A Comparative Textbook Analysis on Measures of Center in Selected School Mathematics Textbooks of Australia and Turkey

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Measures of center have a central role in statistics education. There, however, is not much research on textbook analysis on measures of center, especially at the international level, which is crucial to keep up with the changes in a data-driven globalised world. This study aimed to investigate how selected school mathematics textbooks of Australia and Turkey present the concepts of measures of center to support students' learning. The content analysis was based on three dimensions: (1) the organization and sequence of the concepts of measures of center across grades, (2) conceptualization of measures of center from mathematical and statistical aspects, and (3) the use of context, visual displays, and technological tools regarding measures of center. The first major finding was that the order of presentation of concepts was different in the two countries' selected textbooks. Selected textbooks of both countries introduced measures of center formally rather than starting with an understanding of clump notion (informal understanding of measures of center). The second major finding was that both countries' textbooks focus more on the mathematical aspect of measures of center. Selected Australian textbooks also focus more on the statistical aspect of measures of center than those of Turkey. The third major finding was that the use of context, visual displays, and technological tools also differ. Australian textbooks often used a context with research scenarios about daily life, used a technological package, and related measures of center with a variety of graphs. Findings contribute to our understanding of the differences among countries with different backgrounds in statistics education, pointing to the ways of developing curriculum materials such as textbooks.

Measures of center, particularly the concept of the mean, drew the attention of researchers over the last thirty years. They are descriptive statistics that are used to describe and make sense of the data set (Cai et al., 2002) and used in conjunction with notions of variability, distribution, and informal inference in data modelling situations (English & Watson, 2018; Kazak et al., 2018). Thus, measures of center are central concepts to understand related statistical concepts such as variation, distribution, and inferential statistics, which was often taught in isolation in school mathematics (Kazak et al., 2018) and also to make sense of the information in our daily life (Watson, 2006). Understanding measures of center, particularly mean, is important in dealing with uncertainty in real-life problem contexts (acknowledging inferences from the data might not be completely certain, and being aware of the variation, chance, and data limitations) (English & Watson, 2018). Although measures of center are key concepts in statistics education, both students and teachers have difficulties in conceptualizing them in the context (Jacobbe & Carvalho, 2011; Mokros & Russell, 1995; Watson & Moritz, 2000).

Textbooks as curriculum materials are among the prominent resources for both students and teachers. They are considered as the curriculum resources that may affect teachers' and students' learning and shape instruction (Son & Diletti, 2017). Particularly, textbooks might highly shape the teaching of statistics in the classrooms when teachers do not have enough experience with the statistical content (Weiland, 2019). There have been few studies on the analysis of measures of center in mathematics textbooks, especially in the context of international comparisons. Cai and his colleagues (2002) conducted a comparative study of the United States of America (USA) and Asian school mathematics textbooks based on the procedural and conceptual understanding of the mean. Statistics education has an evolving nature due to the continuing innovations in society (e.g., big data, non-traditional statistical graphs, technological changes), which also allows the use of different methods and pedagogies in learning and teaching statistics (Pfannkuch, 2018). These changes urge us to reconsider curricular learning experiences (Watson, 2017). To gain a fuller picture in the presentation of measures of center in the textbooks, there is a need to extend Cai et al. (2002)'s study by considering recent studies in measures of center conducted in educational settings, which gave clues about learning trajectories for teaching measures of center (Bakker et al., 2006; Büscher & Schnell, 2017; Makar, 2014) and other crucial elements in teaching statistical concepts including measures of center such as contextual features (e.g., real, authentic context) (Gal, 1995; Makar, 2014; Watson, 2016), graphical displays (Pfannkuch et al., 2010), and technological tools (Bakker et al., 2006; Watson, 2016). Furthermore, international comparison studies would inform countries by pointing to the ways of developing their current curriculum materials (Watson, 2017). However, there is little known about statistics education at international level including developing countries (Reston et al., 2014; Smith et al., 2009).

There have been initiatives in making statistics as a part of curricula officially since the 1990s, especially in English-speaking countries (Watson, 2006) (e.g., National Council of Teachers of Mathematics [NCTM], 1989; Australian Education Council [AEC], 1991). Australian curriculum (AEC) (1991), for example, included the section of chance and data in mathematics curriculum covering one-fifth of mathematical content (Watson, 1998). Since 1994 Australian Association of Mathematics Teachers (AAMT) demonstrated efforts for the integration of technology in mathematics education including for learning and teaching of statistics (e.g., providing an online resource for teaching mathematics contents including statistics, *Top Drawer Teachers* by establishing the links with Australian curriculum, see Watson, 2016). In line with early initiatives regarding statistics and probability in English speaking countries Petocz et al. (2018) states that a majority of the research papers in statistics education are written by the researchers in English-speaking countries (e.g., the USA, Australia, the United Kingdom [UK], and New Zealand) and other European countries (e.g., the Netherlands, Spain). Zieffler et al. (2011) support this argument by investigating articles published in the Statistics Education Research Journal (SERJ) from 2002 to 2009. The articles were written by 64 % of the first authors in four English-speaking countries (the USA, Australia, the UK, and New Zealand). Petocz et al. (2018) attributes this result to the economic conditions of developed countries and highlighted the importance of investigating the differences regarding statistics education between developed countries and less-developed countries.

Situating itself into this background, this study aims to investigate the selected textbooks of two countries, Australia as one of the English-speaking countries that had early initiatives in statistics education and Turkey as a developing country that made substantial changes in the curriculum in 2005 to keep with up the changes in the global data-driven world and to adapt the standards of European Union (Bayazıt, 2013; Erbas et al., 2012), including the inclusion of statistics and probability content as one of the distinct strands of curriculum (Yolcu, 2021). Considering Turkish students' performances in the Programme for International Student Assessment (PISA) in which Australia's score on mathematical literacy tasks including statistical content is above the Organisation for Economic Co-operation and Development (OECD) average and Turkey's score is much lower than the OECD average (Thomson et al., 2017; Thomson et al., 2019), this study would point to the ways of developing current textbook materials to provide rich learning opportunities for students, especially after the curricular reforms in Turkey since 2005. Students' performances in PISA and Trends in International Mathematics and Science Study (TIMSS) were linked to learning opportunities provided in textbooks and students' use of textbooks although it might not be a sole factor affecting students' mathematics performances (Fan et al., 2013; Yang et al., 2017). The 1999 Third International Mathematics and Science Study-Repeat (TIMSS-R) was one of the major studies on the role of mathematics textbooks at the international level (O'Keeffe & White, 2017) in which Valverde et al. (2002) regarded textbooks as a mediator (potentially implemented curriculum) between intended and implemented curricula.

This study examines the presentation of measures of center in the selected textbook series of Australia and Turkey by considering conceptualization of measures of center based on research studies on measures of center and considering crucial elements of statistics such as graphical representation, use of technological tools, and context. The research questions are:

1. How are measures of center introduced across middle school grades (grades 5 to 8) in selected Australian and Turkish school mathematics textbooks?
2. How do selected Turkish and Australian school mathematics textbooks (grades 5 to 8) conceptualize the concepts of measures of center?
3. How do selected Turkish and Australian school mathematics textbooks (grades 5 to 8) use context, visual displays, and tools of technology to support students' learning of measures of center?

Literature Review

Significance of Textbook Content Analysis

Textbooks have drawn considerable attention among the education community in the world. They are regarded as the main resources that are frequently used by teachers and students in learning and teaching (Fan & Zhu, 2007; Valverde et al., 2002; Sherman et al., 2020; Wang & Fan, 2021). In particular, teachers who are not familiar with statistical concepts tend to rely on textbooks to cover such concepts (Dunn et al., 2015). Textbooks not only have a significant role for teachers and students; but also, they are considered as reflections of national curricular goals (Haggarty & Pepin, 2002).

Teachers use the textbooks in various ways such as sequencing of the content, deciding what to teach, and planning instruction as it provides a representative of a lesson plan and sample tasks, worked-out examples, and homework content (Reys et al., 2004; Stein et al., 2007). Textbooks have a potential effect on the mathematical content that students are exposed to and on the presentation of textbooks by the teachers and, in turn, students' learning process. Rezat (2011) showed that teachers' use of textbooks affects students' use of textbooks or vice versa, which affects students' learning process. Thus, textbooks are resources used by both teachers and students to shape the enactment of the curriculum (Rezat, 2011). In a broader sense, international comparison studies on textbook analysis also regard textbooks as prominent curriculum materials to potentially affect students' achievement since textbooks are seen as a reflection of implemented curriculum (Valverde et al., 2002). Although textbooks have a crucial role in the learning and teaching of mathematics, for the past twenty years changes in teaching methods and their practice in classroom settings along with technology integration, have raised questions regarding textbooks' quality (O'Keeffe & O' Donoghue, 2015). Thus, the presentation of mathematical content in textbooks is important to understand students' learning opportunities.

