and flexibility. In their study, Uğurel and Bukova-Güzel (2010) examined the conceptual understanding and structure of tasks, their samples and types in terms of the Turkish secondary school mathematics teaching curriculum's perspective and they reached a classification of isomorphic, projective, linear and composite task in terms of their perspective.

Upon task literature, there is a great variety of opinion about what task is, its' features, the product end of it, or its' implementation, and so on. Due to richness in meaning, variations in examples, non-classifications and insufficiency of in-service and pre-service trainings and published documents in Turkey, it can be predicted that both in-service and pre-service teachers have difficulty in designing mathematical tasks (Uğurel et al., 2010). This is also supported by research findings that teachers have a limited perception of mathematical task (Bozkurt, 2012; Özmantar et al., 2010; Uğurel et al., 2010) and some problems with task design (Bal, 2008; Bukova-Güzel & Alkan, 2005; Duru & Korkmaz, 2010; Toptaş 2006). Thus, in Turkey, it is object of interest that at what level pre-service teachers are acquiring the role of mathematical task designers through teacher education. Therefore, the purpose of this study was to evaluate the mathematical tasks that pre-service primary and elementary mathematics teachers designed according to Mathematical Task Design Principles (MTDP) presented by Özmantar and Bingölbali (2009).

Method

Research Design

This is a qualitative research study that used content analysis to examine the mathematical tasks that pre-service primary and elementary mathematics teachers designed. Content analysis is defined as "the systematic set of procedures for the rigorous analysis, examination, and verification of the contents of written data" (Flick, 1998, p. 192; Mayring, 2004, p. 266, as cited in Cohen et al., 2007, p. 475). To get information about how proficient pre-service teachers are in designing mathematical tasks, the principles of task design (Özmantar & Bingölbali, 2009) were used in this study.

Data Sources

The study examines the mathematical tasks designed by pre-service primary and elementary mathematics teachers. For this purpose, a total of 43 mathematical tasks; 19 of which were designed by 36 pre-service primary teachers and 24 of which were designed by 45 pre-service elementary mathematics teachers in a mathematics teaching methods course of their third year of teacher education programs during the spring semester of 2013-2014 academic year were examined. Pre-service teachers designed tasks mostly in groups of two (17 tasks by pre-service primary teachers and 21 tasks by pre-service elementary mathematics teachers) and a few tasks by individually (2 tasks by pre-service primary teachers and 3 tasks by pre-service elementary mathematics teachers) towards their respective grade levels. That is, pre-service primary teachers designed tasks for learning objectives of primary school (1-4. grades) mathematics curriculum (Ministry of National Education [MoNE], 2005) and pre-service elementary mathematics teachers for learning objectives of middle school mathematics (5-8. grades) curriculum (MoNE, 2013). They were free to choose the grade level and learning objectives that their tasks would cover. Before designing tasks, pre-service teachers studied the topics 'what task is', 'types of tasks', 'task design principles' with detailed explanations and some specific examples in the classes.

Framework for Data Analysis

In this study, MTDP framework introduced by Özmantar and Bingölbali (2009) was used in the analysis of data. It is a collection of features that stand out as a result of the examination of the studies on task design and which are commonly mentioned in many studies. In what follows, we present a brief definition of each principle.

1. *Aim of task:* It defines the general purpose of task designed. There are four different task types in the literature: introducing new learning, practicing already learned concepts, overcoming student's difficulties and misconceptions, and raising awareness of the epistemological structure of the field (Özmantar & Bingölbali, 2009, p. 323).
2. *Use of time:* It defines the amount of time required to complete task and the plan of it (Özmantar & Bingölbali, 2009, p. 331).
3. *Classroom organization:* It defines the organization of classroom during task implementation including students' ways of working (i.e., individual work or group work; whole class discussion or teacher's direct instruction, etc.), teacher's way of finishing task (finishing by explaining correct answer to whole class, making explanations by pointing students' errors, explaining the learning objectives formally to whole class, or listening students without making any explanations, etc.) (Özmantar & Bingölbali, 2009, p. 332).

