The Effect of Teaching Approaches on Students' Academic Achievement in Mathematics at the Higher Education Level

Mzomwe Yahya Mazana
College of Business Education
mzomwe.mazana@cbe.ac.tz

Calkin Suero Montero
Uppsala University
csueromontero@gmail.com

Respickius Olifage Casmir
College of Business Education
repickius.casmir@cbe.ac.tz

This study compares the effects of teacher-centered, student-centered, and teacher-student interactive teaching methods on students' mathematics academic achievement. For this purpose, calculus, compound interest, annuities, and depreciation topics were taught to first-year college students in three different teaching method groups: traditional lecture (TL), group discussion (GD), and modified lecture (ML). In the TL group, the lecturer taught mathematical concepts through lectures following a teacher-centered method. In the GD group, students worked in small groups to solve mathematics problems with the lecturer's assistance, following a student-centered teaching method. In the ML group, the lecturer used a combination of lectures and questioning to teach mathematics concepts to students, following an interactive approach. The same lecturer taught all groups and implemented the same post-test at the end of the course. The present analysis uses one-way ANOVA based on this students' cohort (N = 250). The results show a significant difference between the performances of the three groups: the ML group performed better than the GD and TL groups, and low-ability students in the ML group had higher gain scores than students in the other two groups. These findings suggest that an interactive approach (a modified lecture) could help students improve their academic achievement in mathematics.

Introduction

Mathematics teaching is intended to promote the learning of mathematics and, consequently, its performance in examinations. Despite mathematics classroom teaching, there is evidence that undergraduate students struggle with passing mathematics courses (Mazana et al., 2019). Scholars show that many students fail to pass high school and college mathematics courses (Albeshree et al., 2020). According to Intepe et al. (2021), Mazana et al. (2020a), and Mazana et al. (2019), among other factors, students' quality of learning is affected by the teaching methods used by teachers. To determine how students best learn mathematics, it is important to analyze teachers' teaching methods to deliver mathematics instructions (Estébanez, 2017).

Traditionally in Tanzania, like many other Sub-Saharan African countries, the main method of mathematics instruction delivery in higher education has been teacher- or content-focused, which relies heavily on lectures and study notes to deliver content to student populations (Mazana et al., 2023). These lectures are believed to be ineffective in fostering student learning (Paoletti et al., 2018, p. 2). Recently, new student-centered instructional practices have emerged due to the influence of constructivist learning theory. This theory emphasizes students' activity, responsibility, and independence for learning, as opposed to passively receiving information from the lecturer (Baeten et al., 2013; Lahdenperä et al., 2019). Therefore, several recent scholars have called for increased use of student-centered teaching and learning (Lugosi & Uribe, 2020).

Freudenthal (1991), for example, believes that students should be able to solve mathematics problems for themselves, while Schoenfeld (2016) supports mathematics programs that emphasize sense-making. Similarly, Lahdenperä et al. (2019) promote student-centered learning by suggesting that the student-centered course design can promote more favorable learning approaches, greater self-efficacy levels, and positive teaching-learning experiences. In a college mathematics education context, Laursen et al. (2014) argue for an inquiry-based course design, as it eliminates a sizable gender gap that disfavors women students in lecture-based courses. The study by Ting et al. (2019) proposes that a problem-based, collaborative, game-based approach to large-class university mathematics teaching increases conceptual understanding and examination performance. Furthermore, Lesseig and Krouss (2017) report positive outcomes from flipped learning in collegiate mathematics.

Much effort has been invested into developing educational settings in Tanzanian higher education that align with worldwide trends. One major emphasis is on using competence-based pedagogical methods to equip students with the skills needed for national development. Various colleges have seen significant changes in recent years, with most traditional lecture-based teaching being replaced with a competency-based approach rooted in social constructivism, a combination of constructivist theory and socio-cultural theories. Even though learning theories focus on learning rather than teaching, researchers started to examine appropriate instructional methods to encourage students' active knowledge construction (Baeten et al., 2013). According to social constructivists, learning is a social process where students negotiate meaning and co-construct knowledge in a collaborative learning environment (Sharma, 2015). Considering a competence-based approach as a pedagogical framework, mathematics courses are presented using a student-centered approach.

According to scholars Lahdenperä et al. (2019), there are several approaches to achieving student-centered learning. However, all approaches to mathematics instruction have the same goal: to move away from rote learning through traditional lectures (a teacher-centered approach) and toward holistic mathematical understanding. In general, student-centered teaching methods are thought to improve deep and comprehension-oriented student learning (ibid.). According to socio-constructivists, teacher-centered methods fail to ensure students' learning (Beaten, 2013). In an earlier study, Mazana et al. (2023) conducted qualitative research to investigate the teaching approaches used by mathematics lecturers in four higher education institutions in Tanzania. It was found that lecturers frequently mentioned the teacher-centered approach (traditional lecture), but their profile was a mixture of methods (interactive approach). However, the effectiveness of these lecturers' various approaches is unclear. Since lecturers in Tanzanian higher education use these approaches to teach mathematics, researchers must evaluate their effects on higher education students' mathematics achievement. As an example of such evaluation, this study addresses the following research question: "What are the effects of student-centered teaching methods, teacher-centered methods, and teacher-student interactive methods on students' mathematics performance?" A quasi-experiment was conducted to investigate the effect of three instructional methods (traditional lecture (teacher-centered method), guided small group discussion (student-centered method), as well as lecture and questioning/modified lecture (interactive method) on students' academic achievement in business mathematics topics. Specifically, the study answered the following questions:

1. What is the effect of teaching approaches (traditional lecture, guided small group discussion, lecture, and questioning) on students' academic achievement in mathematics?
2. What is the effect of the teaching approaches on students' performance with different mathematical skill levels (high, medium, and low skills) based on the pre-test performance?

