# Characteristics of 3-Year-Old Preschool Children's Evolving Number Knowledge: A One-Year Multiple-Case Study

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There is a considerable body of research on children's number knowledge development, and in particular on children ages 4 years and older and who have some counting experience. Children in early care preschool settings (ages 2 and 3) and who are not yet experienced counters are an understudied population. We conducted a multiple-case study on five 3-year-olds over the course of a year, investigated their challenges and successes as they moved through the developmental progressions for specific number knowledge domains, and determined their key shifts in learning as they related to early predictors of later mathematics achievement. We found interesting variations in their test scores and, with multiple data sources, determined key shifts in learning observed across cases including acquisition of verbal counting skill, one-to-one correspondence, cardinality, and connecting and coordinating their number skills. While the developmental progressions provide teachers with a framework for understanding typical mathematical development, every child's learning path is different. This research provides an in-depth look at 3-year-old preschool children's variability in number knowledge and their key shifts in learning as they relate to early predictors of later mathematics achievement. We share implications for how and when early preschool teachers can support learning.

Three-year-old children's curiosity provides early childhood educators with numerous opportunities to support early mathematics learning. Young children bring informal yet complex mathematical ideas to their early care and preschool classrooms. There is widespread acknowledgement among mathematics educators about the value of capitalizing on this emerging mathematical knowledge as children make connections between their informal knowledge and school mathematics (Clements, 2001). In particular, children's number knowledge development is one of the most important informal-to-formal connections early childhood educators can support. Robust early number knowledge—such as counting, subitizing, numeral recognition, quantity comparison, and part-whole composition and decomposition of numbers—is foundational to later mathematics learning (Siegler & Braithwaite, 2017) and correlated to later mathematics outcomes (Duncan et al., 2007; Geary et al., 2013; Jordan et al., 2010).

Within the considerable body of research on children's number knowledge development, children in early care preschool settings (age 3) are an understudied population, and there is a need to understand the complexity of their number knowledge acquisition (Torbeyns et al., 2015). In this study, we followed five 3-year-olds over the course of a year in their early care preschool classroom, investigated their challenges and successes as they moved through the developmental progressions for specific number knowledge domains, and determined their key shifts in learning. We focused on 3-year-old children because they were not yet experienced counters and were in the early stages of learning to count with meaning.

# Conceptual Framework

## *Developmental Progressions for Verbal and Object Counting*

The early work of Fuson (1988) and Gelman and Gallistel (1978) provide educators and researchers with frameworks for understanding young children's acquisition of verbal and object counting skills. Gelman and Gallistel (1978) named five principles that define the stages of counting—one-one, stable-order, cardinal, abstraction, and order-irrelevance. These principles are the foundation for mathematics education research on developmental learning progressions.

In this study, we relied on Sarama and Clements's (2009) research on learning trajectories, an extension to Fuson's and Gelman and Gallistel's foundational work. They explain learning trajectories in three parts: a goal, developmental progression, and instructional activities. Developmental progressions are useful frameworks for helping educators understand their students' progress toward acquiring critical early number concepts (Sarama & Clements, 2009). The developmental progression is the typical path students take to achieve the mathematical goal. For instance, a developmental progression for counting follows Gelman and Gallistel's principles and begins with the verbal counting words ("one, two, three, four…"), then progresses to one-to-one correspondence in counting (matching only one object to each number word), and then to verbal counting with one-to-one correspondence with meaning (acquiring the cardinality principle that a number word represents the entire set of objects). A child may begin by saying counting words in the correct sequence, but is not yet able to count objects with one-to-one correspondence. With experience, the child gradually improves their verbal counting and maintains one-to-one correspondence as they count four or five objects. After counting correctly and with one-to-one correspondence, an adult might follow up with the question, "So, how many do you have?" The child may not yet be at the developmental progression level of cardinality, thereby re-counting the set of objects instead of saying "There are four." Understanding cardinality means that the child understands that the last number said in the count represents the amount of the set of objects and does not need to re-count the objects to answer the "How many?" question. As children move from verbal counting, to counting with one-to-one correspondence, and to counting with one-to-one correspondence with cardinality, they are progressing through a developmental progression in which each subsequent level of thinking is more sophisticated than the last.

*Predictors of later achievement*. In addition to this line of mathematics education research, cognitive scientists' research pinpoints critical milestones, providing educators with key concepts and skills to anticipate as young children develop mathematical knowledge (e.g., Geary et al., 2018; Le Corre & Carey, 2007; Watts et al., 2014). Within the verbal and object counting progression, cardinality —an understanding that the last number said represents the amount in the set—has emerged as an important topic in recent research on predictors of later mathematics achievement. For example, Nguyen et al. (2016) studied different types of counting preschoolers used. Basic counting in their study included: verbal counting, maintaining one-to-one correspondence, numeral recognition, and perceptual subitizing. Advanced counting included: counting with cardinality, counting forward or back from a given number, and conceptual subitizing. They found that students who used advanced counting at the end of preschool were more likely to exhibit high achievement on mathematics assessments at the end of elementary school.

Geary et al. (2018) conducted a series of studies to assess preschool children's understanding of 12 quantitative competencies and the school-entry number knowledge that predicts later mathematics outcomes. Their results showed that the age that preschool children acquired an understanding of cardinality was strongly related to their later number system knowledge and mathematics achievement. This finding indicated that cardinality is a key anchor for preschoolers' mathematics learning. In a follow-up study, Geary et al. (2019) investigated the prerequisite knowledge for understanding cardinality and assessed preschoolers' approximate number knowledge, ordinal number knowledge, verbal counting, enumeration, and numeral recognition. They found that IQ and letter recognition along with quantity discrimination, verbal counting, and enumeration were factors that influenced children's initial understanding of cardinality and predicted the age at which children understood the cardinality principle.

Overall, these studies suggest that individual differences in numerical knowledge are present as early as preschool and strongly influence mathematics learning. Specifically, preschool-age children's acquisition of counting (and its related sub-skill, cardinality) and number relationships (and its related sub-skill, quantity comparison) is foundational for their mathematical development.