4. *Students' prior knowledge:* It defines the students' content knowledge as well as their knowledge about each aspect of task designed (i.e., students' ways of working, materials, etc.) (Özmantar & Bingölbali, 2009, p. 338-339).
5. *Multiple start point:* It defines designing task having the opportunity of different starts for students (Özmantar & Bingölbali, 2009, p. 333).
6. *Comprehensiveness:* It defines designing task for students of different achievement levels and skills (Özmantar & Bingölbali, 2009, p. 333).
7. *Appropriateness of material:* It defines designing task by carefully selecting materials, determining the roles of materials play to achieve the learning outcomes of the task, students' ability of using materials or their prior knowledge about using materials, materials' opportunities or limitations, problems can be about using materials (Özmantar & Bingölbali, 2009, p. 336).
8. *Supply of material:* It defines whether materials can be supplied by students, accessibility of materials, or alternative materials (Özmantar & Bingölbali, 2009, p. 336).
9. *Instruction of material:* It defines instructions and explanations about how materials are used (Özmantar & Bingölbali, 2009, p. 336).
10. *Student's role:* It defines roles of students assigned by teacher. These roles are included in the parts of understanding of instructions, use of materials, organization of students, students' difficulties, interventions, and measurement and evaluation of task designed (Özmantar & Bingölbali, 2009, p. 337-338).
11. *Teacher's role:* It defines roles of teacher determined by them. These roles are included in the parts of understanding of instructions, use of materials, organization of students, students' difficulties, interventions, and measurement and evaluation of task designed (Özmantar & Bingölbali, 2009, p. 337-338).
12. *Student's misconception:* It defines prediction of the misconception(s) that student may have about the conceptual content of the task designed and determining the kind of orientation while giving student the opportunity of confronting their misconception during implementation of the task. (Özmantar & Bingölbali, 2009, p. 339-340).
13. *Student's difficulty:* It defines determining the difficulties which may arise from each aspect or some combinations of the aspects of implementation of the task designed and making a plan of it (not understanding of instructions, misusing of materials different from their purposes, not appropriateness of students' sittings according to task designed, or unexpected situations during implementation, etc.) (Özmantar & Bingölbali, 2009, p. 339-340).
14. *Measurement and evaluation:* It defines determining the kind of methods by which teachers assess the intended improvements of students during and after implementation of task designed (Özmantar & Bingölbali, 2009, p. 340).
15. *Flexibility:* It defines designing task that permits changes in case of unexpected situations during implementation of task designed (i.e., whether continuing it or not, appropriateness of time allocated, classroom organization, using of materials, etc.) (Özmantar & Bingölbali, 2009, p. 341-342).

Analytical Coding Scheme and Analysis

The coding scheme used in this study was derived from the principles of mathematical task design framework (Özmantar & Bingölbali, 2009) since mathematical tasks designed by pre-service primary and elementary mathematics teachers were decided to be examined according to this framework in the study. Each task was coded as "applicable" or "non-applicable" so that it included each design principle or not, respectively. While coding the tasks based on each principle, the definitions and detailed explanations given by Özmantar and Bingölbali (2009) were concerned. Before initiating the coding, the authors met in a training session to discuss the sample coding done individually on a few tasks for about two hours. In this training session, different opinions were further discussed and decided on agreement. Following this training session, two coders coded all tasks independently and after finishing coding session, *inter-rater reliability* was calculated between coders. Reliability was calculated as the number of agreements divided by the number of agreements and disagreements and multiplied by 100 (Miles & Huberman, 1994). The inter-rater reliability score was 89% for this study. All conflicts between the coders were resolved by consensus.

To clarify and discuss how we determined whether the task included the corresponding design principle or not, we present some examples of the statements selected from several tasks indicating that design principle in Table 1.

Table 1
Sample Statements Appropriate for Task Design Principles

Design principle	Task	Sample statement
Aim	Operation wheel	It is a design task for practicing already learned concepts.
Use of time	Addition jigsaw puzzle	The time allocated for task is totally 1 lesson hour.
Classroom organization	Operation cube	The task is for individual work.
Student's prior knowledge	Tombola	Students' computational skills are assessed.
Multiple start point	Digit calendar	The starting question for the task is determined by casting lots.
Comprehensiveness	Equality system	All students are involved.
Material appropriateness	Whole numbers	Counting stamps made of carton are used.
Material supply	Geometry tree	Cardboard and scissors are used.
Material instruction	Different objects	Squares are cut and then fixed from their corners by toothpicks.
Teacher's role	Garden having a pool	Active involvement of students should be provided.
Student's role	Circled operations	Answers questions asked on the material.
Student's misconception	Squarehead	It is emphasized that there is not always a fixed ratio in patterns.
Student's difficulty	Fractal tree	Pythagorean relation is remembered.
Measurement and evaluation	Wheel of circle	Twenty questions on the wheel of fortune are posed to students randomly.