Related Literature

Teaching Approaches and Their Effect on Students' Achievement in Mathematics

Trigwell et al. (1994) suggested five teaching methods for undergraduate science teaching. These methods can be divided into three broad categories: teacher-centered, student-centered, and teacher-student interaction.

A teacher-centered approach to teaching relates to the traditional conception of instruction, wherein knowledge acquisition is the primary objective of the learning process (Keiler, 2018). In this learning arrangement, the teacher is the center of learning, distributing knowledge regarding the subject matter to students who are considered receptors of that knowledge (Moate & Cox, 2015). This educational setting involves the instructor delivering information to students, who listen, write notes, and communicate with the instructor unless they are allowed to pose or respond to questions (Phungphol, 2005). Teachers who take a "teacher-centered" approach strongly emphasize conventional lecturing, independent student work, and summative examinations to impart knowledge and gauge student progress. Teachers may employ techniques such as lecturing, demonstrating, and showing and telling to convey information to students (Mazana et al., 2023). However, lecturing has been criticized because it causes students to adopt superficial learning strategies (Lahdenperä et al., 2019). Lectures are also thought to have failed to motivate students to learn because the vast majority of them do not develop an understanding of course content or the skills necessary to succeed in mathematics (Paoletti et al., 2018). According to Camac (2018), the teacher-centered approach is not intended to educate all students but only those who adapt to it. Students' interests and learning motivation can be affected by teachers who rely heavily on lecturing because they may become accustomed to being passive learners (Moate & Cox, 2015). Results presented in a meta-analysis by Freeman et al. (2014) show that students who learn through traditional lectures are 1.5 times more likely to fail their mathematics exams when compared with those who learn through active learning strategies. Lew et al. (2016) investigated the effectiveness of the lecture in an advanced mathematics course through a case study. Students could not comprehend the concepts their lecturer was attempting to convey, demonstrating the ineffectiveness of the lecture method. However, when lecturers are supplemented with innovative teaching methods, it can help students become more engaged in their learning, allowing them to access and apply what they have learned in their daily lives better (Moate & Cox, 2015).

Student-centered or "active" learning approaches relate to educational practices that move the focus from the teacher and instruction to the student and learning (Du Plessis, 2020). The student-centered approach originates from constructivist learning theory. It represents a shift from traditional teacher-centered instructional practices (Moate & Cox, 2015), whereby the focus of activity shifts from the teacher to the students, thus providing students with more responsibilities for their learning (Camac, 2018). This approach includes four essential features: active responsibility for learning, proactive management of the learning experience, independent knowledge construction, and teachers as facilitators (McCabe & O'Connor, 2014). Consequently, the respective authoritative-passive roles of the teacher and student are altered by placing the learner as a dynamic collaborator in the teaching and learning process (Du Plessis, 2020). Teachers who use student-centered pedagogy view knowledge through social and relational processes and prioritize students' processes of constructing personal knowledge and understanding rather than rote mastery of course content (Moate & Cox, 2015). An earlier study by Mazana et al. (2023) found that instructors adopt a student-centered strategy to encourage student participation in class, tailor lessons to individual students, and foster stronger instructor-student bonds. Their research demonstrates that these lecturers actively include students in class through group work, guided discussions, student-as-instructor roles, and inspirational techniques. Research about student-centered approaches advocates for this method in order to increase active learning and promote interest, critical thinking, learning, and motivation among students (Stols et al., 2015; Kizito et al., 2016; Keiler, 2018; Borji et al., 2021; Lugosi & Uribe, 2020). For example, Kogan and Laursen (2014) examined the impact of inquiry-based learning (IBL) on college students' grades and course selection. Their findings showed that the IBL had a sizable and persistent impact on previously low-achieving students' grades. Lahdenperä et al. (2019) compared students' learning approaches, self-efficacy, and learning experiences in a partially student-centered traditional lecture-based class versus apprenticeship, a novel student-centered teaching method. The more student-centered the teaching method, the more favorable learning approaches, higher self-efficacy, and a positive learning experience were found. The study of Heuett (2017) compared the effects of a flipped classroom (a student-centered approach) and traditional lectures (a teacher-centered approach) on 82 undergraduate students in the USA. The study found that the students in the traditional lecture class had lower performance on exams and had lower abilities and understanding of the course material. Lugosi and Uribe (2020) have also reported that active learning strategies (student-centered) positively affect undergraduate mathematics students' average results and pass rate.

The teacher-student interactive approach combines student-centered (active learning) and teacher-centered teaching strategies. According to Ganyaupfu (2013), the interactive approach inspires the students to search for relevant knowledge rather than the lecturer monopolizing the transmission of information to the learners. A recent study by Mazana et al. (2023) highlighted that teachers who use the interactive method employ a mix of strategies, including lecture and questioning, which they call "a modified lecture" or "a lecture and question-and-answer strategy", lecture and sharing, group work and sharing, or a combination of two or three strategies to teach the material and take students through a series of cognitive tactics to solve problems and clarify common mistakes. As such, research evidence on teaching approaches maintains that this teaching method improves students' performance. A recent meta-analysis study shows that using mixed teaching strategies in scientific practice positively impacts achieving cognitive and epistemic teaching goals (Halawa et al., 2020). Ganyaupfu (2013), comparing the effectiveness of the teacher-centered, student-centered, and interactive approaches to mathematics teaching of undergraduate students, found that the student-teacher interactive approach was the most effective for mathematics teaching in higher education. In contrast, the teacher-centered approach was the least effective. This implies that using active learning methods in university mathematics may assist in attracting and retaining students from various backgrounds (Kogan & Laursen, 2014).

Traditional Lecture and Students' Academic Achievement

The traditional lecture is typically a teacher-centered method grounded in behaviorism learning theory. This theory strongly emphasizes altering behavior due to learners' associations with stimulus-response (Lessani et al., 2017). The theory claims that experience-based behavior change is what constitutes learning. The teacher would spend most of the time lecturing in a traditional lecture class. This pedagogical strategy emphasizes the teacher's role as a knowledge transmitter (teaching by telling students what to do). According to various scholars (Freeman et al., 2014; Lahdenperä et al., 2019; Moate & Cox, 2015; Paoletti et al., 2018), this method is ineffective in delivering mathematics instruction.

Guided Group Discussion and Students' Academic Achievement

One of the cooperative learning strategies is group discussion (student-centered). Cooperative learning is an educational strategy in which students work in small groups to maximize their and each other's learning (Mazana et al., 2020b; Johnson et al., 2014). According to Lugosi and Uribe (2020, p. 5), this strategy helps students improve their basic mathematical and general academic skills, problem-solving, critical thinking skills, and confidence in math. Studies on the use of team-based learning, group work, and cooperative learning in education emphasize the importance of collaborative work and discussion, and team-based learning is viewed as an important contributor to the development of higher-order thinking skills. For example, Johnson et al. (2014) compared cooperative learning, competitive learning, and individualistic instructional strategies in a meta-analysis study that found that cooperative learning improved individual achievement more than competitive or individualistic learning strategies. Another study by Nanes (2014) found that using team learning in mathematics classes can improve test scores and grades, mathematical skills, and the overall experience of studying and reading a math text.

Similarly, guided discussion is an instructional strategy that encourages students to think critically, share their perspectives, and actively participate in class (Stols et al., 2008). However, during guided discussions, students' group work is led with a structured protocol that usually includes a variety of stem sentences or questions that steer their arguments (Green, 2017; Fung & Lui, 2016). The current study used structured questions to guide students' group conversions. Previous researchers have emphasized the importance of guided discussion for promoting learning outcomes (Saka, 2011; Olukayode, 2012; Gokhale & Machina, 2018). However, as Stols et al. (2008) put it, students' math skills improve when the teacher leads the discussion. This strategy relies on students' prior understanding of the topic to be covered, either from reading or a brief lecture (CFI, 2023).

In our current study, the initial knowledge of the topic discussed was activated through a short lecture. The instructor promotes student participation during guided discussions by posing questions to the whole class. In our study, the questions to guide the discussions were given as printed hand-outs to the small groups. In this way, students could learn different alternative approaches to solving a problem, e.g., through the different solutions put forward by each small group. Lecturers then confirm the students' understanding and find mistakes or misconceptions, if any. Lecturers also get to know the skills and weaknesses of each student, which helps them choose better strategies to teach to enhance students' learning.

Modified Lecture and Students' Academic Achievement

A modified lecture is a teaching strategy that expertly interweaves lecture and questioning during teaching. Through this strategy, the lecturer uses direct question and answer to lead students to understand new mathematical concepts, to clarify or refine existing concepts, or to facilitate their solving of mathematical problems interactively. Questioning occurs when a problem is presented to students in anticipation of their answers (Dahal et al., 2019). Questioning is one of the strategies that mathematics lecturers can employ to encourage students' thinking, learning, and participation in class. Socrates, for example, helped his student understand the concept of justice by asking him a series of tactical questions that prompted him to reflect and think critically about the subject, allowing him to grasp the concept (Buchanan Hill, 2016). Teachers' questioning facilitates the conceptual development of students, reinforces the acquisition of new knowledge, enhances critical thinking capacity, and scaffolds students' efforts to construct and apply disciplinary knowledge (Hu & Duan, 2019). Teachers have often used questioning to increase student participation, focus on a specific concept, assess students' prior and current knowledge, and determine whether or not the assigned tasks were completed (Callahan & Clark, 2014). Teachers' questioning plays a similarly vital role in students' content learning by activating their background knowledge and facilitating conceptual development. According to Stanberry (2018), strategic engagement for increased learning begins with an interactive lecture (such as a *lecture and questioning* strategy in our study). Students are allowed to participate in the learning process in this arrangement by being asked to recommend the next step in the solution (Lugosi & Uribe, 2020). These researchers also believe that lecturers should use an interactive lecture style to communicate with students, as this strategy reduces students' math anxiety (Lugosi & Uribe, 2020).

Academic Achievement

Academic achievement in the context of this study is a term used to describe how students are evaluated during the learning process through tests and exams. According to Lin et al. (2016), achievement tests are typically used to measure academic achievement. This study measures academic achievement by the students' average scores in the pre-test and post-test. Pre-test scores relate to average scores from the test taken before the experiment and are used as a control for the post-test score. The post-test score is the average score from the test taken after the experiment.

Methodology

Research Design

A quasi-experimental research design was used to compare the effectiveness of the teacher-centered (traditional lecture, TL), student-centered (guided group discussion, GD), and interactive (lecture and questioning/modified lecture, ML) approaches for the teaching of business mathematics to undergraduate students in business courses. According to Ross and Morrison (2013), in educational studies, assigning subjects randomly to treatments is sometimes not viable, especially when classes are formed at the start of the year, as is in our case. These researchers suggest that in such circumstances, the researcher should use pre-testing to establish group equivalence, which was conducted in the current study.

The quasi-experiment was conducted in one of the author's classrooms at a higher education institution in Dar es Salaam, Tanzania. The study compared three business mathematics classrooms, where the control group employed a traditional teacher-centered lecture strategy with individual classroom activities.

The GD condition used a student-centered approach involving a demonstration of the topic and small group guided discussions. In this approach, the lecturer defined concepts and showed how problems are solved through examples, followed by a small group discussion (3–5 students per group) with the lecturer's guidance. Under the GD condition, printed sheets with the tasks for group discussion were given to the students. In this group, the class time was spent on a twenty-minute lecture for teacher demonstrations and briefing on important concepts related to the topic. The remaining time was used for guided discussions. In addition to topic-specific questions, students were taught to use guiding questions that led their discussion. At the same time, the lecturer went around to oversee the proceedings, in which the lecturer listened to a portion of each group's conversions to get a sense of important problem-solving processes. The guiding questions stemmed from Mevarech and Kramarski (1997). They prompted students to reflect on their learning and explain the problem, its relationship to prior knowledge, and possibilities for solving it. The questions included (1) questions testing students' understanding of the problem at hand, (2) questions about how the given problem was related, or connected to, other problems, and (3) questions about how the problem should be solved strategically. By answering these questions, the students were working towards the problem solution. After answering the questions, each small group was given five minutes to explain their answers to the class; other groups would chip in if they disagreed with the approach or with the answer provided. In response, students from the presenting group voiced and defended their thoughts on the problem. The lecturer encouraged the groups to discuss different perspectives and strategies for solving the problem.

Under the ML condition, the experimental group implemented a revised version of the lecture format that featured both lecturing and questioning. Following a quick introduction of main concepts and demonstrations, the instructor uses the whiteboard to answer mathematical problems in a modified version of a regular lecture format. Following this, the instructor gives the students a new problem to tackle, and they will be encouraged to guide the lecturer through the solution (Mazana et al., 2023). In the current investigation, the implementation lasted for one hundred and twenty minutes divided into two parts: the first twenty minutes consisted of a brief lecture that presented the key ideas of the topic and provided one example per concept of how to solve the problems related to that concept. The remainder of the class was spent with the instructor presenting a series of new problems to the students, one for each new idea discussed in the brief lecture, and having the students read through them. Occasionally, one of the students volunteered to read the problems out loud so that the others could comprehend them. After that, the lecturer requested that they describe the formula for solving the problem, also known as *the givens*, and then take the lecturer through the following steps until they solved the problem. At the end of the presentation, the lecturer asked the audience about other strategies to solve the problem. Through this teaching method, the students are actively engaged in the learning process for a large portion of the time as the lecture is transformed into a format that demands interaction with the lecturer.

A normal or traditional lecture style was used for the control group under the TL condition. This implementation consisted of the instructor giving a lengthy presentation on a subject, with minimal to no interaction with the students through questioning, during which the students take notes and engage in independent problem-solving at each lesson's conclusion. Under this condition, students were only given thirty minutes out of one hundred and twenty to dedicate to independent problem-solving exercises.

All three classrooms employed lectures, but they varied in length and how they encouraged student participation and problem-solving. Students in the GD condition run discussions in smaller breakout sessions and as a part of the whole class. In the ML condition, the lecturer and students drove the discussions, while the lecturer controlled the discussion under the TL condition. There was no teacher-student interaction in the TL condition, while in the ML condition, students interacted with the lecturer during the class.

Participants and ethics

The participants of our quasi-experiment were 250 first-year bachelor's degree students with different mathematical background knowledge, pursuing a major in marketing and business administration courses, intake 2021/2022, with 87 students in the control group (TL), 76 in the experimental group (GD), and 87 students in the experimental group (ML). Students must enroll in the business mathematics course as a requirement for business-related bachelor's degree programs in Tanzania. At the college where the experiments were conducted, the MTU07102 course (business mathematics for first-year students) is compulsory for all students majoring in accounting, procurement and supply management, and marketing, and more than 1000 students enroll every year. In order to safeguard their identities, the students' names were treated with confidentiality, and their consent to participate in the study was obtained and respected.

Course content and associated challenges

Business Mathematics is a 15-credit bachelor's degree course offered to undergraduate students of business majors by the Department of ICT and Mathematics. Each week of the course had two two-hour lectures and no tutorial sessions scheduled in the timetable under the curriculum model (semester one, 2021–2022). Still, a lecturer was allowed to spare a few minutes for tutorial activities during one of the lecture sessions in a week. In semester one, there were 26 lectures in total. Students in the experimental groups participated in a one-hour-long tutorial in which they solved mathematics problems in small groups or a question-and-answer session. This course comprises a review of basic mathematics (indices, logarithms, sequences and series, simultaneous equations, quadratic equations), calculus, matrices, and the mathematics of finance (compound interest, present values, annuities, and depreciation). However, in this experiment, we covered the topics described in Table 1.

Table 1
Description of course content

Course Units	Brief description
Sequences and series	In this unit, students learn types of sequences, the common difference of an arithmetic sequence, the ratio of a geometric sequence, the n^{th} term, the sum of a finite and infinite series, and the arithmetic and geometric means.
Calculus	In this unit, students learn the rules and techniques of differentiation and integration and how to compute economic functions such as cost, revenue, and profit/loss in this unit.
Compound interest	This unit introduces the concept of interest, the differences between simple and compound interest, and the interest rate, time, and compound amount calculation.

Course Units	Brief description
Present value	Students learn how to calculate present value and its application to future cash flows. This introduces them to the calculations of discounted cash flows for investments and methods of investment appraisals such as net present values and internal rate of return.
Annuities	This unit introduces students to techniques associated with fixed payments over time. They learn the types, and calculations of the future value, present value, and net present value of annuities.
Amortisation and Sinking fund	Through this unit, students learn methods of debt repayment and setting up funds to meet future obligations, and their application for mortgage repayment, bank loans, etc.
Depreciation	This unit introduces the meaning and methods of depreciation. Students learn to calculate the rate of depreciation and prepare the depreciation schedule.

Students enrolled in this course come from different backgrounds, including the natural sciences, business-related courses, arts, and social studies, at the secondary school level and diploma level, which means they have quite different skills with mathematics, and those with diplomas in business specialties are at a considerable advantage. This course expands on what is taught in secondary schools and at any diploma level, including various mathematical models and applications in real-world business problems. The learning demand is challenging for students from high school or other non-business-related backgrounds, and they need to catch up quickly. Also, most students have difficulties understanding English, the language of instruction in higher education in Tanzania. To make matters worse, the course has traditionally been taught through lectures due to a lack of preplanned tutorials, resulting in insufficient exercises. As a result, in recent years, lecturers have begun to use group discussions, question-and-answer strategies, and other interactive techniques to encourage students' active participation in the learning process. The effectiveness of these strategies, on the other hand, has not been tested or reported. This study evaluates and compares the effectiveness of lecturers' strategies for teaching mathematics.

Data collection and procedure

Data were collected using a pre-test and a post-test. For the pre-test, basic mathematics topics (e.g., indices, logarithms, sequences and series, simultaneous and quadratic equations) were taught to all students in the control group (TL) and experimental groups (GD and ML) using the traditional teaching method until the fourth week, which heavily employed lecturing. After the fourth week's lecture, all the students in the three groups took the same 60-minute pre-test to establish their baseline knowledge before treatment was applied. The mean scores of all groups were compared, and a significant value was checked to establish that all groups were identical.

For the post-test, the control group continued using the same traditional teaching method. In contrast, the two experimental groups were taught using student-centered (guided small group discussion) and interactive learning methods (modified lecture), respectively, until the thirteenth week (end of the semester). Hence, the control and experimental groups participated in nine-week teaching sessions that employed different teaching strategies. The same lecturer taught all three classes. During the quasi-experiment implementation, researchers were aware of the possible unconscious bias, carried out even without intending to do so. Therefore, the lecturer selected for the study was experienced, with over ten years of teaching the same course, and well-acquainted with different teaching methods. This lecturer had attended several in-house trainings and those offered by the National Accreditation Council for Technical and Vocational Training (NACTVET) about teaching methods, developing teaching materials, and undertaking learning assessments. Before implementing the experiments, the lecturer received a refresher course. While implementing the experiments, this lecturer regularly discussed the implemented strategies with colleagues for each topic that needed to be taught. They also discussed the assessments (i.e., the tests/quizzes) and collaboratively prepared the teaching and materials.

At the end of the course, all students took the same post-test (60 minutes). The means of the three groups were again compared, and a significant value was checked. The questions included in the post-test covered the topics discussed in the interventions. i.e., matrices, calculus, and the mathematics of finance, including annuities and depreciation. The pre- and post-tests were created by the mathematics lecturer who taught the classes. The test coverage and difficulty level were also moderated by one of the authors, who teaches at the same college. The pre-and post-tests included the assessment of the learning outcomes as established by the NACTVET.

The pre-test and post-test were composed of five open-ended questions worth 15 points each. Example questions for the pre-test include:

1. Miss Violet started working as a marketing officer for a large manufacturing firm in 2001. Her starting salary was Tshs. 22,000, and her contract promised that she would receive a pay rise of 5% yearly thereafter. Miss Violet plans to retire in 2030. Find the nearest Tshs. Her salary in year 30 and the total earnings from employment for 2001 to 2030, inclusive.
2. The College of Business Education football team buys jerseys to wear to football games." The cost of the jerseys can be calculated using the equation C(x) = 0.1x2 + 2.4x + 25, where C(x) is the price of x jerseys. How many jerseys can they purchase for Sh50,000?

Example questions for the post-test included:

1. A loan of 2,000,000 shillings bearing interest at 25% per annum will be amortized by five equal annual installments at the end of each year. Find the size of each annual installment and prepare the amortization schedule. Another question was:
2. If the product sells for Tshs 240 per bottle of water, the Dar es Salaam drinking water company has a fixed monthly cost of Tshs 150,000 and a production cost of Tshs 180 for each bottle of water produced." "Determine the company's profit function and compute the profit (loss) corresponding to production levels of 22,000 bottles of water and 28,000 bottles of water.

Data Analysis

Descriptive statistics (mean and standard deviation) and one-way ANOVA with a teaching approach as an independent variable accounted for observed differences between pre-test and post-test results. The analysis was done using SPSS version 25. A nonsignificant Levene's test ($F(2,247) = 0.195$, $p = 0.823$) indicated that the homogeneity of variance assumption was met. Although the differences in the pre-test average scores were very small, we tested whether they were statistically significant using the one-way ANOVA test.

The pre-test's SPSS output comprises the criterion's value and a p-value greater than 0.05, signifying that the observed differences are insignificant. This means that the three groups of students are equivalent. Thus, we could be sure that any observed changes in students' scores after the experiment were related to the teaching strategies used and not a cause of any random differences in the groups. Table 2 below shows the average pre-test scores of the students after the pre-test was distributed into different groups and the one-way ANOVA test results.

Table 2
Average scores obtained by students in the pre-test and the ANOVA test

Group	Pre-test		One way ANOVA		
	M	SD	F	df	p
GD – Guided small group discussions (N=76)	6.23	3.04	0.723	2	0.486
ML – Modified lecture (N = 87)	6.26	3.41			
TR – Traditional lecture (N= 87)	5.73	3.22			
Total	6.07	3.23			

Results

The Effect of Teaching Approaches on Students' Academic Achievement in Mathematics

First, we calculated average gain scores for each classroom to find changes in performance within groups between pre-test and post-test results [post-test scores minus pre-test scores]. The average gain scores presented in Table 3 indicate that the scores of students in the ML and GD classes increased by 3.15 and 0.20 (average gain scores), respectively. In contrast, those who attended the TL class decreased by 0.82, meaning that students in this group tended to score 0.82 points below average.

Next, we analyzed whether there were any differences in student achievement as a function of the teaching approaches. A one-way analysis of variance (ANOVA) test was used to examine whether the students who attended classes using the three teaching strategies (TL, GD, and ML) differed in their post-test achievement. The independent variable represented the different teaching approaches, with three groups being represented: the control group (TL), the experimental group (GD), and the marginal group (ML). The dependent variable was the score that students obtained on a post-test exam. See Table 3 below for the means and standard deviations for each of the three groups and the gain scores.

Table 3:
Average scores obtained by students in the post-test and their average gain scores

Source		Post-test		Gain score		One Way ANOVA			
Teaching method	N	M	SD	M	SD	df	F	p	ω^2
TL	87	4.9	3.8	-0.82	3.51	2	44.42	0.000	0.26
GD	76	6.4	3.6	0.20	2.96				
ML	87	9.4	2.7	3.15	3.28				
Total	250	6.9	3.9	0.87	3.68				

The tests for normality, examining standardized skewness and the Shapiro-Wilks test, indicated that the data were statistically normal. However, Levene's F test revealed that the homogeneity of variance assumption was not met ($p = 0.01$). As such, Welch's F test was used. An alpha level of 0.05 was used for all subsequent analyses. The one-way ANOVA of students' post-test scores showed a statistically significant effect, Welch's F (2, 157.4) = 44.42, $p < 0.05$, indicating that the post-test scores of students in different groups were significantly different. The estimated omega squared ($\omega^2 = 0.26$) indicated that approximately 26% of the total variation in average scores on students' post-test scores is attributable to differences between the teaching strategies used.

Post hoc comparisons, using the Dunnett T3 procedure, were conducted to determine which pairs of the three teaching approaches significantly differed. These results are given in Table 4 and indicate that the ML group (M = 9.4, SD = 2.7) had significantly higher post-test scores than those in group GD (M = 6.4, SD = 3.6), as well as the TL students' group (M = 4.9, SD = 3.8). Additionally, students in the GD group had significantly higher post-test scores than students in the TL group.

The results confirmed that the post-test scores of the three groups of students (guided small group discussion, modified lecture, and traditional lecture) were significantly different. This generally suggests that the modified lecture strategy (teacher-student interactive approach) was more effective than the guided small group discussion (student-centered) and traditional lecture (teacher-centered) approaches.

The Effect of the Teaching Approaches on the Average Gain Scores of Students with Different Pre-test Mathematics Achievement Levels (high, medium, and low)

We wanted to know whether the teaching approaches (TL, GD, and ML) had affected the students with different levels of achievement differently. Based on the pre-test scores for the achievement tests, students were categorized into three groups: higher achievers (58 students), medium achievers (68 students), and lower achievers (124 students). Analysis of the descriptive statistics displayed in Table 4 shows that the average scores of lower achievers in the ML class had increased by 7.18, the largest average gain score compared to other groups in any of the three classes. At the same time, the average gain scores of high achievers in the traditional class were the lowest compared to high achievers' gain scores in other groups.

We were also interested in seeing whether the differences in the average gain scores among students in the ML class were significant. We conducted a one-way ANOVA to examine whether the average gain scores of students who attended this class differed significantly. We used the average gain scores of the achievement tests [post-test score minus pre-test score] as the dependent variable and prior math achievement (low, medium, and high) as the independent variable (Table 4). As students of various pre-test achievements demonstrated significant differences in their average gain scores, Welch's (F (2, 35.70) = 22.22, $p < 0.05$, $\omega^2 = 0.42$), these results show that the interactive approach (ML) had a significant effect on students of various prior achievement. The post-hock comparison of student groups using the Dunnett T3 procedure indicates that lower achievers had higher average gain scores (M = 7.18, SD = 3.6, $p < 0.05$) than medium and higher achievers.

Table 4.
Pre-test, Post-test, and gain average score differences of students with different prior

Source	Group	Low	Medium	High	One Way ANOVA

		M	SD	M	SD	M	SD	df	F	p	ω^2
Pre-test	TL	1.57	1.2	5.07	0.8	8.81	2.0				
	GD	2.33	0.8	4.53	0.5	8.46	1.9				
	ML	1.18	1.1	5.07	0.8	8.98	2.0				
Post-test	TL	4.37	2.6	4.37	2.9	6.81	1.5				
	GD	3.72	2.9	5.47	2.7	7.91	3.4				
	ML	8.36	3.5	8.54	2.1	10.34	2.4				
Gain	TL	0.96	2.6	-0.7	2.9	-2.00	2.5				
	GD	1.39	2.7	0.93	2.9	-0.56	3.0				
	ML	7.18	3.6	3.46	2.1	1.41	2.1	2	22.2	0.00	0.42

Discussion

This study was designed to compare the effects of three different teaching approaches on students' academic achievement in mathematics. It resulted in several findings of practical significance, which will be discussed in relation to earlier research in related areas.

Teaching Approaches and Academic Achievement

Researchers argue that different teaching methods have distinct effects on students' academic achievement in mathematics (Ganyaupfu, 2013; Freeman et al., 2014; Lahdenperä et al., 2019; Lugosi & Uribe, 2020). The current study supports this view as we observed that the different teaching approaches affect students' academic achievement. The interactive approach (ML) is significantly better for improving students' academic achievement than the student-centered approach (GD), which, in turn, is better than the teacher-centered approach (TL) (see Table 3). One possible explanation is that students who used the interactive methods had the opportunity to actively participate in the learning process by being asked to suggest the next step in the solution (Lugosi & Uribe, 2020). This active participation could allow the students to engage in high-level critical thinking and reflection on the subject matter, which helped them grasp the concepts. In the teacher-centered (TL) approach, students are passive learners and interact with the lecturer when allowed to do so, while students using the interactive (ML) and student-centered (GD) approaches interact with both the lecturer and peers. It seems that the development of mathematical skills depends on active participation and discussion of ideas and strategies for solving a specific mathematical task with the instructor and peers. These findings extend the novelty of the interactive approach's effectiveness on students' mathematics learning, as reported in previous studies (e.g., Ganyaupfu, 2013). Furthermore, we provide explicit methods describing student-centered and interactive methods. At the same time, other studies are rather general about which teaching methods were used to describe the interactive or student-centered approaches and how this improved their academic performance (e.g., Ganyaupfu, 2013). Furthermore, our study is also different in focus from previously reported works regarding the implementation of guided discussions as it involved small group discussions led by printed questions provided by the lecturer rather than the lecturer asking questions to students to guide them in finding the answer to the given questions. Thus, this strategy provided more avenues for the lecturer to implement a student-centred approach.

The effect of teaching approaches on the achievement of students with different prior mathematical skills

According to Kogan and Laursen (2014), active learning environment teaching methods are beneficial for lower achievers academically. In their research, lower achievers' grades in inquiry-based learning (IBL, a student-centered approach) class were raised by 0.3 to 0.5 grade points compared to their prior performance. The findings in the current research reveal better performance by all groups using the interactive approach. Using this approach (lecture and questioning), we further demonstrate that lower achievers in our study also experienced positive and significant increases in average gain scores compared to medium and high achievers. The results demonstrate a much higher rise in lower achiever grades of about 7.18 grade points compared to those reported by Kogan and Laursen (2014) and Ganyaupfu (2013), implying that combining lecture and questioning in the form of a modified lecture, as applied in this research, is likely to improve lower achiever learning outcomes.

Conversely, we observe decreased performance in the high achievers group under the traditional lecture condition. One explanation is that the lecturer's probing of students' cognitive process toward solving a given problem may strengthen their ability to engage in critical thought, reinforce their learning of new material, and deepen their conceptual understanding (Hu & Duan, 2019). According to Lugosi and Uribe (2020), using an interactive lecture style to communicate with students can help them feel less anxious about math, which might have facilitated lower achievers' good performance. Additionally, public sharing with the lecturer and students with high abilities may serve as mastery experiences that boost the lower achievers' self-efficacy and relate effort to mathematical achievement instead of innate ability (Kogan & Laursen, 2014). According to Ganyaupfu (2013), learning entails researching, formulating, reasoning, and applying appropriate problem-solving strategies; our study shows that this is more effective when students actively participate. Thus, these results support the notion that lecturers should place more value on each student's efforts to build their knowledge and understanding than on the rote memorization of math material.

Implications for future research and practice

Future research should address many of the interesting issues raised by this work. The distinct advantages of ML (interactive approach) experiences for underachievers highlight their potential to aid in overcoming higher failure rates in other courses, such as statistical analysis and computing mathematics, which we could not examine in this study. Particularly fascinating will be the comparison of the long-term effects these active learning strategies have on students' learning achievement in business mathematics. Investigating the impact of other active learning strategies such as group work and sharing and the use of ICT tools could provide further insights into the effective approaches to business mathematics teaching. This study used the quantitative approach to demonstrate the effect of student-centered and interactive approaches on student academic achievement. Mixed-methods designs can both prove the effects and explain why they occurred.

The findings also affect how HEIs should respond when lecturers have trouble adjusting to active learning instructional strategies and encourage lecturers to select non-biased teaching methods in fields like mathematics, where they have knowledge control. Support should be provided for teachers to increase their understanding of various teaching methods, such as interactive or student-centered approaches, that might improve their students' academic achievement. In-house training should, therefore, be regularly provided to ensure that lecturers can adopt and practice various methods to keep students interested and motivated during the learning process.

Conclusion

While lecturers who use student-centered methods in the classroom are frequently asked for evidence to support the approach's educational benefits, lecturers who use traditional lecture methods are infrequently subjected to the same level of criticism (Kogan & Laursen, 2014). The current study shows that while low achievers can benefit more from interactive (active) learning methods, medium and high achievers are less affected. These results provide evidence of the benefit of different teaching strategies and their respective impact on students' academic performance, considering their prior academic achievement. This research outcome also calls for educational officials' support regarding capacity building for lecturers toward the successful implementation of suitable teaching strategies in Tanzanian HEIs.

References

Albeshree, F., Al-Manasia, M., Lemckert, C., Liu, S., & Tran, D. (2020). Mathematics teaching pedagogies to tertiary engineering and information technology students: a literature review. *International Journal of Mathematical Education in Science and Technology* 1609-1628. https://doi.org./10.1080/0020739X.2020.1837399

Baeten, M., Struyven, K., & Dochy, F. (2013). Student-centred teaching methods: Can they optimise students' approaches to learning in professional higher education? *Studies in Educational Evaluation* 39(1), 14–22. https://doi.org./10.1016/j.stueduc.2012.11.

Borji, V., Radmehr, F., & Font, V. (2021). The impact of procedural and conceptual teaching on students' mathematical performance over time. *International Journal of Mathematical Education in Science and Technology* 52 (3), 404-426. https://doi.org./10.1080/0020739X.20

Buchanan Hill, J. (2016). Questioning techniques: A study of instructional practice. *Peabody Journal of Education* 91(5), 660-671. https://doi.org./10.1080/0161956X.2016.1227190

Callahan, J. F., & Clark, L. H. (2014). *Teaching in the middle and secondary schools: Planning for competence.* New York: MacMillan

Camac, K. (2018). Student-Centered Approach Vs. Teacher-Centered Approach: Which Is More Effective for A Graduate Data Analytics Course in an E-Learning Environment?

CFI. (2023). teaching-methods. Retrieved July 27, 2023, from CFI notebook: https://www.cfinotebook.net/lesson-plans/fundamentals-of-instructing/teaching-methods

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approach* (4th Ed.). Sage publications.

Dahal, N., Luitel, B. C., & Pant, B. P. (2019). Understanding the use of questioning by mathematics teachers: A revelation. *International Journal of Innovative, Creativity and Change,* 5(1), 118-146.

Du Plessis, E. (2020). Student teachers' perceptions, experiences, and challenges regarding learner-centred teaching. *South African Journal of Education*, 40(1). https://doi.org./10.15700/saje.v40n1a1631

Eriksson, K., Helenius, O., & Ryve, A. (2019). Using TIMSS items to evaluate the effectiveness of different instructional practices. *Instructional Science,* 47(1), 1-18. https://doi.org./10.1007/s11251-018-9473-1

Estébanez, R. P. (2017). An Approachment to Cooperative Learning in Higher Education: Comparative Study of Teaching Methods in Engineering. *EURASIA Journal of Mathematics Science and Technology Education* 13(5), 1331-1340. https://doi.org./10.12973/eurasia.2017.00673a

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the national academy of sciences*, 111(23), 8410-8415. https://doi.org./10.1073/pnas.1319030111

Freudenthal, H., (2006). *Revisiting mathematics education: China lectures* (Vol. 9). Springer Science & Business Media.

Fung, D., & Lui, W. M. (2016). Individual to collaborative: guided group work and the role of teachers in junior secondary science classrooms. *International Journal of Science Education*, *38*(7), 1057-1076. https://doi.org/10.1080/09500693.2016.1177777

Ganyaupfu, E. M. (2013). Teaching methods and students' academic performance. *International journal of humanities and social science invention* 2(9), 29-35.

Gokhale, A. & Machina, K. (2018). Guided Online Group Discussion Enhances Student Critical Thinking Skills. *International Journal on E-Learning, 17*(2), 157-173. Waynesville, NC USA: Association for the Advancement of Computing in Education (AACE). Retrieved July 28, 2023 from https://www.learntechlib.org/primary/p/173291/

Green, D. (2017, June 22). *The guided group discussion method*. Retrieved from Rider Choices: https://www.riderchoices.com

Halawa, S., Hsu, Y.S., Zhang, W.X., