An Investigation of Pre-service Middle School Mathematics Teachers' Discussion Skills in the Context of Lesson Study

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The purpose of the current study is to investigate how pre-service teachers have used five practices for conducting productive mathematical discussions (anticipating, monitoring, selecting, sequencing, and connecting) in teaching statistics throughout their participation in lesson study. Researchers employed a qualitative research method. The participants of the study were three pre-service teachers as they carried out three lesson study. In the data analysis, the framework developed by Stein et al. (2008) was used. The results revealed that the pre-service teachers did not include all practices in their lesson plans and implementations at the beginning. As the lesson study progressed, the pre-service teachers attempted to include anticipating, monitoring, selecting, and connecting practices. However, the difficulties in terms of the sequencing practice were seen to be continuing. In light of the obtained findings, it was suggested that lesson study should be integrated into teacher training programs in Turkey.

Introduction

Mathematical discussions have been emphasized, especially for the last two decades in mathematics education (National Council of Mathematics [NCTM], 2000; 2013). Mathematical discussion environments allow students to share their ideas, think deeply about the arguments they defend, evaluate these arguments from different perspectives, create mathematical meanings, and evaluate their arguments in the context of how and why questions (NCTM, 2000; 2014). Students' thinking about mathematical problems, formulating their own arguments, defending ideas while discussing these arguments in the discussion environment, and teachers' providing guidance can contribute to the establishment of a targeted learning process and develop conceptual understandings (Kazemi & Stipek, 2001; Manouchehri, 2007; Nathan & Knuth, 2003; NCTM, 1991; Stigler & Hiebert, 1999).

The importance given to the discussion process for students is stated in the particular objectives of the Turkish curriculum as follows: "They can express their thoughts and reasoning and also see the deficiencies and gaps in the mathematical reasoning of others, can use mathematical terminology and language properly to explain and share their mathematical thinking logically, can make sense of the relationships between people and objects and the relationships of objects with each other by drawing on the meaning and language of mathematics" (Ministry of National Education [MONE], 2018, p. 9). Thereupon, it becomes clear that learning environments should be structured in such a way as to put mathematical discussions into the center (Kersaint, 2015; NCTM, 2000; 2014). In addition, determining the roles of both students and teachers in the discussion process to create a rich discussion environment is important (Kersaint, 2015). Thus, to encourage students to participate in the discussion, teachers need to plan and determine the tasks they will use before teaching. For an efficient mathematical discussion, the teacher should focus on questioning and encourage students to ask questions (Piccolo et al., 2008). Inquiry-based questions were stated to be

one of the most important criteria determining the quality of the discussion (Van de Walle et al., 2013). Such questions, which reveal higher order thinking skills, allow students to answer differently. Answering questions differently enables different perspectives to be formed and enriches the discussion environment (Henning et al., 2012). In this way, students evaluate the defended argument in many respects and reach judgments by considering different solutions (Danielowski, 2016).

The critical role that the teacher assumes from the preparation process of the discussion to the end of it also largely determines the quality of the mathematical discussion because the teacher chooses the task that would be suitable for starting a discussion. This chosen task should stimulate higher order thinking skills, allow for solutions to employ multiple strategies, and support understanding mathematics conceptually (Smith & Stein, 1998). As students share their ideas and assumptions, the teacher's role becomes a facilitator that clarifies students' ideas and questions to enrich mathematical understanding, rather than simply presenting mathematical approaches and demonstrating the necessary procedures to solve the given tasks (NCTM, 1991). The open-ended questions planned in the discussion process are purposed to reveal students' thoughts and enrich the discussion (Kazemi & Stipek, 2001; Kersaint, 2015). These unearthed thoughts also provide the teacher with insights about what students are learning and enable them to give feedback that can encourage students to think deeper (Walshaw & Anthony, 2008).

In the existing research, the main emphasis has been on what the teacher should know to create such environments. Therefore, these environments can be defined in more detail (Stein et al., 2008). Studies have stated that teachers' successful mathematical discussion environment goes through five practices. These practices are defined as anticipating, monitoring, selecting, sequencing, and connecting.

- Anticipating involves the teacher's ability to predict different ways of solving a mathematical task. In addition to how students make sense of the relevant task, anticipating also focuses on predicting students' knowledge and skills regarding how to solve the task, what strategies to use, how to interpret it, and what ideas to produce.
- Monitoring involves paying attention to students' mathematical thinking and examining their solution strategies more closely as they work. One way of doing this is to observe students individually or in small groups. Monitoring is more than watching and hearing; the teacher needs to ask questions to reveal the students' thoughts and classify their thoughts.
- Selecting involves the teacher's selecting students' important thoughts. These thoughts are a prerequisite for the targeted objectives to be achieved.
- Sequencing involves the teacher's sequencing the selected students' answers and ideas.
- Connecting involves the teacher's establishing links between students' solutions and meaningful relationships between mathematical ideas (Smith & Stein, 2011; Stein et al., 2008).

NCTM (2013) made a series of suggestions for teachers to facilitate productive discussions. In these suggestions, similar points were emphasized. The existing research highlights that these processes effectively support efficient mathematical discussions (Smith & Stein, 2011).

Teaching Statistics Through Discussion

One of the learning areas where mathematical discussion environments are important is statistics (Ben-Zvi, 2011; Cobb & McClain, 2004). In statistical learning environments, students are expected to experience the entire statistical process from primary education to the end of secondary education, namely, posing problems, collecting, organizing, analyzing, and interpreting data (Bargagliotti et al., 2020; Franklin et al., 2005; MONE, 2018; NCTM, 2000). The process of posing questions, which is the first component of doing statistics, includes defining the problem clearly and creating the question(s) that can be answered with the help of data (Franklin et al., 2005). By knowing what features a statistical problem has, this framework can shape questions (Scheaffer, 2006). The second component, the data collection process, includes planning and implementing this plan to collect the appropriate data. Deciding on suitable data collection methods and collecting data according to these methods is one of the essential skills in this process (Franklin et al., 2005). In the data analysis, suitable methods are selected, and the data are analyzed (Franklin et al., 2005; Bargagliotti et al., 2020). Classification, one of the salient elements of data analysis, involves using tools to represent the data to be analyzed, such as graphs and central tendency and dispersion measures (Bargagliotti et al., 2020). In the stage of interpreting the results, which is the last stage of the process of doing statistics, starting from the analysis made in the third stage, and then they are associated with the initial research question (Franklin et al., 2005; Bargagliotti et al., 2020).

Although the importance of having students to experience the process of doing statistics in learning environments has been emphasized, the focus is on transferring information from teacher to students in traditional statistics classes. Thus, students typically only asked to answer the questions. This causes students to see statistics as challenging to learn and composed of rules and procedures (Ben-Zvi & Garfield, 2004). Effective discussion environments help students create and defend statistical arguments and understand the reasons behind important statistical concepts (Ben-Zvi, 2011; Cobb & McClain, 2004). At this point, teachers' creating and maintaining discussion environments is important for the quality of the teaching process. Despite the importance of discussion in statistics learning environments, creating and maintaining discussion environments where students experience significant statistical problems and where arguments are presented and meanings are openly negotiated is not easy (Ben-Zvi, 2011).

The existing research has revealed that it is challenging for both teachers and pre-service teachers to organize and maintain a discussion environment is challenging for both teachers and pre-service teachers (Bennett, 2010; Franke et al., 2009; Smith & Stein, 2011; Stein et al., 2008). Faculty members responsible of training teachers state that the necessary knowledge and skills related to the effective discussion process can be taught to pre-service teachers (Stein et al., 2008). Although some studies focus on developing pre-service teachers' discussion skills (Ghousseini, 2008), little is known about the process of pre-service teachers putting discussion skills into the center of their lessons (Tyminski et al., 2014). Furthermore, what kinds of professional development programs influence teachers' discussion skills warrant research (Pang, 2016).

Developing Discussion Skills Through Lesson Study

The lesson study model is an effective professional development program in which teachers acquire much knowledge necessary for their professional competence through

teaching practices (Borasi & Fonzi, 2002). The theoretical basis of this model recognizes that cognition is social and argues that learning takes place cooperatively in rich learning environments (Fernandez, 2005). Zhang and Cheng (2011) developed practicum-based micro-teaching that is a type of lesson study model based on the concept of approximations of practice in the conceptual framework proposed by Grossman et al. (2009) for pre-service teachers. As this model allows pre-service teachers to practice in a university and real classroom environment, it provides them with opportunities to achieve meaningful learning goals. The first stage of lesson study model is planning, teaching, feedback, and reflection; the second stage is re-planning, re-teaching, and evaluation; and the third stage is reflection. In the first stage, pre-service teachers plan the lesson and teach in the university classroom. The lesson is then revised considering the feedback received from peers and instructor. In the second stage, the lesson is delivered in the real classroom, which is a more complex environment, and this lesson is evaluated by the co-teacher or supervisor. This model allows pre-service teachers to apply what they have learned in the university classroom to the real classroom environment. In the third stage, reflection is performed after the completion of the first and second stages. Reflection is made on what has been observed and learned in the lesson's planning, implementation, and evaluation.

The lesson study model allows a more detailed characterization of the real classroom environment by providing the opportunity to apply what has been learned in the university classroom environment in the real classroom environment (Zhang & Cheng, 2011). All this can support the development of pre-service teachers' discussion skills (Pang, 2016). In addition, it is pointed out that there is a lot to be learned about the conditions under which mathematical discussions occur to accomplish learning objectives and which situations and tools can support this environment should be investigated in more detail (NCTM, 2013). Furthermore, the emphasis on the role of the discussions in the accomplishment of meaningful learning of statistics (Ben-Zvi, 2011; Cobb & McClain, 2004) indicates the importance of conducting the current study. In this regard, the purpose of the present study is to investigate how pre-service teachers have used the practices (anticipating, monitoring, selecting, sequencing, and connecting) of the discussion throughout their participation in lesson study. To this end, an answer to the following research question was sought. How did the pre-service teachers use the five practices for orchestrating mathematical discussion while teaching statistics throughout the lesson study?

Method

To find an answer to the research question stated above, the case study design was employed. In the current study, the lesson study processes conducted by the pre-service teachers regarding teaching statistics were determined as the case to be researched. The case study allows a more detailed examination of the change in the pre-service teachers' knowledge and experience in its natural process (Stake, 1995).

Participants

Participants were selected based on two criteria. Firstly, it was aimed to select pre-service teachers who successfully completed some specific courses. Among the ones who fulfil this criterion, volunteer pre-service teachers were determined. This allowed an in-depth analysis of the focused cases (Gall, Gall, & Borg, 2007). For pre-service teachers to participate in the present study, they were expected to have successfully completed the following courses: Research Methods, Principles and Methods of Teaching, Statistics and

Probability, Methods of Teaching Mathematics, Teaching Practice. Moreover, the participating pre-service teachers were selected for the present study because the pilot study was also carried out in the same school. Of the participating pre-service teachers, Beyza and Şirin were 21 years old, and Gamze was 22 years old. The grade point average of Beyza was 3.21, the grade point average of Şirin was 3.26, and the grade point average of Gamze was 3.29 out of 4. The names of these participants are pseudonymous.

Context of the Study

The study was conducted in the Department of Mathematics Education at a state university located in Ankara. The pre-service teachers graduating from this department can work as mathematics teachers in middle schools (11-14 years-old). In the department, the medium of instruction is Turkish. In the first two years of the four-year program, the main emphasis is on content knowledge (e.g., General Mathematics, Geometry). In the last two years, the focus is on pedagogical content knowledge (e.g., Methods of Teaching Mathematics, Teaching Practice). The participants of this study were twelve pre-service teachers enrolled in the elective course *Microteaching in Mathematics Education*. However, only the data collected from three pre-service teachers were on the focus of this study. The first reason for selecting these pre-service teachers is that a pilot study was conducted in the school they attended for the course of practicum teaching. Hence, their mentor teacher gained knowledge and experience about the lesson study, and the mentor teacher's students became familiar with the study context. Also, during the observations and discussions made throughout the process, this group took a more active role and expressed themselves better. Hence, they provided richer data compared to other groups.

At the beginning of the process that lasted for 14 weeks, information was given about the content of the course, and discussions were made about the articles on statistics education (Ben-Zvi, 2011; Cobb & McClain, 2004). In the following weeks, information was given about the content of the lesson study, and the pilot study was carried out. Then, the pre-service teachers carried out three lesson study focused on the following objectives in the data handling learning area: construct a pie graph of a data set and interpret the graph; construct a line graph of the data and interpret the graph; show the data related to the research question with a pie graph, a frequency table, a bar graph or a line graph; and make conversions between the representations. In each lesson study, lesson plans were prepared for the university and real classrooms, and then they were implemented and evaluated (Figure 1).

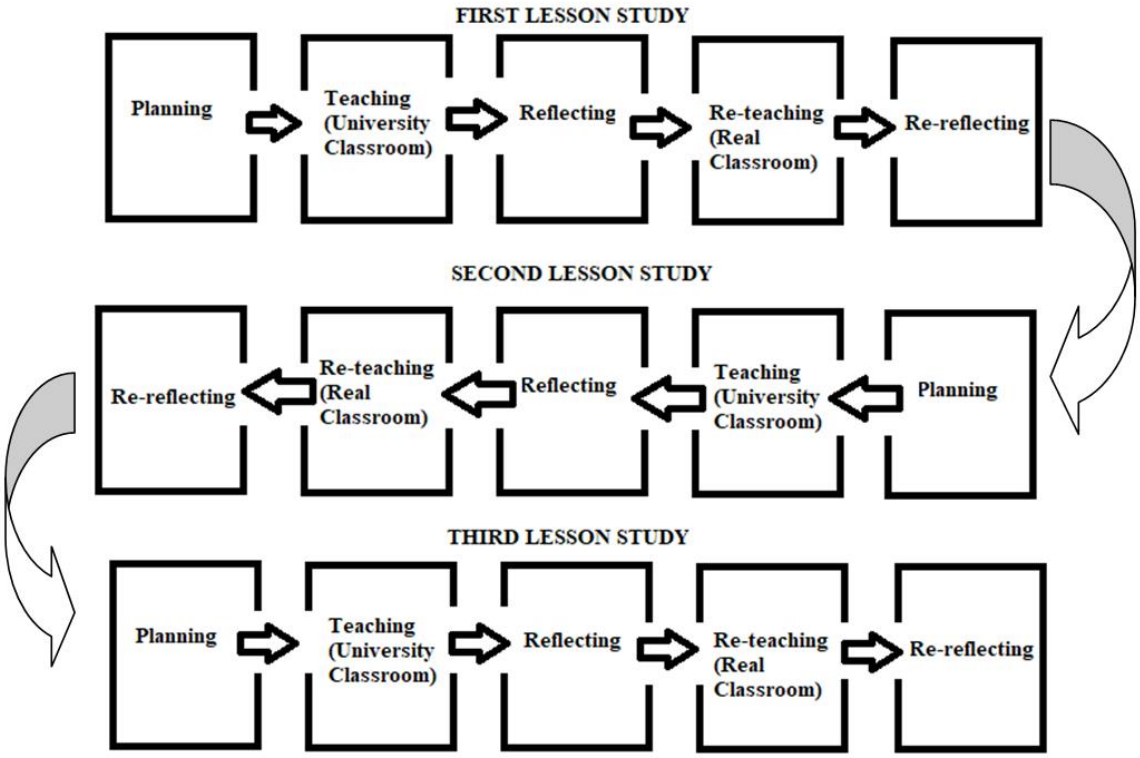

Figure 1. Implementation process (Adapted from Zhang & Cheng, 2011).

The pre-service teachers prepared their lesson plans according to the seventh-grade objectives related to the data handling learning area in compliance with the lesson planning format having four components in the lesson study process (Mathews et al., 2009, p. 506). The prepared lesson plans were first implemented in the university classroom and then in the real classroom. After both implementations, the lesson plans were evaluated, and based on the given feedback, they were revised. The instructor (the second author) and the researcher (the first author) participated in the lesson delivered in the university classroom as observers and evaluated the lesson. In addition, in the microteaching environment, all pre-service teachers in the course evaluated the lesson plans in each cycle and made suggestions about how to improve them. The group members evaluated the lesson plan and made suggestions about how to improve them. On the other hand, in the real classroom, the mentor teacher and the researcher evaluated the lesson.

Data Sources and Data Analyses

In the current study, lesson plans prepared by the pre-service teachers, observation forms, video recordings of lesson planning and revision meetings, video recordings of the lessons delivered in the university classroom and real classroom, field notes taken by researcher, and reflective journals written by the pre-service teachers were used as the data collection tools. In the data analysis, the framework developed by Stein et al. (2008) was used. It was investigated how the pre-service teachers used the practices of anticipating, monitoring, selecting, sequencing, and connecting while designing and implementing their lesson plans (in the university and real classroom). The skills defined for these practices and their definitions are given in Table 1.

Table 1

Analysis Framework

Practices of Discussion	Skills	Definition
Anticipating	Student thoughts	Pre-service teachers' anticipating the difficulties, mistakes, and misconceptions that students may experience, as well as how students will think in their lesson planning.
	Lesson objective	Pre-service teachers' anticipating how to relate their lesson to doing statistics (formulating questions, data collection, data analysis, interpreting results).
Monitoring	Monitoring period	It refers to the period of time given to students to think about the tasks during the lesson implementations.
	Giving right to speak to students and questioning	It refers to allowing students to express their thoughts and questioning to reveal their thoughts.
Selecting	Selecting students' thoughts	It refers to selecting students' thoughts in line with the purpose during the lesson implementations.
Sequencing	Sequencing students' thoughts	It refers to sequencing students' thoughts in line with the purpose during the lesson implementations.
Connecting	Establishing connections between students' thoughts and objectives of the lesson	It refers to establishing connections between students' thoughts and lesson objectives during the lesson implementations.

The obtained data were analyzed within the framework of the skills specified above. Lesson plans prepared by the pre-service teachers, observation forms, video recordings of lesson planning and revision meetings, video recordings of the lessons delivered in the university classroom and real classroom, and reflective journals written by the pre-service teachers were coded by paying attention to the skills determined in the framework for each discussion practice. This analysis process proceeded similarly for each lesson study and was supported with direct quotations where needed. 30% of the data were coded by another math educator working in this field, and the intercoder reliability was 90%. On the points of disagreement, a consensus was achieved through discussion.

Findings

It was revealed that pre-service teachers had various difficulties in anticipating students' thoughts and reflecting their anticipating process on the discussion environment. In the first objective, where they focused on the pie chart, the main aim of the lesson was to construct the pie chart and find its angle measurements. In the relevant task they prepared (Figure 2), it was noted that they did not emphasize doing statistics (research question, data collection, data analysis, interpretation of results).

Number of siblings	Number of people
0	3
1	9
2	6
3 and above	2

Data *on the number of siblings of 20 students who are in the fourth grade of Elementary Mathematics Teaching at A University. Let's create a pie graph based on these data.*

Figure 2. The task used in the first lesson study

The pre-service teachers focused on typical student thoughts about the angle measure and corresponding percentage of the data in the pie chart. They ignored the difficulties that students might experience in terms of what the pie chart elements mean and the relationship of this with doing statistics. The difficulties experienced by the pre-service teachers in anticipating students' thoughts caused them to have challenges in creating and maintaining a discussion environment during the implementations. Although Gamze gave the students time to solve the tasks during the implementation, she was incompetent in questioning how the students thought and taking their thoughts into consideration, that is, she could not monitor. For example, one of the students asked a question about the function of pie and bar charts in doing statistics. Although the pre-service teacher received a student question that would create a discussion environment, she could not maintain this discussion environment. The relevant section from the classroom environment is given below.

The first university implementation

…

Student: Teacher, why do we draw a pie chart; we already know the bar chart.

Gamze: Yes, we know, this is a different kind of representation.

Here, the pre-service teacher was expected to include this student's question in the classroom discussion and to create a discussion environment in which the appropriate graph type to answer the formulated statistical question would be determined. The students would realize that these two graphs have the functions of comparison (bar chart) and part-whole relationship (pie chart) in the doing statistics. However, Gamze did not select this student's question and share it in the classroom environment, which prevented her from sequencing and connecting with the thoughts of other students. In addition to focusing on the procedural aspects of the objective being addressed, their structuring the objective according to certain generalizations caused the classroom implementations not to fulfil their function. Since pre-service teachers have the following generalization *"Temperature is represented only by line graphs"*, they organized lesson plans based on this notion. This caused students to be insufficient in anticipating different ways of thinking. Their inability to anticipate caused the discussion environment to remain limited. The relevant section from the classroom environment is given below (the dialogue is about where the pie chart is used).

The first university implementation

…..

Student 1: If we had the graph of temperature? (Wants to represent the temperatures with a pie chart)

Gamze: When we had the graph of temperature, then you would say a temperature now say a temperature.

Student 1: -2 and 5 degrees.

Gamze: Two different temperatures in two different days? Do you think that concerting these values [into a pie chart] would be meaningful? What do I represent here? We see the numbers of people, don't we. The biggest slice shows the largest number of people, for example. If I drew 25 degrees here [Draws a pie chart], then how would I decide on the slice? Let's assume, I have drawn 25 degrees.

Student 2: Can we decide on it according to the total temperature, teacher? Would it be meaningful?

Gamze: Let's assume that we did it then what would the value 25 degrees indicate to us? There were more slices. What would I have when we had more slices?

….

Student 3: Then, we wouldn't be able to use the pie chart for everything?

Gamze: Yes. Each type of graph has specific places to be used for.

As seen in the section from the classroom environment, although Gamze gave the students the necessary time to complete the task, she was insufficient to make inquiries that would reveal students' thoughts on their heuristics. When the student asked an unexpected question, as given above, pre-service teacher could not make the necessary explanations. Here, what is expected from an expert teacher is that when the part-whole relationship is questioned in the research question, she would state that a context containing the air temperature values of a province can be represented with a pie chart. These and similar difficulties experienced by the pre-service teacher during the implementation process caused her not to be able to monitor, select, sequence, and make connections with the doing statistics.

While revising their lesson plans, the pre-service teachers focused more on anticipating students' thoughts and connecting these thoughts with the doing statistics. They included the roles of bar and pie graphs in doing statistics in their lesson plans. Also, they added explanations about how the elements in the bar graph are related to the elements in the pie graph. In addition, they thought that students might have difficulties reading and interpreting both the bar and pie graphs and shaped their answers accordingly. Although the development of pre-service teachers towards anticipating in their lesson planning was remarkable, they still had various difficulties in implementation. For example, although the pre-service teacher gave the students the necessary time to complete the task, she was insufficient to make inquiries that would reveal students' thoughts about their way of solution. In other words, she could not monitor, select, and sequence students' thoughts as intended. However, the pre-service teacher tried to connect the points that she anticipated during lesson planning with students' thoughts. The relevant section from the implementation environment is presented below.

The first real classroom implementation

…

Gamze: What can you tell about the numbers related to the football team you support by looking at the bar chart (pointing at the chart)?

Student: The number of people supporting the FB football team is 12.

Student: The number of people supporting the GS football team is 6.

…..

Student: If we do not know the number of the supporters of a team but if we know the total number, then we can find the number of supporters of this team.

Gamze: Yes, what else can happen? For example, let me ask you to compare the numbers of the supporters of the teams. What can you tell me?

Student: Yes, for example, the most supported third team is GS.

Gamze: Yes, what can you tell about your friend's answer?

Student: Then, the most supported team is FB.

Gamze: That is what a bar chart does; do you see? I can easily see how many people support this team (pointing to the column of FB). When I look at the lengths of the other bars, I can compare them easily.

…..

Student: Those supporting FB are two times more than the ones supporting GS.

….

Gamze: There is proportion, isn't there? Three times more.

(They start to cut the bars)

Gamze: Has everybody cut? What did you tell me? How many people did you say?

Student: 360.

Gamze: How did you calculate this?

Student: I summed.

Gamze: You summed in fact; what did you do? You put one bar on the other and then on the other, then which total number you obtain?

Student: 360.

….

Gamze: Then, what was the figure you drew?

Student: A circle.

Gamze: And you already know the total angle of a circle.

Student: 360.

Gamze: 360 degrees. That is, there are 360 persons. So how many degrees correspond to these people?

Student: 360.

Gamze: Yes, all of this (pointing to the circle) includes 360 persons. That is, 360 degrees correspond to 360 persons. Then, what is the degree that corresponds to one person?

Student:1 degree.

As seen in the section from the classroom environment, the pre-service teacher tried to connect students' thoughts with the comparison function of the bar in the doing statistics. In addition, she tried to make students understand the subject by connecting important ideas

about the creation of the circle graph and the fact that the sum of the length of bars in the bar graph corresponds to the total angle measure of the pie chart, that is, to the whole. However, it is remarkable that the focus on the fact that the functions of the bar and pie charts in answering different questions or purposes (i.e., comparison among the categories vs. identifying the relative part of the category among the whole) were not mentioned in this environment.

The pre-service teachers focused more on establishing connections with the process of doing statistics, which is the lesson's objective, while preparing their lesson plans for the line graph they addressed within the context of the second objective. In this context, they designed the lesson plans by anticipating the purpose of drawing the line graph. For example, as they anticipated in their lesson plans, Şirin observed that students made mistakes in connecting the lines while constructing the line graph. Thus, Şirin started the discussion with a question that would lead students to think about why the lines were created while constructing the line graph. Although she was able to select student thoughts, she did not sequence different student thoughts. The relevant section from the classroom environment is given below.

> The second university implementation
>
> ……
>
> Şirin: Why do you think we performed these connections [points of change]?
>
> Student: To see? That is, it seems to have decreased in this way, but it also seems to have increased.
>
> Şirin: Yes, we can see the decrease and increase more easily in this way, can't we? From Monday to Tuesday, it decreased in you [Pointing to an activity of a group]
>
> Student: We can see it in the bar, so why do we draw this?
>
> Şirin: Yes, what do you think about your friend's question?...We will talk about this later….

Here, Şirin was expected to make students realize that the main purpose of the change function of the line graph was to answer the research question. Although the question asked by the student, *We can see it in the bar, so why do we draw this?* was a good question for Şirin to make a connection between the formulated question and the function of the graph, Şirin failed to make this connection.

In the revision and evaluation meetings, the pre-service teachers focused on why they could not answer students' questions (e.g., *We can see it in the bar, so why do we draw this?)* and conducted research on this issue. During this research and meetings, they realized that they needed to think about what the power of the line chart means in respect to answering the problem. This resulted in them making some changes to the objective of the lesson. They focused on doing statistics and realized that the discussion process should start with the formulated question. They emphasized the concept of the acquisition more conceptually, set some goals to associate the objectives to the process of doing statistics, and accordingly, started to anticipate students' thoughts. This was a significant turning point for the pre-service teachers. The task they prepared in this context is presented at the Figure 3.

"Uncle Hasan will plant a vegetable in his garden. In order to get the highest yield from this vegetable, the temperature change should be the least for 2 days after the day it is planted. Below is the weather forecast for the next 14 days. Let's find the best days for Uncle Hasan to plant."

1	2	3	4	5	6	7	8	9	10	11	12	13	14
20°C	23°C	25°C	22°C	19°C	23°C	26°C	24°C	25°C	26°C	25°C	23°C	20°C	24°C

Figure 3. The task used in the second lesson study

In this task they prepared, the pre-service teachers stated that for Uncle Hasan's vegetables to be productive, the temperature change should not be too much on the day he plants them and the next two days. Based on this, they asked the students to determine the most suitable days for planting. They monitored the difficulty they anticipated in their lesson plan to be experienced by the students in their implementation. The pre-service teacher who carried out the implementation noticed that some groups drew a bar graph for the research question, which would be more appropriate to be answered by using a line graph as representation while working on the task. Based on this monitoring, Şirin started a discussion emphasizing the function of graphs in doing statistics, so that the students could realize their mistakes. She wanted the students to question whether it would be appropriate to represent the data with a bar chart.

The second real classroom implementation

…

Şirin: Which graphs did we learn?

Student: Bar chart, pie chart, slice, tally.

Şirin: Yes, we have learned these graphs. Ok, what kind of graph is the bar chart? Let's draw a bar chart…is this a suitable graph for the thing I am researching? Then, what was our question? What was our initial problem?

Student: Uncle Hasan is going to plant a vegetable. But the temperature values need to be close to each other for the vegetables.

…

Şirin: … Let's think now. Why did we use the bar chart in our previous question? We used it when we needed to compare data, didn't we? Is this necessary in this problem? (Uncle Hasan task)

Student: No, we need to take increases and decreases into consideration here.

Şirin: Yes, what do you think about your friends' opinions? Is there anyone who has a different idea?

Student: Isn't it necessary to find the lowest temperature change?

Şirin: Then, I won't look at how many times data is repeated. That is, I need to look for a change. Then, is a bar chart suitable?

Student: No.

Student: No, because we cannot see the change in the bar chart.

…

Şirin: Yes, you are right because in our question we are looking for the smallest change. We will choose three days, there won't be much difference of temperature. Then, those who have finished their charts will find these days.

Student: We have found, teacher.

...

Student: 9 10 11 (days), teacher.

Şirin: What was our question? In order to be able to plant in suitable time, the change of temperature needs to be low. Yes, 9, 10,11, thus, we have solved this question….

As seen in the section from the classroom implementation, Şirin created an environment that would enable students to realize the relationship of this process with the formulated question, since she was able to anticipate the difficulty students would have in the process of determining the appropriate graph. In the discussion environment, the students were able to give correct answers. In this connection, although she attempted to select students' responses, she did not attempt to sequence them. However, she aimed to enable students solve the problem by connecting students' thoughts with both the role of graphs in doing statistics and the formulated question (the temperature change should be observed to solve the problem; the bar chart allows comparison, while the line graph allows observing the change).

In the third lesson study process, the pre-service teachers structured the objective they focused on giving more importance to doing statistics and shaped their lesson plans according to students' thinking styles. In this process, their focus was on how graphs play a role in answering the formulated question. What students should pay attention to in the formulated question, how the appropriate graph is determined, and the drawing and interpretation of graphs were included in their lesson plans. During the implementation, they asked the students to work as a group on the task, and they monitored walking around the groups to follow this process. Meanwhile, Beyza observed that some groups focused on keywords and determined and constructed the appropriate graph according to these keywords. The fact that they anticipated the situation while planning the lesson allowed Beyza to establish important connections by purposefully selecting students' thoughts during the discussion. The relevant section from the classroom environment is given below.

The third university implementation

….

Beyza: Ok, can you give me an example? If someone gives me such an example; wants me to do this, then I will use this graph.

Student: For example, when my teacher says that there is a change in temperature or population, then I use the line chart.

Beyza: Humm. Only temperature or population? What is more important, the temperature data or the change?

Student: Change.

Beyza: What is aimed is to emphasize the change of anything.

…….

Student: For example, if we are given a frequency table based on the number of our siblings, we can show this with a bar chart.

Beyza: Ok. You can also show it with a pie chart.

Student: We can show.

Beyza: Which one would you choose according to what?

As seen in the discussion, Beyza aimed to make students realize that the determination of the appropriate graph type is shaped according to the formulated question. In this process, she was able to select the following student thought; *when my teacher says that there is a*

change in temperature or population, then I use the line chart and then she was able to ask the following question: *Only temperature or population? What is more important, the temperature data or the change* as she monitored the difficulties experienced by students while they were working on the task. She aimed to prevent students from determining the appropriate graph based only on keywords (e.g., temperature should always be represented by a line graph) and to connect it with the formulated question. She made students question the fact that the data can be represented with a pie chart or bar chart depending on the research question by asking the following question: *Which would you choose based on what?* However, no attempt of the pre-service teacher was observed to purposefully sequence students' thoughts during the implementation.

In another situation observed in the real classroom environment, the pre-service teacher monitored that the students ignored doing statistics and tried to solve the research question with a pie chart, which would be more appropriate to be solved with a bar chart. The relevant section from the classroom environment is given below.

> The third real classroom implementation
>
> ….
>
> Beyza: ... We talked about the bar chart and the pie chart. Can I clearly separate them from each other? Can I say that a bar chart must be used for this question, or a pie chart must be used for this question? For example, I have a question. We have already done it; we solved the first question with a pie chart and the third question with a bar chart. For example, could not we have solved the question we solved with the bar chart with a pie chart?
>
> Student 2: We could.
>
> Beyza: Why didn't we construct?
>
> Student: We were asked to select president and vice president. As we need to see them in percentage, we calculate their votes. Who got whose vote? Who got the highest number of votes? We need to find this.
>
> Beyza: You are right, the highest number thus we need to compare them. There is no certain distinction between them. …What is important here is to find an answer to the research question.
>
> Student: Then, the research question should be my point of departure.

The pre-service teachers, who anticipated the importance of evaluating doing statistics as a whole in the lesson planning process, had the opportunity to monitor the difficulties experienced during the implementation. This allowed them to select student thoughts and to make the connections that the appropriate type of graph was shaped according to the formulated question and that there were no sharp distinctions between the graphs. However, the pre-service teacher did not attempt to sequence student thoughts.

Discussion, Conclusions, and Recommendations

The current study investigated how the pre-service teachers used the practices of mathematical discussion during their participation in a lesson study. At the beginning of the process, the pre-service teachers operationally addressed the objectives they focused on and anticipated the students' thoughts by considering certain generalizations (e.g., when there is a large amount of data, the pie chart is used). By organizing the tasks, they prepared in this direction, they focused on how the data given in the bar chart could be represented in the pie chart, and also focused on finding percentages and angles. They dwelled on the first types of thoughts that came to the students' minds (e.g., using proportions while finding an angle) and conducted inquiries regarding them. The pre-service teachers' inability to associate the

objective with the doing statistics resulted in their failure to anticipate students' ways of thinking and the potential difficulties they could encounter. The pre-service teachers overlooked both the operational (e.g., which degree a data in the bar chart corresponds to in the pie chart) and conceptual (e.g., meaning of bars in a pie chart and the relationship of graphs with the process of doing statistics) difficulties students could encounter.

During the implementation, although the pre-service teachers allowed the students to work on the tasks in groups to monitor their thoughts, they did not attempt to reveal the students' solutions. Although the pre-service teachers used group work, an essential practice in creating the monitoring environment during the implementations, they did not attempt to elicit students' answers. Though the pre-service teachers asked the students to work on the tasks during the implementation, they could not monitor students' thoughts. While the pre-service teachers were walking around the groups, the students asked some questions such as *Why do we draw the pie chart? We already know the bar chart.* Although such questions were very suitable to start a discussion, the pre-service teachers could not monitor these thoughts. However, through monitoring, teachers can find important opportunities to think about the meaning of students' thoughts and enable all students construct this meaning during the discussion (Brendehur & Frykholm, 2000; Lampert, 2001). This will support the teacher to understand students' thoughts in depth and relate them to the objective of the lesson (Stein et al., 2008). The difficulties the pre-service teachers experienced in anticipating and monitoring resulted in their failure to select and sequence the students' thoughts and connecting with statistical ideas.

The reasons for the difficulties experienced by the pre-service teachers can be evaluated under a few headings. First of these may be the shortages in the pre-service teachers' content and pedagogical content knowledge because the pre-service teachers' lack of knowledge about doing statistics might have caused them to anticipate the course objectives and students' thoughts procedurally and might have caused them to overlook the students' thoughts and the difficulties they could experience about doing statistics. In the literature, similar points have been emphasized and there is a strong correlation between pre-service teachers' knowledge and their anticipation and monitoring students' knowledge (Ball et al., 2008; Garfield & Ben-Zvi, 2008; Heaton & Mickelson, 2002; Hill et al., 2005; Llinares et al., 2016; Young, 2015). Another reason causing pre-service teachers to have the limited ability of anticipating and monitoring is believed to be a lack of experience. Their inadequate experience in teaching might have resulted in their incompetent knowledge and skills about students (Burroughs & Luebeck, 2010; Meiliasari, 2019). In the literature, researchers also reported that while preparing lesson plans, teachers/pre-service teachers do not usually deal with students' thoughts in a detailed and profound manner and that they remain inadequate in unexpected situations related to students (Eskelson, 2013; Meiliasari, 2019; Young, 2015).

The difficulties experienced by the pre-service teachers in anticipating and monitoring resulted in their inability to go through the following stages of the discussion environment. In other words, they could not select and sequence students' thoughts in such a way to bring them to the discussion environment and connect them with statistical ideas. Similar difficulties have also been pointed out in the literature (Eskelson, 2013; Tyminski et al., 2014). Tyminski et al. (2014) stated that pre-service teachers highlight their concerns about the extent to which students can understand rather than selecting and sequencing to enrich the discussion environment and that they cannot be successful in connecting.

With the process progressing, in the revision and lesson planning meetings, the pre-service teachers focused more on the purposes of the objectives and the issue of anticipating students' thoughts. While, at the beginning, these anticipations were more on the operational dimension, they later shifted towards the conceptual dimension. The pre-service teachers were observed to anticipate both the purposes of the objectives and students' ways of thinking by placing doing statistics into the center. For example, while they first focused on the drawing graphs, they later focused on the function of the formulated question and graphs in this process and addressed the difficulties to be experienced by students in their lesson plans and implementations. Considering the students' thoughts within the context of doing statistics caused them to revise the drafts they had prepared in this direction. The development of the pre-service teachers' ability to anticipate positively affected the development of the other stages related to discussion skills. Similar findings have been reported in the literature (Smith & Stein, 2011; Young, 2015). In the literature, the necessity of anticipating is underlined.

The pre-service teachers who were initially inadequate in monitoring attempted to elicit students' thoughts in the later stages of the process. During the second lesson study at the university implementation, the pre-service teacher observed that the students made mistakes while connecting their lines in constructing the line graph. Thus, Şirin started a discussion with a question leading the students to think about why lines were drawn while constructing a line graph. This can be considered as proof of their improvement in monitoring. Similarly, in the third lesson at the university implementation, the question asked by the pre-service teacher, *What is more important, the temperature data or the change?* can be considered an indication of their progress in monitoring students.

The reasons behind the improvement of the pre-service teachers' ability to anticipate and monitoring can be evaluated under a few headings. First of these is the implementations conducted by the pre-service teachers. Their inability to answer students' unexpected questions during the implementations made them realize that they needed to improve their anticipation skills of students' thoughts. In addition, increasing knowledge and skills related to critical student lens in this process is thought to trigger development in anticipating (Fernandez et al., 2003; Young, 2015). Another reason contributing to the improvement of the ability to anticipate is the lesson planning and revision meetings. Meanwhile, in addition to the questioning of other pre-service teachers, expert questioning helped the pre-service teachers to recognize that they need to think about the purposes of objectives and students' thoughts in greater detail. A similar emphasis has been made in the literature (Llinares et al., 2016). The pre-service teachers' recognizing that they need to think in greater detail led them to do more research. As a result of this research, they learned more about how to address students' answers, started to think more comprehensively about students' thoughts, and started to focus on the purposes of objectives more conceptually. The literature emphasizes that evaluating students' understanding of the same or similar concepts/tasks is important for teachers/pre-service teachers (Stein et al., 2008; Wilson et al., 2015). In the final lesson study, their anticipations about how students think increased and shaped their lesson plans accordingly. Moreover, they made more additions to their lesson plans regarding difficulties, mistakes, and misconceptions.

The importance of anticipating the processes about the course/task objectives of both teachers and pre-service teachers was emphasized. Another point noticed in the literature is the effort to develop the mentioned anticipates is stated as an effective factor in the construction and maintenance of classroom discussions (Lampert, 2001; Smith & Stein,

2011; Stein et al., 2008; Stigler & Hiebert, 1999; Tyminski et al., 2014; Young, 2015). Smith et al, (2013) noted that the development of teachers' ability to anticipate is an important factor in improving the quality of instruction.

As the process progressed, the pre-service teachers selected student thoughts more purposefully and connected this purpose with the doing statistics. The reasons for the development of the pre-service teachers in purposefully selecting students' thoughts can be evaluated under several headings. The first of these is their improvement in anticipating and monitoring both the lesson objective and student thoughts. Thinking think deeply about the objectives of lessons and considering student thoughts accordingly support pre-service teachers' selecting student thoughts in a purposeful manner. The literature emphasizes the importance of teachers' handling student responses in a purposeful way (Lampert, 2001; Stigler & Hiebert, 1999). However, similar changes were not observed in their sequencing skills. At this point, the main reason for the difficulties of the pre-service teachers might be the lack of experience. They did not include sequencing in the implementations they carried out probably because they were unaware of the importance of sequencing student thoughts. In this manner, the results of this study are inconsistent with some studies in the literature. For example, in the results obtained by Pang (2016), teachers were more successful in selecting and sequencing as the process progressed, while Eskelson (2013) observed that teachers could not select and sequence.

Although initially the pre-service teachers had difficulty connecting students' thoughts, they later made connections between the purposes of the course/objective and statistical ideas in the later stages of the process. At first, this connection was limited; however, the pre-service teachers were more successful in the last lesson study. For example, in the last lesson study, a clear connection was made with this function. Here, the pre-service teacher asked students to discuss the similarities and differences between the bar and pie graphs, between the pie graph and line chart, and between the bar graph and line chart and in this way, provided important opportunities for students to learn about the functions of different representations and their powers of answering the formulated question. Then, she connected the emerging ideas with doing statistics. This was also an important opportunity for the students having difficulties. For example, in the last lesson study implementation, while she was selecting students' thoughts, she saw that the students associated the purpose of drawing graphs not with the formulated question but with keywords. Later, she started a discussion to question this issue and enabled students who had not made any connection to make this connection.

The literature emphasizes that the teacher's connecting students' answers to each other will make the discussion more effective (Stein et al., 2008). In the present study, one of the most important reasons for the pre-service teachers' success in making a connection with the course objectives is that they clearly stated the lesson's objectives in the planning process and built tasks on these objectives. The development of their knowledge about students' thoughts also supported this. Smith and Stein (2011) stated that the prerequisite for teachers to make successful connections is good foresight. Moreover, the existing research points out that mastering students' thoughts can support connecting the course/objectives with big ideas (Ball, 2001; Brendehur & Frykholm, 2000). In the literature, there are also some studies reporting different results. Research on pre-service teachers and teachers revealed that teachers and pre-service teachers successfully created discussion environments to make connections with the objectives of the lesson (Tyminski et al., 2014). Pang (2016), on the other hand, got teachers to participate in lesson study implementations, and only in the last

week of the five-week process were the teachers able to make successful connections. Eskelson (2013), on the other hand, stated that as the teachers could not anticipate they had great difficulties in making connections. The literature argues that making connections is one of the most challenging practices for teachers (Inoue, 2011; Pang, 2016).

Researchers drew attention to some basic principles in the conceptual learning of statistics (Ben-Zvi, 2011; Cobb & McClain, 2004). One of these principles is to support classroom discussions. Supporting classroom discussions allows students to generate arguments about the ideas involved in statistical research and to make evaluations by focusing on the "why" of these arguments. This makes it possible for students to make sense of important ideas of statistics rather than memorizing them (Ben-Zvi, 2011; Cobb & McClain, 2004). This shows the necessity of teachers' knowing the discussion skills. In this regard, the current study is believed to make some contributions to the literature.

At this point, it is thought that it is necessary to state some limitations of this study. The discussion practices covered within the context of the study were not part of the lesson study activities the pre-service teachers carried out. In other words, no special training on the discussion practices was given to the pre-service teachers. The pre-service teachers were expected to recognize the importance of these practices on their own. In this connection, it can be suggested that training can be given to pre-service teachers on these practices in future studies, and the study should be carried out after this training. For example, the lesson plans they will prepare may include instructions for discussion skills. Finally, the findings show that pre-service teachers' knowledge directly affects their discussion skills. In this regard, pre-service teachers' discussion skills can be examined after the training that will support the development of their knowledge.

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References

Ball, D. L. (2001). Teaching, with respect to mathematics and students. In T. Wood, B. S. Nelson, &J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 11–22). NJ: Erlbaum.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389- 407. https://doi.org/10.1177/0022487108324554

Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education (GAISE) report II*. American Statistical Association and National Council of Teachers of Mathematics.

Bennett, C. A. (2010). "It's hard getting kids to talk about math": Helping new teachers improve mathematical discourse. *Action in Teacher Education, 32*(3), 79-89. https://doi.org/10.1080/01626620.2010.10463561.

Ben-Zvi, D. (2011). Statistical reasoning learning environment. *Revista de Educação Matemática e TecnológicaI beroamericana, 2*, 1-13.

Ben-Zvi, D., & Garfield, J. (2004). *The challenge of developing statistical literacy, reasoning, and thinking.* The Netherlands: Kluwer Academic Publishers.

Borasi, R., & Fonzi, J. (2002). Professional development that supports school mathematics reform. *Foundations series of monographs for professionals in science, mathematics and technology education.* Arlington, VA: National Science Foundation.

Brendehur, J., & Frykholm, J. (2000). Prompting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3*,125–153. https://doi.org/10.1023/A:1009947032694.

Burroughs, E. A., & Luebeck, J. L. (2010). Preservice teachers in mathematics lesson study. *Montana Mathematics Enthusiast, 7*(2), 391–400.

Cobb, P., & McClain, K. (2004). Principles of Instructional design for supporting the development of students' statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.) *The challenge of developing statistical literacy, reasoning and thinking* (pp. 375-396). The Netherland: Kluwer.

Danielowski, J. (2016). *Increasing number sense through mathematical discourse in the primary classroom*. Retrieved from Sophia, the St. Catherine University repository website: https://sophia.stkate.edu/maed/167.

Eskelson, S. L. (2013). *Exploring the relationship between teachers' participation in modified lesson study cycles and their implementation of high-level tasks* [Unpublished doctoral dissertation]. University of Pittsburgh.

Fernandez, C. (2005). Lesson study: A means for elementary teachers to develop the knowledge of mathematics needed for reform-minded teaching? *Mathematical Thinking and Learning, 7*(4), 265-289. https://doi.org/10.1207/s15327833mtl0704_1.

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US-Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education, 79*(2), 171-185. https://doi.org/10.1016/S0742-051X(02)00102-6.

Franke, M. L., Webb, N. M., Webb, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(40), 380–392.https://doi.org/10.1177/0022487109339906.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2005) *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework.* American Statistical Association.

Gall, M. D., Gall, J. P., & Borg, W. R. (2007). Case study research. In A. Burvikovs (Ed.), *Educational research: An introduction* (pp. 446-485). MA: Pearson Education.

Garfield, J., & Ben-Zvi, D. (2008*).* Preparing school teachers to develop students' statistical reasoning. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.) *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education, A* Joint ICMI/IASE study: The 18th ICMI study (pp.187-198). Springer.

Ghousseini, H. N. (2008). *Learning with routines: Preservice teachers learning to lead classroom mathematics discussions* [Unpublished doctoral dissertation]. The University of Michigan.

Grossman, P., Compton, C., Igra, D., Ronald, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross professional perspective. *Teacher College Record, 111*(9), 2055–2100.

Heaton, R. M., & Mickelson, W. T. (2002). The learning and teaching of statistical investigation in teaching and teacher education. *Journal of Mathematics Teacher Education, 5*(1), 35–59. https://doi.org/10.1023/A:1013886730487

Henning, J. E., McKeny, T., Foley, G. D., & Balong, M. (2012). Mathematics discussions by design: Creating opportunities for purposeful participation. *Journal of Mathematics Teacher Education, 15*(6), 453-479. https://doi.org/10.1007/s10857-012-9224-1

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406. https://doi.org/10.3102/00028312042002371.

Inoue, N. (2011). Zen and the art of neriage: Facilitating consensus building in mathematics inquiry lessons through Lesson Study. *Journal of Mathematics Teacher Education, 14*(1), 5–23. https://doi.org/10.1007/s10857-010-9150-z

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 1,* 59–80. https://www.jstor.org/stable/1002169

Kersaint, G. (2015). *Orchestrating mathematical discourse to enhance student learning.* Curriculum Associates, LLC. Retrieved from https://fs24.formsite.com/edweek/images/WP-Curriculum_Associates--Orchestrating_Mathematical_Discourse.pdf.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* Yale University Press.

Llinares, S., Fernández, C., & Sánchez-Matamoros, G. (2016). Changes in how prospective teachers anticipate secondary students' answers. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(8), 2155-2170.

Manouchehri, A. (2007). Inquiry-discourse mathematics instruction. *Mathematics Teacher, 101,* 290-300. https://doi.org/10.5951/MT.101.4.0290

Matthews, M., Hlas, C. S., & Finken, T. M. (2009). Using lesson study and four-column lesson planning with preservice teachers: Lessons from lessons. *Mathematics Teacher, 102*(7), 504-509.

Meiliasari, M. (2019). Mathematics pre-service teachers' anticipation of students' responses: A case study of lesson study for pre-service teachers. In Y. Rahmawati & P. Taylor (Eds.), *Empowering Science and Mathematics for Global Competitiveness* (pp. 472–478). CRC Press.

Ministry of National Education [MoNE]. (2018). *Matematik dersi (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. Sınıflar) öğretim programı* [Primary and middle school mathematics curricula for grades 1, 2, 3,4, 5, 6, 7, and 8]. Ankara, Turkey: MEB.

Nathan, M. J., & Knuth, E. J. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21*(2), 175–207. https://doi.org/10.1207/S1532690XCI2102_03.

National Council of Teachers of Mathematics. [NCTM] (1991). *Professional standards for the teaching of mathematics.* VA: Author.

National Council of Teachers of Mathematics. [NCTM] (2000). *Principles and standards for school mathematics.* VA: Author.

National Council of Teachers of Mathematics. [NCTM] (2013). What are some strategies for facilitating productive classroom discussions? *Discussion research brief,* 1-5.

National Council of Teachers of Mathematics. [NCTM] (2014). *Principles to actions ensuring mathematical success for all.* VA: Author.

Pang, J. (2016). Improving mathematics instruction and supporting teacher learning in Korea through lesson study using five practices. *ZDM Mathematics Education, 48,* 471–483. https://doi.org/10.1007/s11858-016-0768-x

Piccolo, D. L., Harbaugh, A. P., Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics, 19*(3), 376-410. https://doi.org/10.4219/jaa-2008-809

Scheaffer, R. L. (2006). Statistics and mathematics: On making a happy marriage. In G. Burrill (Ed.) *MCTM 2006 Yearbook: Thinking and reasoning with data and chance.* VA: NCTM.

Smith, M. S., Cartier, J. L., Eskelson, S. L., & Ross, D. (2013, April). *Planning and teaching: An investigation of two teachers' participation in collaborative lesson planning activities and the impact of these activities on their instruction*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344-350

Smith, M. S., & Stein, M. K. (2011). *5 Practices for orchestrating productive mathematics discussions.* VA: The National Council of Teachers of Mathematics, Inc.

Stake, R. (1995). *The Art of case study Research*. Thousand Oaks, New Delhi: Sage.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313-340. https://doi.org/10.1080/10986060802229675

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* The Free Press.

Tyminski, A. M., Zambak, V. S., Drake, C., & Land, T. J. (2014). Using representations, decomposition, and approximations of practices to support prospective elementary mathematics teachers' practice of organizing discussions. *Journal of Mathematics Teacher Education, 17*(5), 463–487. https://doi.org/10.1007/s10857-013-9261-4

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516–551 https://doi.org/10.3102/0034654308320292.

Wilson, P. H., Sztajn, P., Edgington, C., & Myers, M. (2015). Teachers' uses of a learning trajectory in student-centered teaching practices. *Journal of Teacher Education, 66*(3), 227–244. https://doi.org/10.1177/0022487115574104

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally*. Upper Saddle River, NJ: Pearson Education, Inc.

Young, J. S. (2015). *Orchestrating mathematical discussions: A novice teacher's implementation of five practices to develop discourse orchestration in a sixth-grade classroom* [Unpublished master dissertation]. Brigham Young University.

Zhang, S., & Cheng, Q. (2011). Learning to teach through a practicum-based microteaching model. *Action in Teacher Education, 33*(4), 343-358.https://doi.org/10.1080/01626620.2011.620523.