Flexibility	Rotating objects	With the same material three dimensional shapes of different objects can be observed.

Note. A sample task is given in the Appendix.

Results

The number of mathematical tasks analyzed for this study was 43, totally. 19 were designed by 36 pre-service primary teachers (PPT) and 24 were designed by 45 pre-service elementary mathematics teachers (PEMT). Table 2 presents the distribution of types of aims required by the design tasks of pre-service primary and elementary mathematics teachers.

Table 2
Distribution of Types of Aims of Tasks Analyzed

Aim	PPT n (%)	PEMT n (%)	Total n (%)
Recalling prior knowledge	1 (5,3)	2 (8,4)	3 (7)
Introducing new learning	2 (10,6)	11 (45,8)	13 (30,2)
Practicing already learned concepts	16 (84,1)	10 (41,5)	26 (60,5)
Overcoming student’s difficulties and misconceptions	0	1 (4,3)	1 (2,3)
Total	19 (100)	24 (100)	43 (100)

Table 2 shows that, in general, all of the tasks designed by pre-service primary and elementary mathematics teachers included aim of task. It also reveals that a vast majority of the tasks designed by pre-service teachers for the aim of practicing already learned concepts (26 out of 43) and a few of them designed for the aims of overcoming student’s difficulties and misconceptions (1 out of 43) and recalling prior knowledge (3 out of 43). Within the pre-service teacher groups, pre-service primary teachers designed tasks predominantly for the aim of practicing already learned concepts (16 out of 19), it is both introducing new learning (11 out of 24) and practicing already learned concepts (10 out of 24) for pre-service elementary mathematics teachers’ design tasks. Across the groups, pre-service elementary mathematics teachers designed nearly half of their tasks for the aim of introducing new learning (11 out of 24), whereas pre-service primary teachers designed almost none of the tasks for this aim (2 out of 19). Furthermore, there are only a few (1-2 or none) tasks designed for the aims of recalling prior knowledge and overcoming student’s difficulties and misconceptions by both groups of pre-service teachers. Table 3 presents the distribution of design principles included in the tasks analyzed.

Table 3
Applicability of Design Principles of Tasks Analyzed

	PPT n (%)		PEMT n (%)		Total n (%)	
Design principle	Applicable	Non-applicable	Applicable	Non-applicable	Applicable	Non-applicable
Use of time	19 (100)	0	24 (100)	0	43 (100)	0
Classroom organization	18 (94,7)	1 (5,3)	21 (87,5)	3 (12,5)	39 (90,7)	4 (9,3)
Student's prior knowledge	17 (89,4)	2 (10,6)	16 (66,6)	8 (33,3)	33 (76,7)	10 (23,3)
Multiple start point	9 (47)	10 (53)	3 (12,5)	21 (87,5)	12 (27,9)	31 (72,1)
Comprehensiveness	13 (68,2)	6 (31,8)	19 (79,2)	5 (21,8)	32 (74,4)	11 (25,6)
Material appropriateness	4 (21,2)	15 (78,8)	10 (41,7)	14 (58,3)	14 (32,6)	29 (67,4)
Material supply	15 (78,8)	4 (21,2)	11 (45,8)	13 (54,2)	26 (60,5)	17 (39,5)
Material instruction	5 (26,5)	14 (73,5)	1 (4,2)	23 (95,8)	6 (14)	37 (86)
Teacher's role	10 (53)	9 (47)	7 (29,2)	17 (70,8)	17 (39,5)	26 (60,5)
Student's role	11 (58,3)	8 (41,7)	4 (12,5)	20 (87,5)	15 (34,9)	28 (65,1)
Student's misconception	3 (15,9)	16 (84,1)	7 (29,2)	17 (70,8)	10 (23,3)	33 (76,7)
Student's difficulty	6 (31,8)	13 (68,2)	5 (20,8)	19 (79,2)	11 (25,6)	32 (74,4)
Measurement and evaluation	7 (37,1)	12 (62,9)	9 (37,5)	15 (62,5)	16 (37,2)	27 (62,8)
Flexibility	12 (62,9)	7 (37,1)	5 (20,8)	19 (79,2)	17 (39,5)	26 (60,5)