Textbook Research

Fan et al. (2013) argued that textbook research is relatively new in mathematics education and 29 % of the textbook research focused on the comparison of textbooks. These studies focused on a variety of mathematical concepts ranging from numbers and operations to geometrical concepts (Avcu, 2019). Several studies on international comparison of textbooks found substantial differences between the textbook series from different countries in terms of content, structure, and the use of the textbooks in classroom settings (Erbas et al. 2012; Jäder et al., 2020; Jones & Fujita, 2013; Pepin & Haggarty, 2001; Valverde et al., 2002). Since this study focuses on the content of measures of center as a key concept in statistics education, this section was restricted to the studies on statistic education in both national and international textbook comparison studies.

There are very few national studies (Bütüner, 2020; Jones et al., 2015; Pickle, 2012; Zapata-Cardona & Escobar, 2019; Weiland, 2019) and cross-national comparisons of textbooks in statistics education (Cai et al., 2002; Wang & Lu, 2018; Yang & Sianturi, 2019). In one of the national studies (in a Colombian context), Zapata-Cardona and Escobar (2019) analysed the nature of task contexts (hypothetical, theoretical, real contexts, without context), and the type of knowledge required to solve tasks (statistical, technological, and reflective knowledge) to promote critical citizenship in Colombian statistics textbooks. The majority of the statistics tasks were hypothetical. They suggested if the tasks are presented within a real context and encourage students to think reflectively, students are more likely to get the opportunity to be a critical citizen and connect statistics with their life. Supportively, in the USA context, Weiland (2019) found selected USA textbooks series used mostly fictional and trivial contextual situations. Jones et al. (2015) also investigated five USA textbook series (grades 1-5) and found most of the statistics tasks emphasize reading graphs (less emphasis on constructing displays) and performing mathematical computations. These studies at the national level call attention to the use of real contexts in statistics education and conceptual understanding of statistics rather than merely focusing on the computational aspect of statistics. There is a further need to compare these findings by comparing textbooks with other countries (Jones et al., 2015).

At the international level, Wang and Lu (2018) investigated the presentation of four learning areas of mathematics (algebra, geometry, statistics and probability, and calculus) in high school mathematics textbooks from seven countries (China, France, Germany, Japan, Russia, the UK, and the USA). One of the major findings of the study was the selected UK textbook series provided the richest content on statistical evaluation and probability among the other countries' textbooks series. The UK textbooks presented content from simple to complex gradually. The series also included information and communication technologies for data analysis and included real-life context, which was unique to the textbooks of the UK. Chinese textbook series were not rich and well-organized in the domains of statistics, probability, and calculus compared to textbook series of western countries. This study highlighted the substantial differences among textbook series from various countries with different cultural backgrounds. One notable difference is that the UK was the first English-speaking country to produce curriculum materials emphasizing statistics and probability (Watson, 1998). Chinese mathematics curriculum included statistics and probability as a distinct topic almost ten years later than some western countries, such as the USA and the UK (Wang & Lu, 2018). These findings seem to be coherent with the arguments of Petocz et al. (2018) and Zieffler et al. (2011) that the majority of studies were written by English-speaking countries such as the UK and Australia, and both of which had early reforms in statistics education along with technological advancements (Moore, 1997). Early reforms in statistics education and technological advancements are crucial factors in shaping curricular materials and lead to striking developments in statistics education. These studies have revealed the importance of analysing textbooks of countries with different backgrounds. These textbook comparison studies can be seen as a way of learning from other countries for educational enhancements (Wang & Lu, 2018).

In an international textbook comparison study, Cai et al. (2002) conducted a comparative study of the USA and Asian school mathematics textbooks based on the procedural and conceptual understanding of the mean. This study found that both countries' textbook series focused more on the mean concept as a computational algorithm. However, the USA textbook series put more importance on the statistical aspect of the mean. It needs to be extended to keep up with the changes in statistics education research by considering research on measures of center. The use of graphical displays (Jones et al., 2015) and context (Zapata-Cardona & Escobar, 2019; Weiland, 2019) have a significant role in the analysis of textbooks in statistics education, especially along with the technology integration in mathematics textbooks (Sherman et al., 2020). Researchers should reconsider and evaluate current statistics curriculums as technology develops and new approaches, methods, or pedagogies are used in statistics education (Pfannkuch, 2018).

Measures of Center

Early studies examined students' understanding of the arithmetic mean concept (Pollatsek et al.,1981; Strauss & Bichler, 1988). These studies focused on the computation of the mean and its computational properties. In further studies, researchers comprehensively addressed the representativeness of the measures of center, which are interchangeably used with the concepts of averages including mean, median, and mode (Makar, 2014; Mokros & Russell, 1995; Watson & Moritz, 2000). The idea of representativeness involves thinking about typicality (Mokros & Russell, 1995; Makar, 2014). For example, asking students to think about, "Is there a typical height for a student in year 3?" (Makar, 2014, p. 61). This would enable students to understand typical as representative by taking into consideration outliers (atypicality). In their study with fourth, sixth, and eighth-grade students, Mokros and Russell (1995) suggested students, firstly, should be given opportunities to develop intuitively idea of representativeness and the mean should be introduced in the late middle school years. However, Makar (2014) discusses the study of Mokros and Russell (1995) still focused on algorithmic approach (e.g., fair share or balance model) and did not pay attention to the understanding of average as a representative of an aggregate.

The measures of center have been reconceptualized and described as representatives of aggregate by several researchers, rather than considering them as individual values and learning trajectories was suggested to develop students' understanding of the measures of center (Bakker et al., 2006; Büscher & Schnell, 2017; Makar, 2014). The study of Bakker et al. (2006), for example, suggested a trajectory that the instruction begins with the notion of the clump that represents the majority of data values in a distribution. Then, it moves to the mean or median concepts informally. In other words, students understand the mean or median as values as group descriptors of the data and start to use formal concepts of mean or median values while comparing two distributions. In this study, students were asked to decide if a fish farmer's claim (genetically engineered (GE) fish grow bigger than normal fish) is correct (p. 3). Each student experienced catching a fish (collecting data) and represented their data on a dot plot. When they were asked to make judgments about data, most students used the clump notion to refer to the majority of values in graphs, by explaining where the clump is. During this process, students also referred to the clump notion when comparing distributions of two types of fish, stating “bigger clump here…the clump is more spread out but bigger…” (p. 4). This activity helped them reason with the center (clump) notion and understand measures of center informally. Then, students were asked to decide if a fish farmer’s claim (GE fish grow bigger than normal fish and their sizes are double) is right. In this activity, by using technological software (TinkerPlots) they moved to the idea of measures of center as a representative of aggregate by building on the notion of clump and measuring it formally, using reference lines and mean values on the graph. This led them to compare distributions with mean while taking into consideration the variation in samples, variations within each data set, and between two data sets. Students should be given learning opportunities to choose sensible measures of center and connect them with the spread notion (Bakker et al., 2006; Çatman-Aksoy & Işıksal, 2021; Kazak et al., 2018; Konold et al., 2015; Zawojewski & Shaughnessy, 2000).

Supportively, even young students (3rd-grade students), who are not familiar with the mean algorithm, developed an informal understanding of the average through informal inferential reasoning (Makar, 2014). Büscher and Schnell (2017) also suggest starting with informal measures, that emerge from the students’ way of thinking, and transition to formal measures of center and spread (mean, median, mode, range), situating itself to the emergent modeling framework (transition from a model of a certain situation to model for formal reasoning of measures). These studies give us clues about how to present these concepts in a meaningful manner in the textbooks. This study does not only examine the concepts of measures of center; but also, examines the presentation of measures of center concerning the key statistical components (relating measures of center with context, technology use, and visual displays).

Conceptual Framework

The current study was shaped based on the framework on textbook analysis (Charalambous et al., 2010), the conceptualization of measures of center in textbooks (Cai et al., 2002; Gal, 1995, Zapata-Cardona, & Escobar, 2019), and other studies related to measures of center.

Charalambous et al. (2010) developed a framework that provided a foundation for analysis of the textbooks in the current study. They identified two major categories for textbook analysis: horizontal and vertical. *Horizontal analysis* refers to the overall examination of textbooks or general aspects of textbooks. It involves investigation of physical aspects of the textbooks (e.g., number of pages, title) and general organization of mathematical content throughout the book (which topics are included, in what order, in which grade level). The horizontal analysis does not provide enough information on how the mathematical content is treated in textbooks to identify what kind of opportunities are provided to students. *The vertical analysis* investigates how a mathematical content (a topic in mathematics) was treated in the textbooks and textbooks are regarded as an environment where knowledge is constructed. It, for example, involves the examination of definitions, rules, representations, worked-out problems, use of context, and cognitive demands of tasks. The current study aimed to examine: (1) organization of measures of center across grade levels (horizontal analysis), (2) conceptualization of measures of center, and (3) use of visual displays, technological tools, and context to support the learning of measures of center (vertical analysis) by examining descriptions, worked-out examples, exercise or practice problems, and activities in the textbooks. Based on the framework of Charalambous et al., this study employs both horizontal and vertical analysis to answer research questions.