### *Using Developmental Progressions to Understand Number Knowledge*

While these developmental learning progressions and predictors of later mathematics achievement provide important frameworks for understanding 3-year-old children's evolving mathematical knowledge, there is considerable individual variability in their skills and development. Detailed microgenetic and case study methods could reveal how children follow or do not follow these developmental pathways, the characteristics of skills-in-transition, and the subtle changes in learning that take place over time (Batchelor et al., 2015). Young children's mathematics must be further examined to better understand the variability in these skills and how and when teachers can best support learning. In addition, studying very young children's (i.e., 3-year-old preschoolers) movement through these developmental paths is still limited (Torbeyns et al., 2015). Hence, there is much to learn about individual student's lived development of number knowledge. Our study uses developmental progressions as a framework to observe important shifts in 3-year-old children's mathematics learning in order to better to understand individual children's evolving number knowledge.

In addition to the large longitudinal studies, much of the research on preschool mathematics investigates curriculum, interventions, and teaching practices as ways to advance children's early mathematics skills. Most of these studies with positive learning effects focus on 4- and 5-year-old preschool students, such as Presser et al.'s (2015) study on Big Math for Little Kids, Ramani and Siegler's (2008) study on using linear board games, and Clements et al.'s (2011) study on the learning-trajectory based TRIAD curriculum. Research on the mathematics learning of 3-year-old children and younger tends to focus on the types of mathematical activities they engage in (Lee, 2012) and how certain behaviors prime them for emergent mathematics (Franzén, 2015; Geist, 2009; Miyakawa et al., 2005). For example, Franzén's (2015) study about toddlers' natural engagement in mathematics without teacher guidance showed that toddlers use their bodies to understand mathematics and Miyakawa et al. (2005) found that 1- to 3-year-olds will develop logico-mathematical thinking when tasks are developmentally appropriate and stimulate interest in solving problems. Our study is not investigating curriculum, interventions, or teaching practices.

Rather, our study is a fine-grain examination into 3-year-old children's individual number knowledge acquisition over a year in an early care preschool setting.

## Methods

We used multiple-case study methods to document and describe 3-year-old preschoolers' challenges, successes, and key shifts in learning as they moved through the developmental progressions for early number knowledge. We selected a multiple-case study approach (Yin, 2009) in order to 1) first, conduct a fine-grain analysis of individual case's test scores and responses over time to make sense of their number knowledge variability, and 2) then, conduct a thematic analysis across multiple cases and across the classroom observations and teacher interviews to better understand characteristics of 3-year-old children's developing number knowledge. The following research questions guided this study: 1) What are the patterns of 3-year-old preschoolers' mathematics assessment scores?; 2) What are the characteristics of 3-year-old preschoolers' mathematics knowledge as it develops over one school year?

### *Participants and Setting*

The participants in this study were five preschool children from one classroom at a university early care preschool center in the western United States. They were selected based on their age (i.e., children closest to age 3) and recent entry to the "senior preschool" classroom (for approximately ages 3-5). Their ages ranged from 2 years 11 months to 3 years 5 months at the beginning of the study. All five children were White females whose parent(s) were students or worked as staff or faculty at the university.

The senior preschool classroom usually had 25 children present each day (full-day sessions, five days per week) with 1 lead teacher, 1 assistant teacher, and an aide. Four of the five participants had been in early care prior to the year of the study in a classroom for children ages 2-3 at the same center.

The lead preschool teacher had a bachelor's degree in early education and was involved in professional development activities related to mathematics instruction, such as attending mathematics education-focused sessions at early childhood conferences and participating in center-based professional development discussions about mathematics education-focused articles, counting collections, and story problems. The lead teacher directed a 15- to 20-minute mathematics-focused activity once a week during small groups. The children used a variety of counters during these activities ranging from realistic looking fruit to more abstract counters such as red and white chips or small rocks to represent items in the problems. The lead teacher emphasized informal opportunities to engage children in counting collections and word problems throughout the week, most often during free play, transitions, and snack time, though this varied by week, by child, and by setting. The teacher indicated that her instruction focused on counting, and especially one-to-one correspondence, for the 3-year-old children in her class across the formal and informal mathematics experiences.

The study's setting occurred in two locations: 1) assessments/interviews were conducted in a quiet observation booth next to the classroom, and 2) classroom observations were conducted near the table where students worked in small-group mathematics-focused activities.

### *Data Sources*

Data for this study were collected using mathematics assessments, classroom observation field notes, and teacher interviews. The multiple sources of data allowed us to understand each child from the perspectives of test scores, the naturalistic classroom setting, and the teacher. It is difficult to obtain reliable mathematics assessment data on young children, hence, the mathematics assessments were carefully selected for this study as a developmentally appropriate tool and the classroom observations and teacher interviews were intended to supplement the assessment data and better understand the characteristics of number knowledge development within the children's daily context and setting.

*Mathematics assessments*. There were two types of mathematics assessments administered during the study: 1) Tools for Early Assessment in Math, Part A (TEAM-A), and 2) Individual Growth and Development Indicators of Early Numeracy (myIGDIs-EN). The TEAM-A is a broad mathematical ability measure that has been empirically validated for both diagnostic and research work (Weiland et al., 2012). It is a one-on-one assessment of PreK-2 children's early mathematics knowledge including five domains based on learning trajectories (Clements et al., 2011). Most items on the assessment engage students with manipulatives, such as counters (e.g., boxes, toy bananas), or pictorial items (e.g., picture of 5 grapes). All items are scored as correct or incorrect with options on some items to assess where students' responses fall on the learning trajectories, which show a progression of knowledge (Sarama & Clements, 2009). The overall score is the number of correct items and is intended to report a student's general mathematical ability compared to same-age peers. In this study, we focused on the results of the TEAM-A pre-K number knowledge items (questions 1-12). We collected data on students' overall mathematics ability at three time points—at the beginning of the study in January, the midpoint in July, and at the end of the study in December—in order to examine changes in students' mathematics understanding over the course of the study.

The myIGDIs-EN is a progress monitoring tool for assessing and monitoring preschool-aged children's (ages 3-6 years) number skill acquisition (Hojnoski & Floyd, 2004). It is individually administered and includes four measures: Oral Counting, Number Naming, Quantity Comparison, and One-to-One Correspondence. Manipulatives are not part of the assessment, but the flip book contains stimuli of numerals or dots. The item scoring is a raw score of correct items, which is norm-referenced with age-based benchmarks. We collected data on children's number skills at three times points (March, May, and September) to examine changes in students number skill acquisition over the course of the study and in between the TEAM-A administrations.