Table 3 shows that, in general, a great majority of the tasks designed by pre-service primary and elementary mathematics teachers included design principles of *use of time* (43 out of 43), *classroom organization* (39 out of 43), *student's prior knowledge* (33 out of 43), *comprehensiveness* (32 out of 43) and *supply of material* (26 out of 43), and only about 40% and less included remaining principles. Within the pre-service teachers groups, pre-service primary teachers involved *use of time* (19 out of 19), *classroom organization* (18 out of 19), *student's prior knowledge* (17 out of 19), *supply of material* (15 out of 19), *comprehensiveness* (13 out of 19), *flexibility* (12 out of 19), *student's role* (11 out of 19) and *teacher's role* (10 out of 19), and pre-service elementary mathematics teachers involved *use of time* (24 out of 24), *classroom organization* (21 out of 24), *comprehensiveness* (19 out of 24) and *student's prior knowledge* (16 out of 24) in their tasks. Across the groups, while pre-service primary teachers considered design principles of *supply of material*, *teacher's role*, *student's role* and *flexibility* for their tasks; pre-service elementary mathematics teachers did not consider much for their overall tasks. It is remarkable that pre-service primary teachers involved more principles (8 out of 14) compared to pre-service elementary mathematics teachers (4 out of 14) in their design tasks.

In general, it is concluded that 10 (53%) of the tasks designed by pre-service primary teachers and 11 (45.8%) of the tasks designed by pre-service elementary mathematics teachers are applicable in mathematics lessons.

Discussion and Conclusions

Selecting and designing appropriate tasks is essential to the success of teaching mathematics (Doyle, 1988; Stein & Lane, 1996). This study examined the mathematical tasks designed by pre-service primary and elementary mathematics teachers in terms of MTDP. It revealed several important findings regarding pre-service primary and elementary mathematics teachers separately as well as overall. Firstly, majority of mathematical tasks designed by pre-service teachers in order to accomplish the aim of practicing already learned concepts and almost none of them designed for the aims of overcoming student's difficulties and misconceptions and recalling prior knowledge. Secondly, across the two groups of pre-service teachers, pre-service elementary mathematics teachers designed tasks as many for the aim of introducing new learning as practicing already learned concepts, but pre-service primary teachers did not. Thirdly, approximately half of the overall designed tasks were applicable in mathematics lessons, although pre-service teachers considered a few design principles for their tasks.

These findings showed that pre-service teachers, in general, preferred to design mathematical tasks for practicing already learned concepts. This finding is consistent with the literature such as Açıl (2011) who obtained primary and mathematics teachers' opinions on concept of task. She found that teachers perceive task as works done for the purpose of practicing in general. This might be the case for pre-service teachers in this study as well. On the other hand, in this study, it was found that pre-service teachers did not prefer to design tasks for the aims of overcoming student's difficulties and misconceptions and recalling prior knowledge. It is emphasized that the kind of teaching approach is mostly shaped by teachers' experiences and the textbooks or other sources that they follow in classrooms (Hiebert & Wearne, 1993; Henningsen & Stein, 1997). Accordingly, pre-service teachers participated in this study may have had a similar experience of teaching mathematics during their school years. There is a similar finding in terms of type of tasks involved in Turkish mathematics textbooks. Kerpiç (2011) and Küçüközer et al. (2008) found that there aren't plenty of tasks designed to overcome students' existing misconceptions or difficulties as well as to recall students' prior knowledge in mathematics textbooks.

Regarding distribution of aims of design tasks across two groups of pre-service teachers, it was found that the number of tasks for the aim of introducing new learning was very distinctive. Apart from their experiences and the textbooks or other sources that they used in mathematics lessons; pre-service primary teachers' knowledge of teaching mathematics could be a factor in designing few tasks for accomplishing the aim of introducing new learning. It is highlighted that the knowledge of teachers (i.e., subject matter knowledge and pedagogical content knowledge) is linked to their choices of tasks and the way they implement them effectively (Stylianides & Stylianides, 2008; Sullivan et al., 2013). In Turkey, in a four-year undergraduate primary education program, there aren't any courses related to mathematics teaching except for mathematics teaching methods course in which the study was conducted. Thus, it can be predicted that pre-service primary teachers are less knowledgeable about teaching mathematics than pre-service elementary mathematics teachers.

In terms of each design principle of mathematical tasks, pre-service teachers mostly involved principles of use of time, classroom organization, student's prior knowledge and comprehensiveness in their tasks. In their study, Henningsen and Stein (1997) found that two most influential factors associated with maintaining student high-level engagement in the tasks are task builds on students' prior knowledge and appropriate amount of time (neither too little nor too much time). In a similar way, Doyle (1983) emphasized that performance on academic work depends on domain-specific knowledge of students. Hiebert et al. (1996) also emphasized students' prior knowledge as the basis for how students engage with a task. It is suggested that teachers must know their students well so that they can progress on the task (Henningsen & Stein, 1997). Moreover, Swan (2008) emphasized building on the knowledge of students already have and making use of whole class teaching, individual work and cooperative small group work as the principles that informed the design and the evaluation of the task. In the present study, it was found that pre-service primary teachers designed most of the tasks in which students can supply materials easily, while pre-service elementary mathematics teachers designed fewer such tasks.