Conceptualization of Measures of Center

Cai et al. (2002) proposed a categorization to analyse the presentation of average (mean) in the mathematics textbooks. This study provides a basis for data analysis in the current study. This categorization suggests two major understandings of average: (a) average as an algorithm and (b) average as a statistic from the perspective of procedural and conceptual understanding. They identified three types of understanding of the average concept: (a) procedural understanding mean as an algorithm, (b) conceptual understanding of mean as an algorithm, and (c) conceptual understanding of mean as a statistic (p. 393). Each type was described by giving examples in Table 1.

Table 1
Conceptualization of Average in Textbooks (Cai et al., 2002, p. 393, 403)

Types of understanding average	Description and examples
Procedural understanding of *mean as an algorithm*	Direct application of the average algorithm (e.g., finding average by using the information given in a visual display (table, graph))
Conceptual understanding of *mean as an algorithm*	Flexible application of the average algorithm (e.g., finding the missing value in the data set when given average and other values in the data set)
Conceptual understanding of *mean as a statistic*	Interpretation and use of the average concept in statistical context (understanding representative nature of the concept) (e.g., comparing mean, median, and mode and deciding which one is appropriate to use)

Knowing arithmetic properties of the average (e.g., the average is influenced by values other than the average) is important to understand the mathematical aspects underlying computations (Strauss & Bichler, 1988) and procedural reasoning enables one to be fluent in calculations (Groth & Berger, 2006). However, the measures of center are not only mathematical objects involving rules, procedures, or algorithms; but also, they are statistics as representative of data sets that help to describe and make sense of the data sets (Cai et al., 2002; Groth & Berger, 2006).

Understanding of measure of center as a representative of distribution is crucial to summarize the data set and compare different data sets (Cai et al., 2002), and to use it to make inferences beyond the data (Makar, 2014). Understanding measures of center as statistic helps students develop a conceptual understanding of measures of center (Cai et al., 2002). Rather than calculating the mean, median, and mode, it is important to encourage students to explain and justify their answers in context, especially in contexts with which they are familiar (Watson, 2006). Connecting measures of center with the spread notion helps to understand the notion of distribution (Zawojewski & Shaughnessy, 2000) and understand the mean might not be enough to describe the data set (Watson, 2006). Making inferences beyond the data is important to understand the main reason for the use of statistics and informal statistical inference should be used to draw context-based conclusions about the data (Makar, 2014). Critical judgment of estimates is important for the development of statistical literacy. Questioning of estimates and claims enables students to develop a questioning attitude towards misleading information, especially in media texts. Newspaper extracts can be used to understand, interpret, and question statistical concepts in social and scientific contexts (Gal, 2002; Kus & Cakiroglu, 2020; Watson, 2006). These studies also provide the foundation for data analysis regarding the conceptualization of measures of center in the current study (see data analysis section).

The Use of Contexts, Visual Displays, and Technological Tools Concerning Measures of Center

Besides conceptualization of measures of center, other crucial elements (context, visual displays, and technology use) were highlighted in the literature. The context of real-world situations has a significant role in statistical thinking (English & Watson, 2018; Pfannkuch et al., 2011). Cobb and Moore (1997) emphasize that "… data are not just numbers; they are numbers within a context." (p. 801). In other words, statistics cannot be separated from the context in which the problem arose. Gal (1995) discussed that students should be able to understand finding the average as a tool to make sense of a variety of situations and contexts rather than to solve computational average problems. Researchers particularly addressed the importance of authentic and meaningful contexts in the teaching of measures of center (Büscher & Schnell, 2017; Gal, 1995; English & Watson, 2018; Makar, 2014; Mokros & Russell, 1995; Watson, 2000).

Regarding the investigation of context in statistics education, Gal (1995) proposed two different contexts for making sense of average: *Generative (active)* (engaging in real problems and gathering the data), which is also called *authentic context* (Makar, 2014), and *interpretive (passive) contexts* (interpreting the social and scientific contexts). It is important to explore real problems given in statistical investigation contexts rather than "'story problems' that are artificially constructed and do not represent real problems" (Garfield & Franklin, 2011, p. 137). In a recent study, Zapata-Cardona and Escobar (2019) also categorized the nature of context in textbooks as follows: task without context (without any connections to real-life), hypothetical context (not real data), theoretical context (conducting simulations), and real context (real data from databases) (see Table 2). Creating realistic contexts enables teachers to understand students' reasoning of averages and students can get opportunities to understand when and how to use averages in actual situations. Rather than computing averages without a context, students should be able to deal with both passive (e.g., interpreting a newspaper extract) and active contexts (Gal, 1994). In this study, these studies provided the basis for analysing the use of contexts in teaching of measures of center in textbooks (see section on data analysis).

Studies in statistics education included tasks that require interpretation of visual displays generated by the use of technological software and as well by manual creation (Bakker et al., 2006; English & Watson, 2018; Kazak et. al., 2018). The use of technological tools (e.g., TinkerPlots, Fathom) seemed to support understanding of measures of center by enabling the transition from the idea of the clump to the informal meaning of measures of center (Bakker et al., 2006) and understanding the aggregate view of the data through juxtaposing individual values (Lee & Hollebrands, 2008). For example, in the study of Bakker et al. (2006) on center and variation, students were asked to create their dot plots manually and reason about these visual displays and also create dot plots through the software, which allowed them to see where the clump is, relate the clump with mean, and also see how the data is spread out. In this sense, using a visual approach in teaching statistics invokes students' visual imagination and develops students' visual comparative reasoning by paying attention to the shifts, overlaps, clusters in the distribution, and relative differences between centers (Pfannkuch et al., 2010).

Table 2
The Nature of Contexts of Tasks in the Textbooks (Zapata-Cardona & Escobar, 2019)

Context of tasks	Description and examples
Task without context	Tasks without context do not have any connections with real-world (e.g., presenting only numbers in a data set and asking to find the mode)
Hypothetical context	Tasks in hypothetical context do not provide information that can be verifiable from the real sources (databases from statistical departments, data from scientific research, or historical records) (e.g., presenting age of a group of people registered for a contest)
Theoretical context	Tasks in theoretical context require stimulating the data by using software or manipulatives (e.g., writing all possibilities of the results of throwing two dice)
Real context	Tasks in real context provide verifiable information from real sources (databases from statistical departments, data from scientific research, or historical records) (e.g., presenting a number of athletes in the National Games in a specific year)

Educational Systems in Turkey and Australia

In Turkey, students start schooling at 5 years 9 months, or 5 years 6 months old if their parents approve, for compulsory education (Ministry of National Education [MoNE], 2019). There are twelve years compulsory education structured as three blocks of four years, four-years primary (Grades 1 to 4); four-years middle school (Grades 5 to 8); and four-year high school (Grades 9-12) (Boz & Yıldırım, 2014). The education system is centralized in Turkey. The national curriculum (MoNE, 2018) is implemented in both private and public schools. Textbooks are prepared in line with the national curriculum. These textbooks are either the textbooks published by the MoNE or the textbooks published by private companies and approved by the MoNE. The Ministry of Education requires schools to use these textbooks and distributes these textbooks free of charge to students. Each student has equal access to these textbooks in both public and private schools. Textbooks published by MoNE are regarded as one textbook series because all of them are approved and published by MoNE, but they are written by different authors across grades.

The education system in Australia is much more complex than that of Turkey. It has six states and two territory governments. Each state has its education system (Henman & Gable, 2015). However, the differences between states and territories in schooling (school structures and age requirements) have been minimized after 2015 according to the latest National Report on Schooling in Australia (Australian, Curriculum, Assessment and Reporting Authority [ACARA], 2020). Primary education included a Foundation year, which is the first year of full-time schooling, and followed Years 1 to 6 in all states and territories except the state of South Australia (Years 1-7 after Foundation year). Students in secondary education are educated between Years 7 to 12 (South Australia, Years 8-12). In most of the states and territories, students start compulsory schooling at six years old except Tasmania (five years), Queensland (six years and six months), and Western Australia (five years 6 months) (ACARA, 2020). States and territories had their curriculum before the curriculum change in Australia in 2014 (Gleeson et al., 2020). In 2014, the implementation of the national curriculum, through the Australian Curriculum Review, was approved by the government (Wiltshire & Donnelly, 2014). Since Australia has six states and two territory governments, different textbook series are used in schools of different states (e.g., Haese Mathematics, Math Quest) in Australia (Jäder et al., 2020; Stacey & Vincent, 2009).

This brief overview of education systems and textbook use provides background about the similarities (textbooks series are based on the national curriculum, starting age for compulsory schooling) and differences between these countries (use of different textbooks). It provides a basis for textbook selection with the assumption of the equivalency between the grades in both countries because of the similarity in the starting ages of schooling (almost six years old) in both countries. The focus of the analysis was restricted to middle school grades' textbooks in line with school structure in Turkey (Grades 5 to 8 in Turkey, corresponding to Year 5 to 8 in Australia). It also informs the limitation of selecting one textbook series by addressing challenging processes in selecting textbooks of countries with different states.