*Classroom observation field notes*. Researchers observed the participants in small-group mathematics activities nearly once per month throughout the study, with a total of eight observations. The lead researcher took field notes about the activities for each observation and the children's mathematical behaviors. The small-group mathematics activities were selected for the classroom observations for two main reasons: 1) these were intentionally planned mathematics experiences to encourage counting and problem solving (Carpenter et al., 2017); and 2) problem solving is an excellent window into children's mathematical thinking (Charlesworth & Leali, 2012). Each session was audio recorded for the purpose of reminding the researcher of events and behaviors that occurred during each small group activity. The purpose of the classroom observation field notes was to collect data on students' use of early number knowledge skills in their naturalistic classroom setting to supplement or augment the test score data.

*Teacher interviews*. A researcher interviewed the lead classroom teacher at three time points during the study (March, June, and September) in order to collect data on the teacher's perspective of the small-group mathematics activities and the participants' mathematics learning. The researcher used a semi-structured interview protocol to collect data on the teacher's main learning objectives in mathematics for that month, typical lesson plans for the small-group mathematics activities, noticings about the children's mathematics learning, and plans for next month's mathematics lessons. Each interview was audio recorded and lasted for approximately 15 minutes.

### *Procedures*

Data were collected at multiple time points across the year (summarized in Table 1). The purpose of this timeline was to obtain achievement data at early 3-years-old, mid 3-years-old, and close to age 4. The time points for data collection were planned to maximize opportunities to capture subtle shifts in learning or use of early number knowledge skills from ages 3 to 4. The study lasted over one year (January to December) during the students' regularly scheduled early care day (8:30am to 5:00pm) with almost all assessments and observations in the mornings. All children participated in typical classroom activities.

Table 1
*Data Collection Time Points*

| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TEAM-A | X | | | | | | X | | | | | X |
| myIGDIs-EN | | | X | | X | | | | X | | | |
| Observations | | X | X | X | X | X | | X | X | | X | X |
| Interviews | | | X | | | X | | | X | | | |

### *Data Analysis*

Case study data analysis consisted of examining, categorizing, and combining the qualitative and quantitative data within each case and across multiple cases (Yin, 2009). We examined the trends in test scores for each participant and across the five cases. We used open and axial coding on children's verbal responses and gestures during the assessments, with the verbal and object counting developmental progressions and numerical cognition predictors as our conceptual framework. We conducted a deductive qualitative analysis of teachers' interviews and classroom observations to accompany the test score and assessment-response analysis in order to more fully understand students' evolving mathematics knowledge.

## Results

The results are organized around our research questions. We first present the patterns of the preschool children's mathematics assessment scores by summarizing their overall scores and more specifically describing each case's scores, responses, and strategies. Then, we address the second research question and present the results on the characteristics of evolving mathematics knowledge based on the assessments, classroom observations, and teacher interviews.

*Patterns of Preschool Children's Mathematics Assessment Scores*

The five cases demonstrated variation in their patterns of achievement on the TEAM-A. A broad view of overall scores showed patterns of increasing scores (Hannah and Jane), decreasing scores (Sam), inconsistent scores (Emily), and emerging growth (Ava). Table 2 summarizes the number of items students scored correct across three different assessment points on the TEAM-A. All names are pseudonyms.

Table 2
*Children's TEAM-A Scores (Number of Items Correct) Across Time Points*

| Student (age at start of study) | January | July | December |
|---|---|---|---|
| Ava (2 years, 11 months) | 0 | 0 | 3 |
| Hannah (3 years) | 1 | 9 | 20 |
| Jane (3 years, 2 months) | 6 | 12 | 19 |
| Emily (3 years, 3 months) | 2 | 1 | 2 |
| Sam (3 years, 5 months) | 12 | 11 | 7 |

The five cases demonstrated different patterns of achievement on the IGDIs-EN assessments ranging from increasing overall scores (Ava, Hannah, Jane, and Sam) to increasing then decreasing overall scores (Emily). A closer examination of the IGDIs-EN subtests showed much more variation in patterns of achievement. Table 3 summarizes the number of items students scored correct on the IGDIs-EN across administrations.

Table 3
*Children's myIGDIs-EN Number of Items Correct Across Time Points*

| Student | March | May | September |
|---|---|---|---|
| Ava | Overall: 10<br>OC: 5<br>OO: 1<br>QC: 4<br>NN: 0 | Overall: 14<br>OC: 5<br>OO: 9<br>QC: 0<br>NN: 0 | Overall: 26<br>OC: 14<br>OO: 5<br>QC: 6<br>NN: 1 |
| Hannah | Overall: na<br>OC: na<br>OO: na<br>QC: na<br>NN: na | Overall: 31<br>OC: 5<br>OO: 5<br>QC: 12<br>NN: 9 | Overall: 38<br>OC: 13<br>OO: 4<br>QC: 14<br>NN: 7 |
| Jane | Overall: 37<br>OC: 15<br>OO: 10<br>QC: 9<br>NN: 3 | Overall: 43<br>OC: 10<br>OO: 15<br>QC: 14<br>NN: 4 | Overall: 62<br>OC: 29<br>OO: 11<br>QC: 15<br>NN: 7 |
| Emily | Overall: 4<br>OC: 3<br>OO: 1<br>QC: 0<br>NN: 0 | Overall: 7<br>OC: 5<br>OO: 0<br>QC: 2<br>NN: 0 | Overall: 4<br>OC: 4<br>OO: 0<br>QC: 0<br>NN: 0 |

| | | | |
|---|---|---|---|
| Sam | Overall: 33<br>OC: 10<br>OO: 4<br>QC: 12<br>NN: 7 | Overall: 41<br>OC: 12<br>OO: 3<br>QC: 19<br>NN: 7 | Overall: 56<br>OC: 30<br>OO: 5<br>QC: 13<br>NN: 8 |

*Note.* IGDI Subtests: OC – Oral Counting, OO – One-to-One Correspondence, QC – Quantity Comparison, NN – Number Naming

Overall, Ava and Sam's patterns in assessment scores differed on the IGDIs-EN and TEAM-A. Emily's scores were low on each assessment across the year, and multiple data sources (e.g., observations and teacher interviews) were needed to better understand Emily's mathematics progress, which are reported in a subsequent section. Hannah and Jane showed steady mathematics progress on the assessments across the year when considering overall scores on each test. A more detailed analysis of the subtests and individual items as well as the multiple data sources revealed differences in the characteristics of their developing mathematics knowledge. The next section of the results addresses this analysis.

### *Characteristics of Preschool Children's Developing Mathematics Knowledge*

Our second research question was about the characteristics of 3-year-old preschool children's mathematics knowledge as it develops over one year. To answer this question, we conducted a more detailed analysis of the patterns of mathematics achievement by using learning progressions as the conceptual framework. We considered how individual items and subtests assessed levels along the learning progressions and characterized students' mathematics knowledge in terms of learning progression levels and emergent themes.

This section is organized by learning progression domains (i.e., counting, comparing and ordering quantities, and numeral recognition and representation), and within each of these, other emergent characteristics of students' developing mathematics knowledge are presented (e.g., key shifts in learning). Finally, we present the results of the observations and teacher's interviews within the themes of One-to-One Correspondence and Problem Solving (Sam and Jane), One-to-One Correspondence and Organizing into Sets (Ava and Hannah) and Developing One-to-One Correspondence in Play (Emily).

*Counting developmental learning progressions.* Table 4 summarizes the children's counting developmental learning progressions (Sarama & Clements, 2009) based on their correct/incorrect responses on individual TEAM-A items, scores on the IGDIs-EN Oral Counting (OC) and One-to-One (OO) subtests, and verbal responses and gestures during the assessments.

Table 4
*Summary of Counting Learning Progressions Progress Across Time Points*

| Student<br>Age in Jan<br>Age in July<br>Age in Dec | Pre-Counter or Chanter[1] | Verbal Reciter | Reciter to 10 | Corresponde r | Counter of Small Numbers | Producer of Small Numbers |
|---|---|---|---|---|---|---|
| Ava<br>2yr 11mo<br>3yr 5mo<br>3yr 10mo | | Jan TEAM<br>March IGDIs | Emerging in May IGDIs & July TEAM | May IGDIs, Sept IGDIs, & Dec TEAM | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Hannah<br>3yr<br>3yr 6mo<br>3yr 11mo | | Jan TEAM | July TEAM | Emerging in May IGDIs, July TEAM, & Sept IGDIs | Emerging in July TEAM (to 4) | Dec TEAM (to 7) |
| Jane<br>3yr 2 mo<br>3yr 8 mo<br>4yr 1 mo | | | | March IGDIs & May IGDIs | Jan TEAM, July TEAM & Sept IGDIs | Dec TEAM (to 7) |
| Emily<br>3yr 3mo<br>3yr 9mo<br>4yr 2mo | Jan TEAM, March IGDIs, & July TEAM | May IGDIs, Sept IGDIs, & Dec TEAM (up to 4) | | | | |
| Sam<br>3yr 5 mo<br>3 yr 11mo<br>4yr 4 mo | | | | March, May, & Sept IGDIs<br>Dec TEAM | Jan TEAM & July TEAM | |

[1]Note: The category definitions are based on Sarama and Clements (2009) developmental learning progressions. Pre-Counter: names some number words, not in a sequence; Chanter: sing-song numbers but without meaning; Reciter (Verbal): counts but not always correct sequence; Reciter (10): verbally counts to 10 with some one-to-one correspondence; Corresponder: uses one-to-one correspondence between count words and objects, but not yet consistent cardinality; Counter (Small Numbers): counts objects to 5 with cardinality; and Producer (Small Numbers): can produce a group of four objects.

*Oral counting (verbal reciting): Emily and Sam.* This analysis revealed oral counting as a theme in Emily and Sam's data, though in different ways. While Emily's overall assessment scores were low with little improvement, this analysis indicated that Emily progressed from the Pre-Counter/Chanter level to the Verbal Reciter level over the year. For example, in the January and March assessments, Emily would often name some number words or sing a few number words but not connected to the action of counting. Whereas, in the May, September, and December assessments, Emily showed more evidence of verbal counting by using separate number words (rather than just singing or saying a singular number word), attempting to sequence individual count words, and using the count words to count items by pointing to the objects (i.e., making meaning of what it means to count). Emily's progress was within the meaning of counting and there was emerging evidence of some skills in oral counting.

Rather than progressing from one level to the next, Sam moved between the Corresponder and Counter of Small Numbers levels of the counting progression across the test administrations. This analysis showed that Sam's most evident growth was her oral counting to 30, though inconsistent one-to-one correspondence prevented access and correct responses to items with more than five objects.

*Shift to one-to-one correspondence: Ava.* While Ava scored better on the myIGDIs-EN than on the TEAM-A, this analysis showed converging evidence of Ava's movement through the counting learning progression, showing her growth in counting as a Verbal Reciter to a Corresponder. Specifically, by the December TEAM assessment, Ava was able to show her ability to count up to 6 objects with one-to-one correspondence, though not yet with cardinality. A transcript from the July and December TEAM-A administration provides evidence of Ava's shift to one-to-one correspondence at the December time point when she was 3 years, 10 months old. This shift contributed to Ava's ability to engage in and access more tasks during the December assessment. Figure 1 provides the transcript from the first two TEAM items where the interviewer placed four play grocery boxes in a horizontal row. For item 1, the interviewer asked Ava to count the boxes, assessing her counting and one-to-one correspondence. For item 2, after Ava counted the boxes, the interviewer checked for cardinality by asking, *So, how many altogether?*

| **July**<br>**Items 1 and 2 (4 boxes)** | **December**<br>**Items 1 and 2 (4 boxes)** |
|---|---|
| **I:** Okay [Ava], will you help me shop? I'm going to show you some food boxes and I want you to tell me how many I have, okay? (places four boxes) How many boxes are there altogether?<br>**Ava:** One – two – three (touching each of the first two boxes 1-to-1, she then glides over the last two as she says three).<br><br>**I:** So, how many boxes are there?<br>**Ava:** One – two – three – four – five (not touching the boxes at all, only feet visible in the video, propped against the table near the boxes). | **I:** Please help me shop. I'm going to show you some food boxes. Tell me, how many food boxes are there?<br>**Ava:** One – two – three – four (touching each with 1-to-1 for all four blocks).<br><br>**I:** So, how many?<br>**Ava:** One – two – three – four – five – six – seven (not touching the blocks, hands not in the video). |
| Item 1 counting 1-to-1: 0 (incorrect)<br>Item 2 cardinality: 0 (incorrect)<br>Developmental progression: Verbal Reciter | Item 1 counting 1-to-1: 1 (correct)<br>Item 2 cardinality: 0 (incorrect)<br>Developmental progression: Corresponder |

*Figure 1.* Transcript of Ava's progression from Verbal Reciter to Corresponder

*Coordinating one-to-one correspondence and cardinality: Jane and Hannah.* Jane and Hannah had two different paths to the Producer of Small Numbers level of the counting learning progression. While Jane started further along the progression due to her early knowledge of cardinality (January), it took time for her to learn to coordinate that understanding with one-to-one correspondence (July). An analysis of Jane's patterns of performance on individual items suggested that her improved oral counting to 29 by September (including the ability to even count backwards from 10 in December) and improved coordination of one-to-one correspondence with her early acquisition of cardinality led to a shift in her learning. This was evident in her ability to access and correctly respond to Producer of Small Numbers items by December.

Hannah's major shift in learning occurred between the July and December TEAM assessments as she moved from an emerging Counter of Small Numbers to a consistent Counter of Small Numbers and Producer of Small Numbers. The analysis showed that developing the skills of one-to-one correspondence and cardinality led to a critical shift in her ability to access more difficult tasks and with more consistent and coordinated accuracy. Figure 2 provides the transcript from Hannah's January, July, and December TEAM-A administrations that highlight this movement among the levels in the counting learning progressions in the context of the "counting boxes" items. In January, Hannah showed emerging evidence of one-to-one correspondence by touching objects when asked to count, but did not attach number names to the individual objects. In July, Hannah showed an understanding of cardinality but difficultly maintaining one-to-one correspondence with more than three objects. The December portion of the transcript showed Hannah's coordination of one-to-one correspondence and cardinality led to correct responses for Items 8 (counting with one-to-one) and 9 (cardinality).

| **January**<br>**Items 1 and 2 (4 boxes)** | **July**<br>**Items 8 and 9 (6 boxes)** | **December**<br>**Items 8 and 9 (6 boxes)** |
|---|---|---|
| **I:** Will you help me shop? I'm going to show you some food boxes. Please tell me how many I have.<br>**I:** (after a long pause) You can count them. Want to count out loud for me?<br>**Hannah:** (touches each box with her finger)<br>**I:** There you go. You touched each one of them, didn't you?<br>…<br>**I:** So, [Hannah], how many are there altogether?<br>**Hannah:** Three. | **I:** Okay, tell me how many food boxes I have?<br>**Hannah:** One – two – three – four – five – six – seven – eight (placing them in a line again, and touching 1-to-1 for the first three, touching each of the remaining three, but counting double numbers four-five and six-seven for the next two blocks, finishing with a 1-to-1 for eight).<br>**I:** Wow! So how many altogether?<br>**Hannah:** Eight. | **I:** Can you count those food boxes and tell me how many I have?<br>**Hannah:** One – two – three – four – five – six (touching each with 1-to-1).<br><br>**I:** So how many is that Hannah?<br>**Hannah:** Six. |
| Item 1: 0<br>Item 2: 0<br>Pre-Counter because no verbal counting words (though touching each box shows evidence of emerging understanding of one-to-one correspondence and what it means to count objects) | Item 1: 0<br>Item 2: 0<br>Reciter to 10<br>(shows evidence of cardinality, but without accurate one-to-one correspondence) | Item 1: 1<br>Item 2: 1<br>Counter of Small Numbers |

*Figure 2.* Transcript of Hannah's progression from Pre-Counter of Small Numbers

## *Comparing and Ordering Quantities Developmental Learning Progressions*

There were six total items on the TEAM-A for assessing students' comparing and ordering quantities knowledge. Each of these six items assesses one aspect of the learning progression. The IGDIs-EN QC assessed students' knowledge of quantity comparison (i.e., which square has more dots) on as many items as they could complete in one minute. Overall, there were fewer opportunities to assess students' comparing and ordering quantities knowledge than there were for counting. Table 5 summarizes students' performance in this domain on the TEAM-A combined with their performance on the myIGDIs-EN Quantity Comparison (QC) subtest. This provided insight to their early number understandings beyond counting.

Table 5
*Summary of Comparing and Ordering Quantities Progress Across Time Points*

| | Perceptual Comparer | First-Second Ordinal Counter | Early Comparer of Similar Items | Counting Comparer to 10 | Mental Number Line (Small Numbers) | Serial Orderer to 5 | Mental Number Line to 10 |
|---|---|---|---|---|---|---|---|
| Item | myIGDIs-EN QC test (60-sec dot comparisons) | TEAM: Who is first in line? | TEAM: Are these the same amount? | TEAM: Which is more: 11 apples vs 9 boxes? | TEAM: Which is biggest: 5, 6, or 4? | TEAM: Put numerals in order from least to greatest. | TEAM: Which is biggest: 7, 9, or 5 |
| Ava | Mar: 4<br>May: 0<br>Sept: 6 | December | | | | | |
| Hannah | Mar: n/a<br>May: 12<br>Sept. 14 | July<br>December | January,<br>July,<br>December | July<br>December | December | December | December |
| Jane | Mar: 9<br>May:14<br>Sept: 15 | January,<br>July,<br>December | January,<br>July,<br>December | July<br>December | December | | |
| Emily | Mar: 0<br>May: 2<br>Sept: 0 | January<br>December | | | | | |
| Sam | Mar: 12<br>May: 19<br>Sept. 13 | January,<br>July,<br>December | | | | | |

[1]Note: The category definitions below are based on Sarama and Clements (2009) developmental learning progressions. Perceptual Comparer: compares collections (either very small collections or quantities quite different in size) and knows one has more than the other; First-Second Ordinal Counter: names the first and sometimes second objects in a sequence; Early Comparer of Similar Items: compares small collections (1-4) or similar items, sometimes using number words; Counting Comparer to 10: compares using counting (up to about 10); Mental Number Line (small numbers): uses number knowledge of small numbers to determine number relationships using numerals (TEAM item: Which is biggest: 5, 6, or 4?); Serial Orderer to 5: orders numerals 1-5; Mental Number Line to 10: uses number knowledge to determine number relationships using numerals (TEAM item: Which is biggest: 5, 6, or 4?).

Overall, only Hannah and Jane moved beyond the TEAM-A First-Second Ordinal Counter item. By the December administration, Hannah was able to accurately respond to the more challenging items that required her to use her number knowledge to determine number relationships using numerals (i.e., without pictorial supports), including the item that involved a comparison of larger numbers 7, 9, and 5. It was notable that Hannah's progress with these individual items corresponded with her counting progress. Specifically, on the July TEAM-A, Hannah's performance on the counting items aligned with the Corresponder level and she was able to accurately solve the quantity comparison item for comparing 11 apples and 9 boxes (i.e., Counting Comparer to 10). On the December TEAM-A, Hannah's performance was categorized as Producer of Small Amounts, denoting her solidified cardinality and understanding of numerals/number words on the counting learning progression, which corresponds with her performance on the comparison items that involve numerals only (i.e., Mental Number Line and Serial Orderer). Similar to Hannah, Jane's improved performance on quantity comparison items across time points corresponded with her improved coordination of one-to-one correspondence and cardinality.

### *Numeral Recognition and Representation*

Table 6 summarizes students' performance on numeral recognition and representation items. The myIGDIs-EN NN subtest that assessed numeral recognition skills presents students with numerals, 1-20, and asks them to name the numeral with a number word. The TEAM-A item that assessed numeral representation asked students to match pictures of grapes with their corresponding numerals for quantities of 1 to 5.

Table 6
*Summary Performance on Numeral Recognition and Representation Items*

| Student | TEAM-A Match Numerals | IGDIs-EN NN subtest |
|---|---|---|
| Ava | Jan: --[1]<br>Jul: --<br>Dec: 0 | Mar: 0<br>May: 0<br>Sept: 1 |
| Hannah | Jan: --<br>Jul: 3<br>Dec: 5 | Mar: --<br>May: 9<br>Sept. 7 |
| Jane | Jan: 0<br>Jul: 4<br>Dec: 5 | Mar: 3<br>May: 4<br>Sept: 7 |
| Emily | Jan: --<br>Jul: --<br>Dec: -- | Mar: 0<br>May: 0<br>Sept: 0 |
| Sam | Jan: 5<br>Jul: 3<br>Dec: 3 | Mar: 7<br>May: 7<br>Sept. 8 |

[1]Note: The symbol -- denotes that students were not assessed on that item due to five incorrect items prior to the numeral representation item. Hannah did not take the March IGDIs-EN, hence -- represents that those data were missing.

Overall, the data show that Hannah, Jane, and Sam had some numeral recognition and representation by age 3 years 6 months, 3 years 4 months, and 3 years 5 months respectively.

### *Characteristics of Mathematics Knowledge: Observation and Teacher Perspectives*

The results of the nine classroom observations and three teacher interviews are presented in terms of overlapping themes within these data sources and across cases. Key themes in the observations and teacher interviews included Organization, Counting, One-to-One

Correspondence, Story Problems, Use of Magnitude, and Play. Within each case and across cases, some themes were more prominent. The results are organized in terms of overlapping themes: One-to-One Correspondence and Problem Solving (Sam and Jane), One-to-One Correspondence and Organizing into Sets (Ava and Hannah), and Developing One-to-One Correspondence in Play (Emily).

*One-to-one correspondence and problem solving: Sam and Jane.* Both in the observations and interviews with the teacher, Sam and Jane consistently attempted the teacher's story problems or counting collections in small group settings. Within this small-group problem-solving setting, the teacher stated that she noticed their verbal counting and one-to-one correspondence and intentionally used these instructional moments as opportunities to improve their one-to-one correspondence when counting objects involved in the story problem or task. During the observations, for instance, it was observed that Sam accurately counted objects with one-to-one correspondence up to 4 during Observations 2 and 3, up to 5 during Observation 5, and up to 6 and with cardinality in Observation 9. In Jane's observations, she was seen counting into the teen numbers (such as to 14 in Observation 3), but with inconsistent one-to-one correspondence as she counted objects. However, in Observations 6 and 8, she was observed successfully solving story problems when groups of objects were within 5.

*One-to-one correspondence and organizing into sets: Ava and Hannah.* Ava and Hannah enjoyed using the objects during the story problems in small groups, and in both the observations and teacher interviews, they tended to focus more on organizing the materials rather than solving the problems unless prompted by the teacher. For example, in three different observations, Ava gathered as many of the materials as possible, rather than the number of materials in the story problem and worked on lining up the objects. While that seemed to occupy Ava's attention, there was one observation that noted her attempt to count 15 objects. She counted accurately with one-to-one correspondence up to 7. In the teacher's September interview, she provided multiple examples of Ava's tendency to organize quantities in play, such as in the kitchen area organizing the plates into groups of 3.

Hannah attempted the problems more often than Ava in these small group settings, especially if the numbers were within 5, but also showed an inclination to focus more on organizing items (such as organizing items in 2s by colour) than on solving the problem. One observation noted that Hannah counted with one-to-one correspondence up to 10 when asked to produce a set of seven objects. Hence, while incorrect in the requested activity (make a set of 7 for the story problem), she exhibited her ability to count with one-to-one correspondence to 10. In the September interview with the teacher, she noted Hannah's desire to make sure everyone had a fair share during snack. Both in these examples from the teacher and in the observations, Hannah uses magnitude words to describe quantities, such as "lots" and "too many" rather than counting or naming quantities with their number name.

*Developing one-to-one correspondence in play: Emily.* Emily showed more instances of counting with some one-to-one correspondence when the teacher strayed from the given story problem and more playfully engaged Emily in counting, with questions such as, "How many cookies do you have there? Will you give me two?" or "Look, Emily! I have 5 fingers, and I put down 2. How many am I showing you?" The teacher's interview also noted that Emily was more apt to count during free play when a teacher engaged with her and when the counting was part of the play activity. Unlike the other students, the teacher noted that Emily struggled to engage in the story problems and counting collections, so she decided to work with Emily on learning numerals for the quantities using a number puzzle and a numeral board as part of her play with a teacher.

## Discussion

This research provides an in-depth examination of 3-year-old preschool children's evolving number knowledge over one year and key shifts in their early number learning. We chose to focus on an understudied population, preschool children who began the study at age 3, in order to better understand the number knowledge development of students who were in the early stages of learning to count. The results of this study identified and described variations in preschoolers' number knowledge test scores and the characteristics of their developing number knowledge.

### *Variations in Number Knowledge Test Scores*

The patterns of assessment scores over a year varied across the cases and across the two tests. Hannah and Jane's TEAM-A scores followed a steady growth pattern, while Ava, Emily, and Sam's TEAM-A scores needed further analysis and other supporting data to understand their evolving mathematical knowledge. There was less variation on the preschoolers' myIGDIs overall test scores (Ava, Hannah, Jane, and Sam all had increasing scores over the year), however, a closer examination of the subtests showed that this was due to improved oral counting across the cases, which was to be expected because verbal and object counting experiences were both emphasized in the classroom context and are the appropriate topics for instruction at this age (Sarama & Clements, 2009). The results showed more variation, even divergent results, on the one-to-one, quantity comparison, and numeral knowledge myIGDIs subtests, likely because these are new and developing knowledge for this age group.

Further, there were variations in scores between the two tests for two cases. Specifically, Ava and Sam did noticeably better on the myIGDIs test, mainly due to their improved oral counting, while Hannah, Jane, and Emily had similar patterns on both tests. This was interesting in that the myIGDIs was a progress monitoring tool for symbolic number knowledge (i.e., number words, numerals) and its application to counting and comparing quantities of static pictorial dots (i.e., one-to-one correspondence and comparing quantities) while the TEAM-A items were mostly set within contexts for counting and quantity comparison of quantities of objects or pictorial representations of food items (e.g., counting 6 boxes in a cart at the grocery store; comparing who, bear or cat, has more grapes). We interpreted this to mean that Hannah and Jane were further along with their counting knowledge at the beginning of the study and were better able to attempt problems in which they were asked to apply their early verbal and object counting knowledge in context-rich situations. In Emily's case, neither test captured her early number knowledge well because she was still at the emerging stage of verbal counting and had difficulty accessing the tasks that involved object counting, quantity comparisons, and numerals.

The results showed that oral counting was an area of growth for all five cases. As students improved their verbal counting skills and concepts, they were better able to access object counting, quantity comparison, and numeral tasks on these assessments. This is consistent with learning progressions that indicate children's predictable path of learning the count list, one-to-one correspondence, and cardinality and that each level in the learning progression supports the development of subsequent, yet related, concepts and skills (Sarama &

Clements, 2009). In contrast to oral counting, one-to-one correspondence items and subtests across both tests resulted in the most inconsistent scores for all five cases, which we interpreted to mean is a critical emerging and developing skill at this age. Overall, the test scores provided a broad overview of the variation in children's number knowledge development, but further analysis was needed to understand the individual cases' evolving number knowledge because of the "complex, multicomponential nature of counting skill" (Batchelor et al., 2015, p. 132).

### *Characteristics of the Cases' Early Number Knowledge Development*

In terms of the characteristics of 3-year-old preschool children's evolving number knowledge, the developmental progressions provided a useful framework for understanding typical mathematical development, though children's learning paths and timing across these paths differed. Using the predictors (Geary et al., 2018; Koponen et al., 2019) and developmental learning progressions (Sarama & Clements, 2009) as frameworks to analyze assessment data, and supported by naturalistic classroom observations and teacher interviews, we found key shifts in the preschoolers' mathematics knowledge. The shifts in learning observed across cases in this study included: knowing the sequence of number words (verbal counting); one-to-one correspondence; cardinality; and connecting and coordinating the concepts and skills of counting, one-to-one, and cardinality.

*Verbal counting.* The characteristics of Emily and Ava's number knowledge differed from the other three cases in that they were earlier in their verbal counting learning progression paths. Emily accessed the verbal counting tasks over the year of the study and showed an important shift from *pre-counter/chanter* to *verbal counter*. Emily's counting shift took the forms of sequencing more counting words in order and showing more intention of counting objects, both occurring later in the study. Ava made a shift (at about 3 years 5 months) as she moved from *reciter* to *counting sequentially.* Ava's shift was characterized by using her *reciter* knowledge to more deliberately count objects with one-to-one correspondence and become an *emerging corresponder*.

While verbal counting was not considered a key learning shift for Jane and Sam in our analysis, both of these cases showed a clear development of counting fluency, but in different ways. Jane began the study able to count as high as 15 (in March), then exhibited development in her verbal counting fluency by counting through the teens to 20 accurately and even counting backwards by the end of the study. Sam showed fluency in counting to 30 by the end of the study. Jane and Sam both exhibited an understanding of verbal counting early on in the study, though their one-to-one correspondence and coordination of number skills took time to develop as consistent skills over the year. In Jane's case, the classroom observations revealed that her counting fluency into the teens encouraged her to count larger and larger sets of objects during the small group problem solving setting, which provided her a space to continuously practice applying her verbal counting skills to meaningfully correspond counts with objects and consider cardinality of a set. Similarly, Sam was observed as consistently engaging in these teacher-provided tasks and putting her fluent counting to work in object-counting-focused tasks where she could build her correspondence and cardinality.

*Correspondence and cardinality*. Sam, Jane, and Hannah showed an understanding of cardinality by age 3 years 6 months. This is important as Geary et al.'s (2018) study showed that children with cardinal knowledge at around 3 years 10 months up to 4 years 2 months had better number system knowledge outcomes by the beginning of first grade. Overall, Jane and Hannah's test scores and learning progression levels followed the pattern of number knowledge development we would expect to see based on the learning progressions (Gelman & Gallistel, 1978; Le Corre & Carey, 2007; Sarama & Clements, 2009) and the research that indicates the importance of early acquisition of cardinality understanding (Geary et al., 2019). While Jane showed early acquisition of cardinality and high oral counting (to 15) early in the study, she needed multiple experiences to improve her one-to-one correspondence. Once Jane's one-to-one correspondence became more consistent, she then could more regularly solve tasks for producing amounts and comparing quantities. This differed for Hannah, in that she showed one-to-one correspondence skills early in the study, and acquisition of cardinality characterized her key shift in early number knowledge learning. This analysis supports Batchelor et al.'s (2015) findings that cardinality understanding is distinct from counting skills and cardinal knowledge may not be initially stable across tasks. This was especially true in the case of Sam.

Previous studies show that verbal counting and cardinality are predictors for later mathematics success (e.g., Geary et al., 2018; Koponen et al., 2019), and even that the age in which students acquire cardinality has long-term implications for success (Geary et al., 2019). However, there is not much empirical evidence on the predictive value of one-to-one correspondence for later mathematics achievement. Our results indicate that the acquisition of consistent one-to-one correspondence was followed by shifts in the ways Jane, Hannah, and Ava more intentionally accessed tasks or completed object counting and comparison items. While one-to-one correspondence may not be considered a predictor of later mathematics achievement, in our study, it emerged as a construct that led to shifts in performance (particularly, for Jane) and access to more challenging tasks (for Hannah and Jane on the assessments and for Sam in the classroom observations). By the end of the study, Ava showed emerging corresponding skills, which aligned with her more deliberate attempts in counting objects and comparing quantities.

*Connecting and coordinating early number knowledge skills and concepts*. In Hannah and Jane's cases, once one-to-one correspondence and cardinality were more consistently acquired, this coordination among the constructs led to shifts in learning progression levels. In both cases, Jane and Hannah's analyses showed that consistent and coordinated one-to-one correspondence and cardinality led to access of more difficult counting tasks as well as corresponded with their improved accuracy of quantity comparison and numeral knowledge tasks. In the cases of Hannah and Jane, both showed evidence of cardinality by age 3 years 6 months, but the improved coordination of cardinality with one-to-one correspondence and verbal counting by the end of the study led to their skills in completing *producer* level tasks.

It was more difficult to see a clear shift in Sam's TEAM-A data, as the analysis of her data indicated that she moved between the *corresponder* and *counter of small numbers* learning progression levels on the TEAM-A assessment over the year. The myIGDIs-EN analysis showed her quantity comparison and numeral knowledge, which did not show up as clearly on the TEAM-A. Finally, the classroom observations showed growth in her counting small amounts over time as she was observed successfully counting amounts of four early in the study and counting amounts of six later in the study. While Sam's shifts in learning on the TEAM-A were not as evident, the other data sources showed that she was developing her coordination of early number knowledge skills (i.e., oral counting, numerals, cardinality, comparing quantities) in order to attempt and solve various tasks.

## *Implications*

Overall, the children's coordination of number knowledge skills provides further evidence of the interconnected nature of early number knowledge skills (Batchelor et al., 2015; Sielger & Braithwaite, 2017) and the importance of multiple and regular early number sense experiences over time. These results have implications for the early childhood classroom, indicating that educators of 3-year-old children should provide multiple counting experiences that emphasize the interconnectedness of verbal counting, one-to-one correspondence, and cardinality. Further, while children may show acquisition of one-to-one correspondence one day, they may show fragile knowledge of the same skill on a different day. This back and forth of skill acquisition is to be expected, especially for 3-year-old children, as it takes time and multiple experiences to develop early number knowledge skills. Hence, multiple opportunities to count the same set of objects with opportunities to re-organize and re-count the same set of objects and count different sets of objects and pictures is critical for this age group (Carpenter et al., 2017). Early childhood educators can use the developmental progressions to help them predict and watch for shifts in learning, but also support the coordination of these skill levels.

Another implication, based on the multiple data sources in this study and specifically in Sam's case, is that quantity comparison is naturally present in play and guided object counting tasks, but could be more emphasized as a necessary experience in 3-year-old early care classrooms. Quantity comparison was under-emphasized in the classroom observations in this study, and it seemed to be an anchor for Sam's number knowledge learning. We wonder if Sam's progress on the TEAM-A would have shown more consistent growth if she had more support coordinating her intuitive magnitude knowledge (as appeared in the myIGDIs-QC) and formal numeral naming (myIGDIs-NN) with object counting knowledge. Batchelor et al.'s (2015) study showed the importance of developing the relationship between counting concepts and magnitude comparison, which seemed to be a missed opportunity for Sam.

A final implication, stemming from Ava and Sam's differing performance on the TEAM and myIGDIs-EN, is that early childhood educators should attend to connections among different representations of numbers, for example connections between verbal number words, numerals, and iconic images of a quantity (Rodríguez et al., 2018).

## *Limitations*

While case study research contributes to the existing research base by providing a detailed and context-specific investigation into young children's early number knowledge development, there are important limitations to acknowledge in case study research. First, the study took place in one early care center on a university campus with only five participants, all of whom are White females. Therefore, it is unlikely that the resulting characteristics of 3-year-old children's evolving number knowledge documented in this study can be generalized to different types of preschool environments and populations of students. Second, the classroom observation data only captured teacher-directed lessons and therefore, we did not capture the mathematics and teaching interactions that occurred during free play, transitions, and snack times, which are very important times in the children's day for mathematical exploration. Third, we relied on assessments to understand young children's developing mathematics knowledge. While we carefully selected developmentally appropriate and valid and reliable tools for this age group, these assessments focus on evaluating correct and incorrect responses, rather than students' reasoning (Lee, 2014; Lee & Md-Yunus, 2016) or spontaneous attention to quantities and number (Franzén, 2015; Hannula-Sormunen et al., 2015; Rathé et al., 2019). Finally, our study focused on early number knowledge, which we acknowledge limits our understanding of our cases' broader mathematics learning because we did not analyze other dimensions of their early mathematical knowledge such as geometric and spatial thinking (Milburn et al., 2019).

### *Conclusion*

Careful consideration to 3-year-old children's early number skills will help early childhood educators better understand the complex and interconnected nature of preschoolers' evolving number knowledge development. Future research efforts could focus on pedagogy to support 3-year-old students' mathematics learning as well as early educators' skills in supporting students' number development in meaningful and developmentally appropriate ways.

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