It is also important to note that pre-service teachers did not involve majority of the design principles in their tasks. The finding of few tasks having multiple start point principle is indicated by Ashline and Quinn (2009) and Bell (1993) which is mathematically rich tasks are featured open-ended design so that all pupils in the class can find a suitable challenge within it.

In terms of appropriateness and instruction of material, few pre-service teachers involved these principles in their tasks. Henningsen and Stein (1997) found that students did not make much progress on the task because they were unsure about how to use the tangram pieces within the expectations of the task and played with the tangram pieces in off-task ways during the implementation of the task. Therefore, it is important to answer these questions during designing tasks: Which materials will be used?, How they will be used?, What kind of explanations will be made for students before using materials?, Do students have the ability or prior knowledge for using materials?, Will materials be supplied for students?, Are materials accessible for students? (Özmantar & Bingölbali, 2009).

Similarly, less than half of the tasks designed by pre-service teachers involved teacher's role explicitly. Doerr (2006) found that the actions of teacher toward listening to students and understanding the multiplicity of ways that students might think about the mathematics of the task supported extensive student engagement with the task and led to revise and refine their own mathematical thinking. Besides, Henningsen and Stein (1997) found that teaching behaviors such as scaffolding, modeling high-level performance, and consistently pressing students to provide meaningful explanations are influential in assisting students to engage at high levels with the task. In a similar way, Doyle (1988) argued that teachers should be especially attentive to the extent to which meaning is emphasized and the extent to which students are explicitly expected to demonstrate understanding of the mathematics underlying the activities in which they are engaged. In this study, findings indicated that pre-service teachers should consider the role of teacher while designing tasks and define explicitly. Similar to teacher's role, in the present study, it was found that few pre-service teachers involved students' roles in their tasks. Balacheff (1999) differentiated between rules concerned with a particular task which is made explicit in the class and a set of obligatory practices established as such by their use and which is established implicitly. Thus, it is important to define roles of students have explicitly in different steps of the task during task design.

In terms of student's misconceptions or difficulties, a number of tasks were involved by pre-service teachers. This shows that pre-service teachers did not consider carefully possible misconceptions or difficulties that students might have, or they did not have enough knowledge of the misconceptions or difficulties of conceptual content of the task. Doerr (2006) emphasized that teachers should have the knowledge of typical early conceptions or possible misconceptions that students might have. Researchers have reported that teachers tend to ignore the difficulties and misconceptions that students present during implementation of the task (Stylianides & Stylianides, 2008; Swan, 2007). However, it is emphasized that teaching for conceptual understanding is more effective when teachers confront difficulties rather than seeks to avoid or pre-empt them and expose and discuss common misconceptions (Swan, 2008). In this regard, it is important to consider the concepts that students might have difficulty or misconception while designing tasks by pre-service teachers.

It was found that less than half of the tasks designed by pre-service teachers were involved measurement and evaluation principle. The extent to which the learning objectives are achieved with the implementation of the tasks is one of the main topics that should be considered during the design of tasks by pre-service teachers. It is suggested that teachers should especially think about the formative assessment process and determine the techniques that they will use in order to measure students' progress on the task before implementation of the task (Özmantar & Bingölbali, 2009).

Lastly, pre-service teachers had little concern about flexibility of tasks during designing process. This finding is supported by Lieberman's (2009) view that teachers often expect the lesson to go as planned when they first teach it. So, it can be predicted that they are close to alternatives in an unconscious way. Similarly, in their study Henningsen and Stein (1997) found that the teacher's orchestration of students' engagement with the task in a way that strictly followed the task set-up caused the decline in students' engagement to procedural forms of thinking.

In general, as given in detail in the previous paragraphs, findings revealed that almost half of the tasks designed by pre-service teachers were applicable. Although it was found that pre-service teachers did not consider some of the design principles for their design tasks, it could be concluded that they were applicable since all or majority of the tasks were involved aim of task, use of time, classroom organization, student's prior knowledge and comprehensiveness principles by pre-service teachers. Moreover, in the present study, it was found that there was no difference between pre-service primary and elementary mathematics teachers in terms of applicability of their design tasks in mathematics lessons. This finding is consistent with what Sullivan et al. (2009) found in their study that primary and secondary teachers were equally likely to create student-focused investigative type lessons.

There are a wide range of resources in which mathematics teachers can find suitable tasks for students (e.g., textbooks and guidebooks, and numerous enrichment materials) (Zaslavsky, 2007), which are commonly designed by non-teachers. However, it is claimed that the teacher is not a neutral conduit for tasks but is also a designer (Watson, 2008). In this study, we attempted to portray overall pictures of pre-service primary and elementary mathematics teachers in terms of mathematical task design so that teacher educators can improve their teacher educator programs to support them in designing good tasks. Based on the results of the study, in order to enable pre-service primary and elementary mathematics teachers, who will be future teachers, to design successfully mathematical tasks, as well as the knowledge given on instructional task plan, task implementation, task management and task types in courses such as Classroom Management, Mathematics Teaching I-II, and Special Teaching Methods I-II which are offered in various teacher education programs of education faculties, pre-service teachers can also work on task design in courses such as Instructional Technologies and Material Development, and Planning and Evaluation in Teaching. In addition, with various elective courses such as Investigation of Mathematics Curriculum, Mathematics Textbook Analysis, and Activity Design, which can be included in teacher education programs, classroom environments where pre-service teachers can examine and discuss mathematical task design principles in depth can be created. Moreover, this study can be extended in a way to encompass the implementation dimension of the designed tasks since how teachers implement them are also crucial (Arbaugh & Brown, 2005). In future studies, particular attention can also be given to explore cognitive demands in mathematical tasks as well as designing tasks specific to some content areas of mathematics.

Acknowledgements

In this study, pre-service primary teachers designed mathematical tasks as part of the project supported by Bartin University Scientific Research Projects Commission (Project Number: BAP-2012-2-57).

References

Açıl, E. (2011). *İlköğretim öğretmenlerinin etkinlik algısı ve uygulanışına ilişkin görüşleri* [Elementary school teachers' opinions on the activity concept and implementation] [Unpublished Master's Thesis]. University of Gaziantep.

Arbaugh, F., & Brown, C. A. (2005). Analyzing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education, 8*, 499-536. https://doi.org/10.1007/s10857-006-6585-3

Ashline, G., & Quinn, R. (2009). Using mathematically rich tasks to deepen the pedagogical content knowledge of primary teachers. In B. Clarke, B. Grevholm, & R. Millman (Eds.), *Tasks in primary mathematics teacher education: Purpose, use and exemplars* (pp. 197-214). Springer. https://doi.org/10.1007/978-0-387-09669-8

Bal, A. P. (2008). Yeni ilköğretim matematik öğretim programının öğretmen görüşleri açısından değerlendirilmesi [The evaluation of new mathematic curriculum in term of teachers' perspectives]. *Ç. Ü. Sosyal Bilimler Enstitüsü Dergisi, 17*(1), 53-68.

Balacheff, N. (1999). Contract and custom: Two registers of didactical interactions (P. Herbst, Ed. & Trans.). *The Mathematics Educator*, *9*(2), 23-29. (Original work published 1988)

Bell, A. (1993). Principles for the design of teaching. *Educational Studies in Mathematics*, *24*, 5-34. https://doi.org/10.1007/BF01273293

Bozkurt, A. (2012). Mathematics teachers' perceptions of mathematical activities. *Education and Science, 37*(166), 101-115.

Bukova-Güzel, E., & Alkan, H. (2005). Yeniden yapılandırılan ilköğretim programı pilot uygulamasının değerlendirilmesi [Evaluating pilot study of reconstructed Turkish elementary school curriculum]. *Kuram ve Uygulamada Eğitim Bilimleri, 5*(2), 385-417.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). Routledge. https://doi.org/10.4324/9780203029053

Doerr, H. M. (2006). Examining the tasks of teaching when using students' mathematical thinking. *Educational Studies in Mathematics*, *62*, 3-24. https://doi.org/10.1007/s10649-006-4437-9

Doyle, W. (1983). Academic work. *Review of Educational Research*, *53*, 159-199.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, *23*(2),167-180.

Duru, A., & Korkmaz, H. (2010). Öğretmenlerin yeni matematik programı hakkındaki görüşleri ve program değişim sürecinde karşılaşılan zorluklar [Teachers' views about a new mathematics curriculum and difficulties encountering curriculum change]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 38*, 67-81.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524-549. https://doi.org/10.2307/749690

Herbst, P. (2008). The teacher and the task. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the joint meeting of PME 32 and PME-NA XXX* (Vol. 1, pp. 125-131). México: Cinvestav-UMSNH. Retrieved from https://www.pmena.org/pmenaproceedings/PMENA%2030%202008%20Proceedings%20Vol%201.pdf

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse and students' learning in second grade arithmetic. *American Educational Research Journal*, *30*(2), 393-425.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P. Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, *25*(4), 12-21. https://doi.org/10.3102/0013189X025004012

Kerpiç, A. (2011). *Etkinlik tasarım prensipleri çerçevesinde 7. sınıf matematik ders kitabı etkinliklerinin değerlendirilmesi* [Evaluation of the 7th grade mathematic textbook activities within the framework of principles of activity design] [Unpublished Master's Thesis]. University of Gaziantep.

Küçüközer, H., Bostan, A., Kenar, Z., Seçer, S., & Yavuz, S. (2008). Altıncı sınıf fen ve teknoloji ders kitaplarının yapılandırmacı öğrenme kuramına göre değerlendirilmesi [Evaluation of six grade science textbooks according to constructivist learning theory]. *Elementary Education Online*, *7*(1), 111-126.

Lieberman, J. (2009). Using lesson study to develop an appreciation of and competence in task design. In B. Clarke, B. Grevholm, & R. Millman (Eds.), *Tasks in primary mathematics teacher education: Purpose, use and exemplars* (pp. 11-25). Springer. https://doi.org/10.1007/978-0-387-09669-8

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded source book*. Sage Publications.

Ministry of National Education [MoNE] (2005). *İlköğretim matematik dersi öğretim programı ve kılavuzu: 1–5. Sınıflar* [Elementary mathematics curriculum and manual: Grades 1–5]. Devlet Kitapları Müdürlüğü.

Ministry of National Education [MoNE] (2013). *Ortaokul matematik dersi (5, 6, 7 ve 8. sınıflar) öğretim programı* [Middle school mathematics curriculum (grades 5, 6, 7 and 8)]. Devlet Kitapları Müdürlüğü.

Olkun, S., & Toluk-Uçar, Z. (2007). *İlköğretimde etkinlik temelli matematik öğretimi* [Activity-based mathematics teaching in elementary education] (3rd ed.). Maya Akademi.

Özmantar, M. F., & Bingölbali, E. (2009). Etkinlik tasarımı ve temel tasarım prensipleri [Task design and basic design principles]. In E. Bingölbali & M. F. Özmantar (Eds.), *İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri* (3rd ed., pp. 313-348). Pegem Akademi.

Özmantar, M. F., Bozkurt, A., Demir, S., Bingölbali, E., & Açıl, E. (2010). Sınıf öğretmenlerinin etkinlik kavramına ilişkin algıları [Elementary teachers' perception of instructional activities]. *Selçuk Üniversitesi Ahmet Keleşoğlu Eğitim Fakültesi, 30*, 379-398.

Silver, E. A., Mesa, V. M., Morris, K. A., Star, J. R., & Benken, B. M. (2009). Teaching mathematics for understanding: An analysis of lessons submitted by teachers seeking NBPTS certification. *American Educational Research Journal*, *46*(2), 501-531. https://doi.org/10.3102/0002831208326559

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*(2), 455-488.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*(1), 50-80. https://doi.org/10.1080/1380361960020103

Stylianides, A. J., & Stylianides, G. J. (2008). Studying the classroom implementation of tasks: High-level mathematical tasks embedded in 'real-life' contexts. *Teaching and Teacher Education: An International Journal of Research and Studies*, *24*(4), 859-875. https://doi.org/10.1016/j.tate.2007.11.015

Sullivan, P., Clarke, D., & Clarke, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal*, *21*(1), 85-105.

Sullivan, P., Clarke, D., & Clarke, B. (2013). *Teaching with tasks for effective mathematics learning*. Springer. https://doi.org/10.1007/978-1-4614-4681-1

Swan, M. (2005). *Improving learning in mathematics: Challenges and strategies*. Sheffield, England: Department of Education and Skills Standards Unity.

Swan, M. (2006). *Collaborative learning in mathematics: A challenge to our beliefs and practices.* London: National Institute of Adult Continuing Education.

Swan, M. (2007). The impact of the task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, *10*, 217-237. https://doi.org/10.1007/s10857-007-9038-8

Swan, M. (2008). A designer speaks. *Educational Designer*, *1*(1), 1-17. Retrieved from https://www.educationaldesigner.org/ed/volume1/issue1/article3/

Swan, M. (2011). Designing tasks that challenge values, beliefs and practices: A model for the professional development of practicing teachers. In O. Zaslavski & P. Sullivan (Eds.), *Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning* (pp. 57-71). Springer. https://doi.org/10.1007/978-0-387-09812-8

Toptaş, V., (2006). İlköğretim matematik dersi (1-5) öğretim programının uygulanmasında sınıf öğretmenlerinin karşılaştıkları sorunlarla ilgili görüşleri. In *Ulusal sınıf öğretmenliği kongresi: Bildiri kitabı* (Vol. 1, pp. 277-285). Kök Yayıncılık.

Uğurel, I., & Bukova-Güzel, E. (2010). Matematiksel öğrenme etkinlikleri üzerine bir tartışma ve kavramsal bir çerçeve önerisi [A discussion on mathematics learning tasks and a conceptual framework proposal]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 39*, 333-347.

Uğurel, I., Bukova-Güzel, E., & Kula, S. (2010). Matematik öğretmenlerinin öğrenme etkinlikleri hakkındaki görüş ve deneyimleri [Mathematics teachers' opinions and experiences related to learning tasks]. *Buca Eğitim Fakültesi Dergisi, 28*, 103-123.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Allyn & Bacon.

Watson, A. (2008). Task transformation is the teacher's responsibility. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proceedings of the joint meeting of PME 32 and PME-NA XXX* (Vol. 1, pp. 147-153). México: Cinvestav-UMSNH. Retrieved from https://www.pmena.org/pmenaproceedings/PMENA%2030%202008%20Proceedings%20Vol%201.pdf

Watson, A., & Mason, J. (2007). Taken-as-shared: A review of common assumptions about mathematical tasks in teacher education. *Journal of Mathematics Teacher Education, 10*, 205-215. https://doi.org/10.1007/s10857-007-9059-3

Zaslavsky, O. (2007). Mathematics-related tasks, teacher education, and teacher educators: The dynamics associated with tasks in mathematics teacher education. *Journal of Mathematics Teacher Education, 10*, 433-440. https://doi.org/10.1007/s10857-007-9060-x

Appendix.

A Sample Task: Viewpoints

In this task, students build a building and draw the left, right, front, and back direct views of it. Before coming to class, students bring styrofoam, toothpicks and a box cutter with them. First, they prepare 2 cm cubes of which the buildings are made by cutting styrofoam with a box cutter. Then, they build the buildings by using toothpicks to stick cubes together. They can also build their buildings from a building plan which might be given by the teacher. After students build their buildings, they draw the front, right, left, and back direct views on a sheet of square grid (or square dot grid) paper as shown in Figure 1.

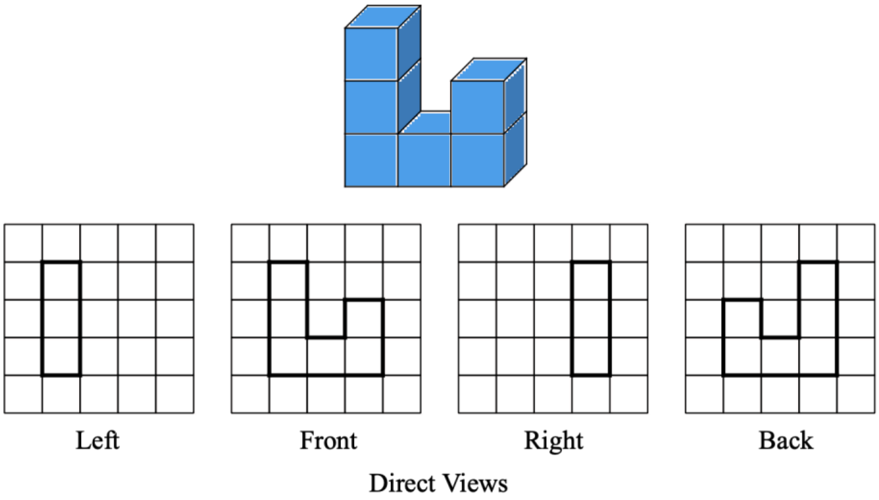

Figure 1. "Viewpoints" task

Ask students whether they notice any similarity between couples of different direct views. In the second part of the task, students are given a right and front view (or a left and back view) and they build the building that has those views. A more challenging task might be to draw perspective views of these block buildings on isometric dot grid paper.