Method

Textbook Selection

The purpose of the study was to investigate how measures of center are presented in the selected school mathematics textbooks of Australia and Turkey. To investigate the presentation of measures of center, students' editions of middle school mathematics textbooks (Grade 5 to 8) were selected (see Table 3). Turkish textbooks published by the Turkish Ministry of Education were selected because they are official textbooks distributed to students free of charge by the Ministry of Education and used by most of the students in public schools (Erbas et al., 2012). Textbook selection is a challenging process for the authors, especially in countries offering different textbooks such as the USA (Wang & Lu, 2018). Similarly, in Australia, different textbook series (e.g., Haese Mathematics, Math Quest) are used. In the current study, the Haese Mathematics textbook series (Haese et al., 2014; Haese et al., 2015a, Haese et al., 2015b; Haese et al., 2019) were selected because of the accessibility of textbooks. Moreover, they were also mentioned by the various researchers in mathematics education (Ellis et al., 2016; Sheppard, 2008; İncikabı & Ulusoy, 2019; Walsh et al., 2017), considered as one of the popular mainstream books in Australia (İncikabı & Ulusoy, 2019), and widely used in Australia, particularly in South Australia (Sheppard, 2008). These studies also support the textbook selection. Although the investigation of several textbook series is appreciated to reach generalized results, which is beyond the scope of this study, this study focused on the textbooks of one publisher (Grades 5 to 8) from Australia and Turkey. This study does not aim to generalize the findings from the selected textbook series to other Australian mathematics textbook series. This case shows an example of an in-depth analysis of textbooks regarding the teaching of measures of center in the selected textbook series.

Table 3
Mathematics Textbooks Selected in the Current Study

Country	Textbooks	Publisher
Australia	Mathematics for Australia 5	Haese Mathematics (Haese et al., 2014)
	Mathematics for Australia 6	Haese Mathematics (Haese et al., 2015a)
	Mathematics for Australia 7	Haese Mathematics (Haese et al., 2015b)
	Mathematics for Australia 8	Haese Mathematics (Haese et al., 2019)
Turkey	Middle School Mathematics 5	Ministry of National Education (Çırıtçı et al., 2019)
	Middle School Mathematics 6	Ministry of National Education (Çağlayan et al., 2019)
	Middle School Mathematics 7	Ministry of National Education (Keskin-Oğan & Öztürk, 2019)
	Middle School Mathematics 8	Ministry of National Education (Böge & Akıllı, 2019)

This study does not aim to compare the curriculum standards of both countries. Textbooks are regarded as the potentially implemented curriculum (Valverde et al., 2002). However, it is crucial to know about curriculum standards regarding measures of center to explain differences between textbooks of both countries. The Turkish Mathematics Curriculum from Grades 1 to 8 (MoNE, 2018) included learning objectives regarding measures of center in sixth and seventh grades. Students are expected to: (1) calculate and interpret the range of a data set; (2) calculate and interpret arithmetic mean of a data set; (3) use arithmetic mean and range in interpreting and comparing two data sets in sixth grade, and find and interpret mean, mean, and mode in a data set in seventh grade. The ACARA (2015) included learning objectives regarding measures of center in Years 7 and 8. Year 7 students are expected to "calculate mean, median, mode, and range for sets of data. Interpret these statistics in the context of data" (p.16), "describe and interpret data displays using median, mean and range" (p.16), whereas Year 8 students are expected to, "investigate the effect of individual data values, including outliers, on the mean and median" (p.16). Additionally, students are also expected to interpret data sets in contexts in Years 5 and 6 (by finding the most occurred value in the data set) even though it (mode) is not precisely mentioned in the learning objectives. The online platform of the Australian Curriculum also provides additional resources (real data and sample activities) for learning and teaching of these concepts.

Data Analysis

Content analysis was conducted to analyse the definitions or explanations, worked-out problems, practice problems, and activities or opening problems in the textbooks. The content analysis mainly involves summarizing the data through coding and categorization of the text in a meaningful way (Cohen et al., 2018). The analysis included examining: (1) the organization and sequence of the concepts of measures of center across grades, (2) the conceptualization of measures of center, (3) the use of context, visual displays, and technological tools concerning the measures of center.

The units of analysis of the current study included all textbook features; (1) definitions and explanations, (2) worked-out examples (problems with solutions as a representative situation), (3) exercise and practice problems (problems without solutions; exercise problems appear within the chapter and practice problems appear at the end of the chapter), and (4) activities, research projects, or any similar kind of section (e.g., discussion, opening problem) (activities and research projects involve student-centered tasks that require active engagement). Textbooks of both countries were imported into the qualitative data analysis software (MAXQDA software). The author, fluent in both Turkish and English, coded each unit of analysis in the textbooks by using a coding scheme developed based on the previous studies and the data that emerged from this study (Table 4 and Table 5). Thus, the coding process was not independent of the studies on learning and teaching of statistics and textbook research in statistics.

Each unit of analysis was related to the mean, median, mode, and/or range concepts addressed in the statistics chapter of textbooks. Even if the content concerning these concepts is not presented under the topic of measures of center and spread in the textbooks, they were also analyzed to provide a comprehensive picture of the way of presentation of these concepts. For example, in a selected Australian textbook (Year 8), there were worked-out examples and practice problems concerning the mean concept under the topic of sample size although the mean concept is described under the topic of measuring the centre and spread in the same book. Similar examples were also observed in the textbooks of other grade levels. There was a total of 90 coded units of analysis in Turkish textbooks (55 units in sixth grade and 35 units in seventh grade) and 214 coded units of analysis in Australian textbooks (39 units in fifth grade, 53 units in sixth grade, 48 units in seventh grade, 64 units in eight grade). To determine these units of analysis, the author examined 99 pages (only 19 of 99 pages were related to measures of center and spread) from all selected Turkish school textbooks combined and 94 pages from all selected Australian school textbooks combined.

Coding Scheme

The coding scheme of the study that guided the data analysis involves two parts: conceptualization of measures of center and the use of crucial elements (context, visual displays, and technological tools) concerning measures of center.

Conceptualization of Measures of Center

Categorization by Cai et al. (2002) to analyse the presentation of average concept provided the basis for data analysis in this study, which was explained in the section of literature. By considering other studies on measures of center (see literature and Table 2) and emerging data of the current study, the conceptualization of measures of center was grouped into two categories: *measures of center as a mathematical algorithm* and *measures of center as a statistic*. Measures of center as a mathematical algorithm refers to procedural understanding of the mean, median, and mode, and focuses on their arithmetic properties. Measures of center as a mathematical algorithm was explained through three codes: (a) Finding the measure of center and spread (e.g., using the add-and-divide algorithm for finding the mean, finding mode as the most frequent number or category, median as the middle value in the data set), (b) performing arithmetic properties (e.g., changing particular values in the data set and seeing how mean, median, and mode changes), and (c) relating measures of center in a computational sense (the relational understanding between mean, median, and mode based on mathematical rules without understanding its statistical aspect (e.g., the rule of mean is more affected by outliers compared to median). Examples regarding each code were presented in Table 4 (see Appendix A).

The second categorization regarding the conceptualization of measures of center is measures of center as statistics. Measures of center as statistics was explained through four codes: (a) Interpreting in context (making sense of mean, median, and mode in the context); (b) Understanding the notion of distribution (e.g., understanding the measures of center as a representative of a distribution, relating measures of center with spread concept and the distribution); (c) Making inferences from the data sets (using measures of center to make decisions from the data including the comparison of different groups of data and/or make inferences from a sample); (d) Critical judgment of decisions or estimates (questioning the reliability of measures of center). Examples regarding each code were presented in Table 4.

The Use of Contexts, Visual Displays, and Technological Tools Concerning Measures of Center

Gal (1995) and Zapata-Cardona and Escobar (2019)'s categorizations guided the analysis regarding the use of context to teach measures of center in textbooks (see section on literature). The use of visual displays and technological tools in the textbooks were also analyzed. In the current study, visual displays refer to graphical displays and tabular displays. The use of visual displays in the textbooks was analysed under three categories; (a) Ready Displays (students are given a sort of visual display, e.g., table, graphs). (b) Constructed Displays (students are asked to construct a visual display), (c) No Visual Displays (there is not any visual display or students are not asked to construct a visual display). The use of technological tools refers to the use of any digital tools such as Excel, TinkerPlots, Fathom, or other kinds of software in the teaching of the notion of measures of center. The use of technological tools was analysed as follows; (a) No use or emphasis of technological tools (any technological tool is not highlighted in a particular definition, worked-out problem, etc.), (b) Use of technological tools (any technological tool (e.g., Excel, TinkerPlots, Fathom, or any software) is highlighted or students are asked to use them). The examples of each coding were presented in Table 5.

It is worth noting that at times more than one code was assigned to a unit of analysis. For example, there was one practice problem requiring finding the median and mode, interpreting them, and as well as making a prediction from the data set. In the following example from the eighth-grade textbook of Australia (p. 393), students are asked to find the median and mode that is considered as an example for the category of measures of center as algorithm (see Figure 1). Besides, the practice problem also encourages students to interpret these measures and make predictions beyond the data set by asking them to decide what number of shoes for each size.

