increasing graph suggests the *y*-values also get bigger), and contextual applications were helpful, but not necessary, to facilitate this type of discussion. Overall, the data supports that the expository content in the textbook series frequently used

Recall that a slope occurrence might be coded with more than one slope component. To look at the contextual applications by slope component, it is important to recognize this overlap when interpreting Table 10. The results indicate *Determining Property* rarely occurred with *applications*, compared to the other slope components. Discussions and sample problems in the expository text using slope to determine relationships between lines typically occurred without any applied context, with no consideration of what a parallel (or perpendicular) relationship would mean in an applied real-world situation.

Table 10
Relative Frequency of Contextual Applications by Slope Component

Slope Component	Contextual Applications
CP (n=134)	56%
R (n=132)	66%
BI (n=77)	68%
DP (n=28)	4%
S (n=19)	63%

Summary

The UCSMP curriculum develops slope across all seven of its secondary mathematics textbooks and utilizes real-world applications frequently. Our analysis highlighted the importance of considering more than just students' experiences with slope in precalculus (e.g., looking not only at precalculus textbooks but also prior books in the curriculum series) when determining their preparation for calculus. A natural progression of covariational reasoning and slope occurrences was seen as we looked across the curriculum. A glance at the precalculus curriculum alone might have suggested a lack of covariational reasoning and few combinations of slope components to prepare students for calculus. However, when viewed considering what occurs across the entire curriculum of the series used in this study, it seems likely that the term *slope* is often used in precalculus textbooks as a taken-as-shared term (with a robust, but previously developed, meaning) without a thorough rehashing all the notions of slope developed in the courses leading up to precalculus.

Our analysis suggests that slope is primarily and consistently developed with *Constant Parameter*, *Ratio*, and *Behavior Indicator* interpretations in the UCSMP textbook series. Moreover, these three components of slope are often connected to each other and frequently incorporate covariational reasoning. Together, these ideas suggest that the UCSMP positions students to be prepared to understand the definition of a derivative as a function which provides the instantaneous slope (or rate of change) of the original function at a given point. The series' grounding across the seven textbooks of slope as a *Behavior Indicator* with connections to other slope components also suggests students should be well positioned to understand the first derivative test's approach of using the sign of the derivative for determining open intervals over which a function is increasing and decreasing.

Slope was less frequently described as a *Determining Property*. Occurrences of slope involving *Determining Property* were rarely connected to covariational reasoning and real-world contexts. The lack of emphasis on determining property could impact students' understanding of both antiderivatives and the Mean Value Theorem. As students in calculus develop an understanding of the relationship between families of antiderivatives of a function, they need a notion of *Determining Property* connected with covariational reasoning. They need to understand that two linear functions with the same slope will have the same change in the output variable for a set change in the input variable and should be able to connect this to corresponding graphs that are parallel because they produce the same rise for a set run. This extends to antiderivatives in calculus where a student can interpret two antiderivatives as having the same derivative function, and, hence, the same instantaneous rate of change, and parallel tangent lines, at points with the same input values, as illustrated in Figure 4. Extending this thinking across all *x*-values in the domain of the two functions would lead to the notion of a family of antiderivatives as consisting of all vertical translations of a given function. Calculus also builds on students' notions of *Determining Property*, often using both *visual* and *nonvisual* approaches, when the Mean Value Theorem is introduced, since it requires considering equivalent average and instantaneous rates of change visualized as slopes of secant and tangent lines, respectively.

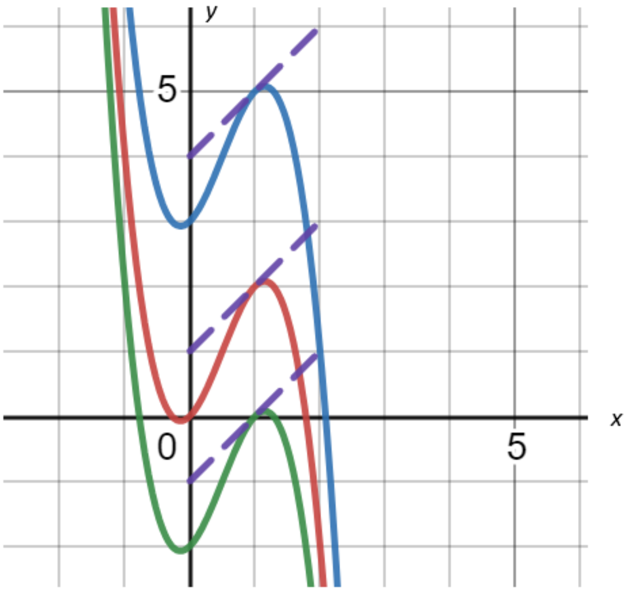

Figure 4. A family of antiderivatives with parallel tangent lines for any common input

Slope was infrequently connected to *Steepness* in this textbook series. While often oversimplified as an application of slope in a static, physical setting (e.g., building a ramp), it is important that students develop notions of slope as *Steepness* that are connected to their other, more dynamic notions of slope (Nagle & Moore-Russo, 2013a). From a calculus perspective, a derivative is often conceptualized as a measure of a function's sensitivity to change. This requires students to have a strong foundation connecting slope as a *Ratio* and *Steepness*, where steepness is not just a physical attribute but also a description of a linear relationship's sensitivity to change in outputs for a change in inputs. Note that this is achieved by also building students L3 covariational reasoning. In calculus, this idea is extended to studying concavity of nonlinear functions where we want to compare how this steepness changes to relate physical characteristics (concavity) to functional characteristics (e.g., rate of change increasing/decreasing). A strong foundation connecting slope as a *Ratio* and *Steepness* that allows for an awareness of the sensitivity to change in outputs for a change in inputs, is also important in understanding linearization in calculus. Finally, *Steepness* is important to be able to interpret regression lines in statistics and has been shown to be an area where students struggle to apply their prior knowledge of slope (Nagle et al., 2017).

Conclusions

This study only considers a single textbook series, and there are findings that are specific to the series. However, it also provides a window as to what might occur in other series. Future research should look at the development of slope across the most popular textbook series, including those that have not received as many accolades as the UCSMP curriculum. Future research should also consider how the curricular material is enacted by the instructor; it would be interesting to know if teachers who use UCSMP textbooks supplement them to augment the students' exposure to certain slope components or more real-world problems.

The UCSMP curriculum has been lauded as a high-quality, reform-oriented series that emphasizes contextual applications. Recently, Rezat et al. (2021) described mathematical content as one of three objects of change through curriculum materials (with pedagogical approaches and mathematics as a subject). In-depth investigations of how a particular mathematical concept is developed across the series can provide new insight for the curriculum developers to consider when evaluating whether their materials are achieving the desired changes. In terms of this series' coverage of slope in its expository material, our findings support the textbooks' claims regarding emphasis on contextual applications. However, there are slope components, such as *Determining Property* and *Steepness*, that may need additional attention and intentional coverage by secondary teachers using this curriculum. While the series did incorporate several contextual applications (with a large portion that were *functional*, rather than *physical* contexts), there are textbooks in the series that have fewer *applications*. While that might be expected of a *Geometry* textbook, it is rather surprising for the *Functions, Statistics, and Trigonometry* textbook. This information can support curriculum developers in considering not just contextual *applications* of the concept of slope, but also in considering whether those applications accompany the various components at different points in time across the secondary curriculum.

There are findings in this study that reach beyond the UCSMP curriculum that can be of value both for the research community as well as for practicing instructors. First, students' exposure to slope in several courses across the secondary curriculum, not just in precalculus courses, inform the ideas of slope students bring to a calculus course and will influence how prepared students are to build upon their ideas of slope in a meaningful way. Secondly, this study reiterates how important it is to have a robust notion of slope, which is likely why the topic occurs across the curriculum. Secondary instructors should consider how students are developing their understanding of slope across grade levels. Without an intentional consideration of where in the curriculum different notions of slope are developed, it is more likely that students will arrive in calculus classes "capable of using the slope formula to find the rate of change; however, ...unable to interpret the meaning of rate of change in a contextual situation, with a given graph, or more importantly with nonlinear situations" (Teuscher & Reys, 2012, p. 373). The robustness of students' notions of slope also depends on the extent to which the various components have been developed in tandem, whether *visual* or *nonvisual* approaches have been emphasized, how *covariational reasoning* has been incorporated, and the extent to which students have had opportunities to reason about slope in real-world *applications*.

Third, and finally, although slope is sometimes oversimplified as a physical property of a linear graph, our analysis of slope across this textbook series has illustrated that it is a multi-faceted concept, with different components and approaches emphasized at various stages of the precalculus curriculum, and with important implications for students' preparation for calculus. Thus, improving students' preparation for calculus requires taking a holistic view of developing a flexible, robust understanding of slope across the curriculum; this will help us know possible ways students might build understanding of derivatives, derivative applications, and even antiderivatives from their existing notions of slope.

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