In a survey, 25 randomly selected women were asked to give their shoe size. The results were:

7.5 7 6.5 8.5 9 10 11 6.5 8.5 9 8 8 9.5
8.5 7.5 5 7 9 8.5 8 8 9.5 10 6 7.5

a Find the range of the data. Explain what this means.
b Find the median of the data.
c Find the mode of the data. Explain what this means.
d If you were a shoe store owner, would you stock the same number of shoes for each size? Explain your answer.

Figure 1. Coding of a practice problem as finding measures of center and as well as interpreting them and making predictions (Australian mathematics textbook, Year 8, p. 393).

Reliability of Coding

To ensure reliability, a second coder, who is a researcher in mathematics education and fluent in English and Turkish, independently coded each part of the textbook series of both countries. Before starting the coding, the author explained the units of analysis of the study and described a pre-determined coding scheme. The agreement between the two coders was 93.7 %. According to Miles and Huberman (1994), agreement with the percent above 80 provides reasonable reliability. The researchers discussed the codes and categories until reaching a consensus on the coding scheme.

Table 5
Crucial Elements in Learning of Measures of Center

Elements	Examples of coding
Use of Context	
No Context	"Consider the data set: 7, 8, 0, 3, 0, 6, 0, 11, 1. Find the i) mean ii) median iii) mode of this data." (Australian textbook, Year 7, p. 343).
Story Context	"The ages of a group of students are given: 11, 14, 12, 13, 12. When a student joins this group, the median age of the group is 12. According to this, which of the following cannot be the age of the new participant in the group." (Turkish textbook, Grade 7, p.284)
Hypothetical Realistic Context	"In a survey, 25 randomly selected women were asked to give their shoe size. The results were: 7.5, 7, 6.5, 8.5, 9, 10, 11, 6.5, 8.5,7.5, 5, 7, 9, 8.5, 8, 8, 9.5, 10, 6, 7.5. If you were a shoe store owner, would you stock the same number of shoes for each size?" (see Figure 1)
Real Context	"In this Activity, you will grow wheat over a 21 day period in a controlled experiment. You will use 6 grains of wheat in each of 4 plots" Every Monday, Wednesday, and Friday, record the mean height of any germinating seeds for each plot … " (see Figure 5)
Visual Displays	
Ready Displays	see Figure 6
Construct Displays	"The number of items bought by customers at a convenience store were: 3, 5, 5, 8, 5, 3, 5,9, 7, 4, 5, 8, 7, 7, 6. Draw a dot plot of the data. Calculate the mean, median, and range of the data, and indicate these values on plot" (Australian textbook, Year 7, p. 34)
No Visual Display	see Figure 1
Technological Tools	
No Use	see Figure 1
Use of technological tools	"The number of bedrooms in the houses on a street is listed below: 2, 3, 1, 3, 2, 2, 4, 3, 2, 2, 5, 1, 2, 1,3, 1. Find the range of this data set. "You can use the statistics package or your calculator to find the measures of centre and spread" (Australian textbook, Year 8, p.393)

Findings

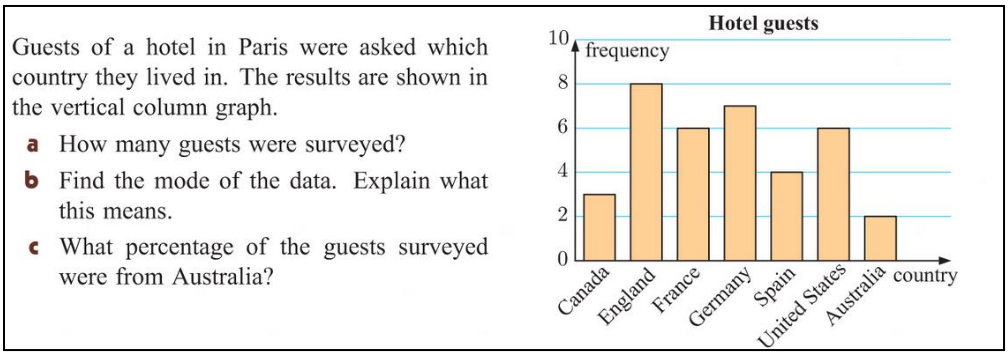
Organization and Sequence of Measures of Center Across Grades

The organization and sequence of the concepts refer to how the concepts are introduced in the textbooks and in which grade level students are engaged with these concepts. Table 6 shows that in selected Australian textbooks content regarding measures of center (mean, median, and mode) starts one year earlier than Turkish textbooks by introducing the mode concept at fifth grade. Then, sixth grade students are taught the mode and the mean. After sixth grade, all concepts including range concept (a measure of spread) are related to each other. On the contrary, Turkish textbooks introduce the range concept earlier than Australian textbooks and related it only with the mean. While mean is related with the range concept in sixth grade, then mean, median, and mode concepts were presented in seventh grade without relating these measures with the spread notion. In eighth grade, students are not taught any content of measures of center and spread. The selected Australian textbooks seemed to follow a more consistent approach regarding the relation of concepts in the presentation of measures of center across grades. Turkish textbooks relate the mean with the range in sixth grade; however, in sixth and seventh grades relating measures of center with spread is not observed.

Table 6
Presentation of Concepts in the Selected Australian and Turkish Textbooks across Grades

Country	5th Grade	6th Grade	7th Grade	8th Grade
Australia	Mode	Mode, mean	Mean, median, mode, range	Mean, median, mode, range
Turkey	-	Mean, range	Mean, median, mode	-

Secondly, regarding the organization of measures of center, the selected textbooks from Australia and Turkey also seemed not to follow a visual approach from an understanding of the idea of the clump to a more formal understanding of measures of center (formal definition of measures of center as an algorithm). In the Australian sixth-grade textbook, for example, students are introduced to the concept of mean and asked to think about what “average” means in daily life by giving examples such as “the average speed of a car”, “the average score for a test” (p. 311), which would elicit how students conceptualize the average and make connections with everyday language. Then, the mean concept is formally described (what it means and how it can be found), stating “In statistics, the mean or average of a set of numerical data is an important measure of its centre” (Year 6, p. 312) and the add-and-divide algorithm was presented for finding the mean. After this explanation, a worked-out example is presented to apply the formula to a particular set of numbers without a context. Then, students are asked to solve exercise problems in context, which also involve questions related to a graphical representation (see Figure 2). At the end of the chapter, they are engaged with an investigation part or a statistical experiment. Detailed examples are presented in the conceptualization of measures of center section.


Guests of a hotel in Paris were asked which country they lived in. The results are shown in the vertical column graph.

a How many guests were surveyed?

b Find the mode of the data. Explain what this means.

c What percentage of the guests surveyed were from Australia?

Figure 2. Interpretation of the mode and use of graph (Australian mathematics textbook, Year 8, p. 383).

Similar to the selected textbooks of Australia, Turkish textbooks start with a question or a daily life example in sixth and seventh grades. For example, seventh grade students are presented with a real-life context about the mean of a citizen's age and an activity about the students' grades. Then, the mean concept is introduced with a formal explanation (add-and-divide algorithm). After the explanation, students are asked to solve a worked-out problem or exercise problem, which mostly focuses on the application of this algorithm. Students are also given the rules regarding the relationship between measures of center. It seemed that there are attempts to introduce the measures in a context before the formal explanations. However, it is difficult for students to make sense of distribution as an aggregate rather than a collection of individual values and measures of center as a characteristic of the distribution. To achieve this goal, it is important to start with an informal understanding of the clump in the distribution and its relationship with measures of center (Bakker et al, 2006). Detailed examples are presented in the conceptualization of measures of center section. These organizations of concepts in both countries show that selected textbooks attempt to introduce the notion of average in specific situations of everyday life and then directly move to the formal aspect of the mean.

Conceptualization of Measures of Center in Textbooks

The conceptualization of measures of center is organized under two headings: measures of center as a mathematical algorithm; measures of center as a statistic. Table 7 summarizes the findings of conceptualization of measures of center.

Measures of center as a mathematical algorithm. Conceptualization of measures of center as a mathematical algorithm was interpreted under three dimensions: (1) finding the measures of center and range by using a mathematical formula, (2) exploring their properties (e.g., how the value of the mean, median, mode changes when a value is omitted from data set or added to the data set), (3) comparing measures of center without understanding their statistical meanings.

Both countries' textbooks focused more on finding the mean, median, mode, and range by using a particular formula or rule. According to Table 7, the percentages were above 80 (Australia) and 60 (Turkey) for each section of the textbooks except for activities. Australian textbooks, for example, often consisted of problems such as "Find the i) mean, ii) median, iii) mode of the data" (Year 7, p. 343). Whereas the mean is defined by the add-and-divide algorithm, median and mode were explained by the rules of middle value of an ordered data set and the most occurring category or score in the data set respectively. Similarly, Turkish textbooks defined the mean as, dividing the total of all individual values in a data set by the number of values (add-and-divide algorithm, see Figure 3), the median as the number in the middle of the data set where the numbers are ordered from big to small or small to big, and the mode as the most occurring number in the data set (seventh-grade textbook). The selected textbooks from both countries dominantly included the content regarding measures of center as an algorithm, particularly finding measures of center and spread by using the formula, compared to measures of center as a statistic (see section on measures of center as statistics).

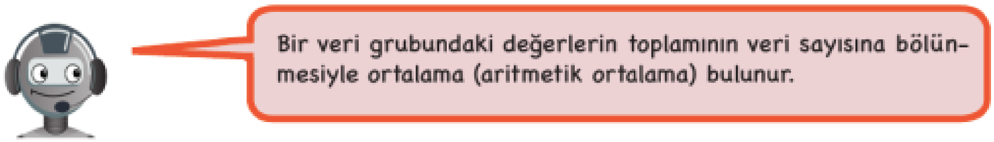


Figure 3. Description of arithmetic mean: “The mean (arithmetic mean) is found by dividing the total of all values in a data set by the number of values” (Turkish seventh-grade textbook, p. 263).

Table 7
Conceptualization of Measures of Center in Australian and Turkish Textbooks

Conceptualization of measures of center	Definitions and explanation		Work-out problems		Exercise and practice problems		Activities	
	n	%	n	%	n	%	n	%
Turkey								
Measure of Center as Algorithm								
Using formula	9	60	15	78.9	34	72.3	2	33.3
Performing properties	3	20	2	10.5	8	18.1	1	16.7
Relating procedurally	2	13.3	0	0	2	4.3	0	0
Measure of Center as Statistic								
Interpreting concepts	1	6.7	0	0	1	2.1	0	0
Making inferences	0	0	4	21.1	6	12.8	2	33.3
Understanding distribution	0	0	1	5.3	1	2.1	1	16.7
Critical judgment of estimates	0	0	0	0	0	0	0	0
Australia								
Measure of Center as Algorithm								
Using formula	18	90	24	92.3	123	86	4	36.4
Performing properties	0	0	2	7.7	6	4.2	1	9.1
Relating procedurally	1	5	0	0	4	2.8	0	0
Measure of Center as Statistic								
Interpreting concepts	2	10	2	7.7	25	17.4	1	9.1
Making inferences	0	0	0	0	14	9.8	5	45.5
Understanding distribution	1	5	0	0	7	4.9	4	36.4
Critical judgment of estimates	0	0	1	3.8	6	4.2	1	9.1

[1]Note: The total percentages are not equal to 100 % in some cases in the table since more than one code was assigned to a unit of analysis (e.g., one exercise problem). For example, there was one exercise problem focusing on both finding the mean as an algorithm and interpreting it.

Both countries' selected textbooks also encourage students to perform their arithmetical properties when a value is added or omitted from the data set and to decide when to use mean, median, and mode concepts in a given data set. Turkish textbooks focus more on these two components of measures of center (performing arithmetic properties and relating measures of center procedurally) compared to Australia (see Table 7). These were addressed from the procedural aspect in both countries' textbooks. Here is an explanation from the seventh-grade mathematics textbook of Turkey:

> The arithmetic mean is affected by the change of each data. Mode is more useful when the number of values in the data is large. Unlike the arithmetic mean, the mode is not influenced by outliers. Median is not affected by values that are too large and too small. (p. 267)

This explanation focuses on when to use the measures of center. It, however, focuses primarily on procedural understanding and it is given as a rule, which involves finding the mean, median, and mode at particular conditions (e.g., outliers). It is not discussed how these values represent the data and which one is the suitable measure as a representative of the data. Furthermore, contrary to this definition, the mode is defined as the most common (frequent) category or number within a data set (Groth & Bergner, 2006).

Measures of Center as a Statistic

Conceptualization of measures of center as statistic requires to understand measures of center as a representative of the aggregate in the data set. Four processes were identified in the textbooks: (1) Interpreting concepts, (2) making inferences from the data, (3) understanding the notion of distribution (relating measures of center with the data set and with the spread notion), and (4) critical judgment of decisions or estimates.

Australian textbooks involved more tasks on interpreting measures of center, particularly on the interpretation of the mode in the exercise and practice problems (Table 7). An example from the eighth-grade textbook is presented in Figure 2. In this example, students are asked to work with categorical data represented in the bar graph, find the mode of the data set, and interpret the mode in the context of guests of a hotel being interviewed. The textbooks also included problems that encourage students to make decisions or inferences from the data. Whereas Turkish textbooks often present data sets with only a few data values, Australian textbooks include the data set with more values to enable students to make efficient inferences from the data (seeing trends and patterns in data). van Dijke-Droogers et al. (2020) argue that students make big inferences from small samples. The introduction of growing sample sizes would enable students to decide if the sample size is large enough to make valid inferences from the data (Bakker et al., 2017). In the sixth-grade textbook of Turkey, there is an exercise problem as follows: "Veysel's written examination scores for mathematics were 80, 60 and 70; Cengiz's scores are 50, 90 and 80. Find out which one did better at math class." (p. 149). The introduction of data sets with only three individual values might not be helpful to achieve this goal, considering Turkish students' difficulty in seeing trends in data (Güler & Didiş-Kabar, 2021), unless the textbook does not aim to encourage students to compare different sample sizes (sample vs. large) to make valid inferences from the data. Here is another example from the eighth-grade textbook of Australian textbook including a data set with more data values compared to the Turkish textbook and asking students to make inferences from the data:

In a survey, 25 randomly selected women were asked to give their shoe size. The results were:

7.5	7	6.5	8.5	9	10	11	6.5	8.5	9	8	8	9.5
8.5	7.5	5	7	9	8.5	8	8	9.5	10	6	7.5	

If you were a shoe store owner, would you stock the same number of shoes for each size? (p. 393)

Regarding the relation of measures of center with distribution, Australian textbooks focused more on choosing the suitable measure of the data set in the practice problems and activities. The textbooks of seventh and eighth grades (Year 7) also relate measures of center with the spread as follows: "When we analyze numerical data, we want to be able to describe how the numbers are distributed. To help us understand a distribution, we want a measure of its centre, and also how the data is spread on either side of this centre." (p. 341). The terms of middle or centre of the data give clues about the aggregate view of the data. Turkish textbooks similarly encourage students to relate the mean with range concept to make decisions from the data in sixth grade. However, the data set as distribution is not covered.

Compared to other statistical aspects of measures of center, critical judgment has less emphasis in the textbooks of both countries. Turkish textbooks did not include the problems related to questioning of the measures of center. In the Australian textbooks, students are asked to judge the reliability of the measures of center. In Figure 4, students are presented with a data set with four values in the eighth-grade mathematics textbook of Australia. Students are encouraged to think about the reliability of the average when they are given a data set with small sample size. Critical reading of media texts regarding measures of center was not observed in the textbooks of both countries.

Maura is interested in finding the average membership size for gyms around Australia. She randomly selects four gyms, which have the following membership numbers:

280 173 227 88

a Use this result to predict the average number of members for gyms around Australia.

b Comment on the reliability of your prediction.

Figure 4. Critical judgment of average concept (Australian textbook, p. 400).

The Use of Context, Visual Displays, and Technological Tools

The use of context. The use of context was analysed based on four dimensions: (1) No context, (2) Story context, (3) Hypothetical realistic context, (4) Real context. The results were summarized in Table 8.

Table 8

The Uses of Context Regarding Measures of Center in the Australian and Turkish Textbooks

Uses of context	Turkey		Australia	
	n	%	n	%
No Context	37	42.5	38	19.38
Story Context	19	21.8	9	4.59
Hypothetical Real Context	29	33.3	148	75.5
Real Context	2	2.3	4	2.04

Turkish textbooks (42.5 %) involve tasks without context more than those of Australian textbooks (19.38 %). In other words, it is more common to see the descriptions or problems including only numbers. For example, the following worked-out example from the Turkish book was presented without context. It only presents a series of numbers and students are asked to find the mode in the given data set (see Figure 5).

Let's Solve Together 7
Let's find the mode of this data set: 100, 100, 90, 85, 80, 80, 80, 75,100
Solution:
100, 100, 90, 85,80, 80,80, 75,100
The most frequent numbers in this data set are 100 and 80. Therefore, the modes are 100 and 80.

Figure 5. Worked-out example without a context (Turkish textbook, Grade 7, p. 266).

Seventy-five-point five percent of analysed units (148 out of 196 units) in the Australian textbooks involve hypothetical realistic context in the presentation of measures of center and spread whereas 33.3 % of them (29 out of 87 units) included hypothetical realistic context in Turkish textbooks. Furthermore, the nature of this context differs in the two countries' textbooks. In Australian textbooks, the context in the worked-out examples and problems was often used in a scenario as if one collects data about a real-life issue and analyses it, which emphasizes the statistical research process. This is important to help students to make sense of the data and the nature of statistics. The following example is a part of the practice problem from the seventh-grade Australian textbook: "The students in a class were asked how many events they compared in at the school's swimming carnival. The results are displayed on a dot plot. Find the i) mean, ii) median, iii) mode of the data …" (p. 343). In Turkish textbooks, on the other hand, the topics of hypothetical contexts are limited to the number of sold things (e.g., fridges, mobile phones, tickets), which makes it difficult to relate to students' lives.

Regarding the use of real context, both countries' textbooks involved real context even if the number of usages is relatively low. Australian textbooks involve real context, particularly in the Research and Activity sections (Figure 6). Contrary to the nature of real context, story contexts neither reflect a real problem nor involve dealing with real data. They are regarded as passive contexts with artificial problems. Turkish textbooks included more story context (21.8 %) in teaching of measures of center compared to Australian textbooks (4.59 %). Here is an example from a story context from the seventh-grade textbook of Turkey: "The ages of a group of students are given: 11, 14, 12, 13, 12. When a student joins this group, the median age of the group is 12. According to this, which of the following cannot be the age of the new participant in the group." (p.284). This context involves arbitrarily given numbers regarding the age of a group of students and does not engage an active involvement with real data and real problems regarding why students are finding the median and the age of the student. On the contrary, Figure 6 shows the beginning of an activity regarding the use of real context in which students are asked to conduct a statistical experiment about finding the mean height of the germinating seeds under different conditions.

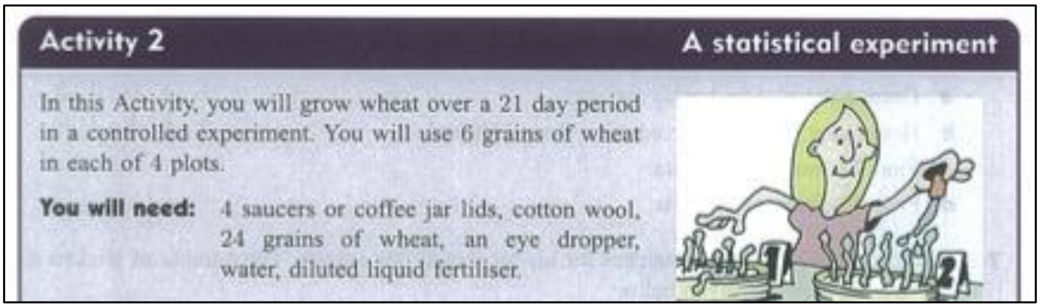


Figure 6. Australian textbook, Year 6. Real context in a statistical experiment (p. 314).

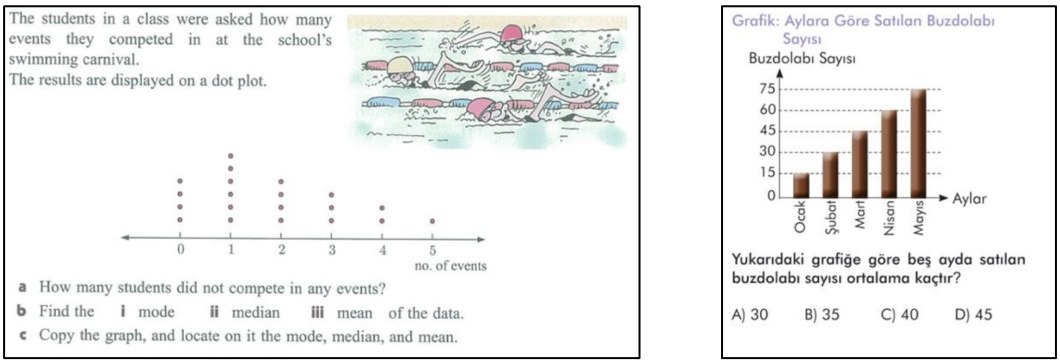
The use of visual displays. Visual displays refer to any graphical (e.g., column, dot, pie chart, stem-and-leaf) or tabular representation (tally and frequency table). The analysis was conducted based on the number of visual displays, which are either already presented in the textbooks, or students are asked to construct. Note that only the number of visual displays in the presentation of measures of center and spread was presented (see Table 9).

Table 9
The Uses of Visual Displays in the Turkish and Australian Textbooks

Uses of visual displays	Turkey		Australia	
	n	%	n	%
Ready Displays	26	30.2	61	37.4
Construct Displays	0	0	53	32.5
No Visual Display	60	69.8	70	42.9

[2]Note: The total percentages are not equal to 100 % in Australian textbooks in the table since an exercise problem involved both ready and constructed displays or more than one type of graph.

Table 7 shows that 69.8 % of coded units do not involve any visual displays in Turkish textbooks. It is more common in the selected Turkish textbooks compared to the textbooks of Australia. In the Australian textbooks, students are asked to find the mean, median, mode, and range by using the information presented in the visual displays; but also, they are asked to construct a variety of graphical representations. On the contrary, students are not asked to construct a visual display in the context of measures of center, and visual displays are limited to the bar graph in Turkish textbooks (Figure 7).



(a) (b)

Figure 7. The use of visual displays: (a) An example from Australian textbook, (b) An example from Turkish textbook.

The selected Australian textbooks also encourage students to relate mean, median, mode, and range with the graphical representations by asking them to locate each value on the graphs. For example, in Figure 7a, students are asked to find the mean, median, and mode by using the data values presented on the dot graph and to locate them on the graph (Year 7, p. 343). Dot plots are considered as a useful visual representation to make the transition to histograms and enable students to read off each data point. However, students were only asked to locate the values on the graph without making sense of these measures rather than focusing on the bump and outliers in the data and making sense of the measures of center. On the other hand, in the selected Turkish textbook (see Figure 7b), students are asked to find the mean of the number of fridges sold in five months, which are presented on the bar graph in the sixth-grade textbooks of Turkey (p. 148). The bar graph is considered the easiest way of presentation of data regarding categorical variables and categories can be organized without an order. However, in this example, students are not asked to relate measures of center with data visually and it is hard for students to imagine the distribution in this example with very limited data values. It simply requires reading of the values on the graph and making calculations.

The use of technological tools. The use of technological tools refers to the use of any digital tools such as Excel, TinkerPlots, Fathom, or other kinds of software in the teaching of the notion of measures of center. Australian textbooks seemed to put more emphasis on the use of technological tools (6 out of 193 coded units) and provided an application, named statistical package and calculator, in the electronic version of the textbooks from fifth grade to eight-grade. In the interactive user interface of this application, students flexibly experience recording the data (discrete, continuous, or categorical), constructing a variety of graphs, finding measures of center and spread (range, standard deviation). On the other hand, there is only one worked-out example (1 out of 85 coded units) regarding the use of a technological tool in the seventh-grade textbook of Turkey. The example is as follows, "The grades that students in a class of ninth graders got from a mathematics course are as follows: 60, 63, 70, 70, 75, 90, 95, 97, 100. Let's calculate the arithmetic mean, mode, and median of these grades using a spreadsheet program." (p. 268).

Discussion and Conclusions

This paper investigated how the selected Turkish and Australian school mathematics textbooks presented the measures of center to support students' understanding of the measures of center by taking into consideration of the use of contexts, visual displays, and technological tools. This study shows a case that shows substantial differences between the selected textbook series of Turkey, that had reforms in curriculum contents including statistics and probability in 2005, and Australia among English-speaking countries that made early reforms in statistics education (AEC, 1991), and conducted the majority of studies in statistics education (Petocz et al., 2018).

The selected textbooks of Australia and Turkey introduced the mean, median, mode, and range concepts at different grades. The selected textbook series of Australia started with the mode concept, which is a relatively early inclusion in the textbooks compared to Turkey. Then, they moved to the mean and median concept, which is close to the natural progression of students regarding measures of center "from *mosts*, to *middles*, to the use of the mean as *typical* or as a *fair share*, to a *representative* for an entire data set" (p. 156) (Biehler et al., 2018). This difference can be attributed to the differences in national curriculum because the ACARA (2015) suggested sample tasks on the mode concept regarding the objective of interpreting data in context in fifth grade even though it did not specify precisely the mode concept. The Turkish curriculum (MoNE, 2018) introduced mean and range concepts in sixth grade and the mode concept in seventh grade. Turkish textbook series seemed to be strictly consistent with 2018 curriculum (MoNE, 2018). On the other hand, the textbooks of both countries seemed not to provide a learning progression from the identification of clumps in the data to the informal understanding of the measures of center and then to its formal understanding, as suggested by the studies on measures of center (Bakker et al., 2006; Makar, 2014). Furthermore, they did not develop a visual approach to identify the clump in the data representation and to understand the shape of the distribution (Pfannkuch et al., 2010). This conclusion puts importance on examining the learning progressions or trajectories in designing textbooks (Olson, 2014).

Both countries' textbooks put more emphasis on measures of center as an algorithm compared to the meaning of measures of center as a statistic. Both countries' curriculums (ACARA, 2015; MoNE, 2018) involved learning objectives regarding the calculation of measures of center and spread. Thus, it is common to see the add-and-divide algorithm in both countries' selected textbooks. It might not be surprising that students perform better in finding the average and range (Çatman-Aksoy & Işıksal-Bostan, 2021), or conceptualized it from an algorithmic approach (Mokros & Russell, 1995). The selected Australian textbooks also emphasize the statistical aspect of measure of center more compared to the selected textbooks of Turkey. Textbooks of Australia ask students to locate mean, median, mode, and range in the distributions, simply read off the data values in the graphical representations, or calculate mean and median by adding a value to the data set, or considering the effect of the outliers. However, these still have a focus on mathematical aspects. It is crucial to build the understanding of the essence of measures of center as a representative of the aggregate and relate it with distribution and variation concepts (Bakker, 2004; Pfannkuch et al., 2010; Zawojewski & Shaughnessy, 2000).

Another major conclusion is related to the use of context, visual displays, and technological tools to support the learning of measures of center in the textbooks. The textbooks of both countries include definitions or problems without context. It is more common to see in the selected Turkish textbooks. More than 50 % of coded units (e.g., problems, explanations, activities) were without context or included story context. It, however, is not possible to think of statistics without context (Cobb & Moore, 1997). The statistical concepts should be developed within contextual problems rather than simply learning a specific concept and then applying it. Developing the concept in contextual problems encourages students to make a judgment in that context and use their background knowledge of the context (Pfannkuch et al., 2011). Hypothetical realistic contexts are often used in Australian textbooks within a scenario of statistical research about a real-life issue. This might help students to make sense of the data and nature of statistics more intuitively. On the other hand, the inclusion of real contexts and real data into the textbooks is important to make sense in the students' lives (Zapata-Cardona & Escobar, 2019), which is rarely observed in the selected textbooks of both countries.

Graphical representations are seen as helpful in learning of measures of center (Watson et al., 2018). Australian textbook series put more emphasis on relating the mean, median, mode, and range with a variety of visual displays. Such a difference in the textbook series of both countries can be attributed to the differences in learning objectives in curriculum because the ACARA (2015) involved a particular learning objective of "describe and interpret data displays using median, mean and range" (p. 16) in seventh-grade. On other hand, the Turkish Curriculum (MoNE, 2018) involved learning objectives regarding interpreting and constructing graphs; but did not involve objectives regarding relating visual displays with measures of center and spread. Turkish textbooks focus on bar graphs to support students' understanding of measures of center and ask students to read the data given in the graphs. It could be also supported by the use of technological tools through which students can interactively deal with distribution, understand the aggregate view of the data, and the measures of center as the representative of this aggregate (Bakker et al., 2006; Lee & Hollebrands, 2008). The selected Australian textbooks provide students opportunities to deal with the data through an application of statistics in the digital book, which is considered as an example of technology integration in textbooks (Sherman et al., 2020). Recently, the textbooks have a living nature for learning statistics by embedding the technology into their content (Biehler et al., 2013).

This study investigated the way of presentation of measures of center in the textbooks by extending the previous studies on the arithmetic mean (Cai & Moyer, 1995; Pollatsek et al., 1981; Strauss & Biehler, 1988). This paper investigated not only the learning and teaching of measures of center in terms of mathematical and statistical aspects but also the relation of them with crucial elements of statistics like context, visual displays, and technological tools. Furthermore, there are very few studies on the investigation of statistical concepts in textbooks, particularly cross-cultural comparisons of textbooks in statistics education. Even though the findings of the study are limited to the selected school textbooks of two countries, this study contributes to the statistics education literature modestly by providing an in-depth analysis of the presentation of measures of center in textbooks of countries with different backgrounds in statistics education, informing international readers from different countries.

The striking differences between the selected textbooks of Australia and Turkey can be attributed to Australia's early inclusion of statistics and probability content into the curriculum (Watson, 1998), its economic and technological developments, and the fact that the majority of studies in statistics education have been conducted by researchers in English-speaking countries (the USA, Australia, the UK, New Zealand) and other European countries such as the Netherlands (Petocz et al., 2018; Zieffler et al., 2011). The findings are also consistent with the study of Wang and Lu (2018) in which they emphasized the differences in content presentation in statistics (well-organized content presentation, integration of technology, and use of real-life context) between the UK and China. Developments in the content presentation of Chinese textbooks were included about 10 years later than western countries such as the UK and USA. This study similarly points to the ways of textbook development in these countries in terms of capturing the statistical aspect of measures of center, using real context with a wide range of topics, relating measures of center with spread through visually and technologically enhanced tasks. While some countries still have difficulties in updating curriculum materials, some of them are in their second or third wave of updating curriculum materials. For example, in the Australian context, Watson (2017) compares curriculum standards of Australia, New Zealand, and the U.S. and suggests ways of developing curriculum materials in Australia (Watson, 2016). In this regard, there is a need to increase international collaborations for the benefit of all citizens in the world (Ben-Zvi & Makar, 2016).

The findings of the study also provide suggestions for teachers in task selection regarding measures of center and spread. Teachers can select tasks or appropriate tasks from the textbooks (Remillard, 1999; Sherin & Drake, 2009). In both selecting and appropriating the content or tasks from the textbooks to teach measures of center, teachers would consider several key issues about the tasks. These are: (1) using tasks with realistic contexts that require the examination of the distribution graphically, to make sense of the measures of center as a representative of the aggregate in the data set (English & Watson, 2018; Lee & Hollebrands, 2011), (2) starting with tasks or content that follow an informal approach to learning measures of center (understanding the notion of the clump visually) rather than initially learning formal measures of center (Bakker et al., 2006), and (3) relating measures of center with the concepts of distribution, spread, and inference (Kazak et al., 2018).

Limitations and Future Research

The findings of the study should be interpreted within its limitations. This study is limited to the selected school textbooks of two countries, which are widely used in Turkey and one of the mainstream textbook series of Australia. Different textbook series are used in Australia and data analysis in the current study was located on one textbook series. Therefore, it is important to note that the conclusions of the study are limited to the selected school textbook series. It does not suggest generalizability at the international level. Although curriculum standards of both countries regarding measures of center were taken into consideration in interpreting the findings, this study did not examine to what extent textbooks are consistent with the national curriculum standards of Turkey and Australia. To learn more about the learning opportunities provided to students for understanding the concepts of measures of center, in future studies researchers should also investigate how the teachers implement these curriculum materials in their classrooms and how students using these textbooks learn the concepts of measures of center. Future research would also include more textbook series from different countries including developing and developed countries to gain more insight into the cross-cultural comparisons of textbooks. This study does not advocate one country's textbooks over another. Rather, it aimed to identify the similarities and differences in their approaches in the presentation of the measures of center, which are key concepts in statistics education. The findings of this study could shed light on the studies on textbooks content analysis in statistics education.

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Appendix A:

Table 4

Conceptualization of Measures of Center in the Textbook Analysis

Codes and categories	Examples of coding	Relevant terms from literature
Measures of Center as Algorithm		Procedural and conceptual understanding of mean as an algorithm (Cai et al., 2002); average as an algorithm (Mokros & Russell, (1995)
Finding the mean, median, and mode	"Consider the data set: 7, 8, 0, 3, 0, 6, 0, 11, 1. Find the i) mean ii) median iii) mode of this data." (Australian textbook, Grade 7, p. 343).	Direct application of average algorithm (Cai et al., 2002)
Performing arithmetic properties	"The average of the two numbers is 80. What is the average when the number '20' is added to this data set?" (Turkish textbook, Grade 6, p. 144)	Flexible application of average algorithm (Cai et al., 2002); properties of average (e.g., the average is influenced by values other than the average) (Strauss & Bichler, 1988)
Relating measures of center procedurally	"The arithmetic mean is affected by the change of each data. Mode is more useful when the number of values in the data is large. Unlike the arithmetic mean, the mode is not influenced by outliers. Median is not affected by values that are too large and too small." (Turkish textbook, Grade 7, p. 267)	Comparing and contrasting mean, median, and mode in the unistructural/concrete level or multistructural level (including only definition or process telling) (Groth & Bergner, 2006)
Measures of Center as Statistic		Conceptual understanding of mean as a statistic (Cai et al., 2002); representativeness (Mokros & Russell, 1995; Makar, 2014)
Interpreting in context	"Guests of a hotel in Paris were asked which country they lived in. The results are shown in vertical column graph … (b)Find the mode of the data. Explain what this means." (see Figure 2)	Understanding and interpreting average in context (Watson, 2006)

Codes and categories	Examples of coding	Relevant terms from literature
Understanding the notion of distribution (relating measures of center, measures of spread)	"When we analyze numerical data, we want to be able to describe how the numbers are distributed. To help us understand a distribution, we want a measure of its centre, and also how the data is spread on either side of this centre." (Australia textbook, Grade 7, p. 341).	*Relating measures of center with variation* (Bakker et al., 2006; Cai et al., 2002; Zawojewski & Shaughnessy, 2000; Watson, 2006); *understanding average as a representative of a distribution* (aggregate) (Makar, 2014)
Making inferences from the data sets	"Maura is interested in finding the average membership size for gyms around Australia. She randomly selects four gyms, which have the following membership numbers: 280, 173, 227, 88 Use this result to predict the average number of members for gyms around Australia Comment on the reliability of your prediction" (see Figure 3)	Explorations of average through informal inferential reasoning (Makar, 2014); Comparison of mean, median, and mode to make appropriate decisions (Cai et. al., 2002)
Critical judgment of decisions or estimates		Questioning the claims that are inappropriate (without proper statistical justification) (Watson, 2006); Knowledge of critical questions (Gal, 2002)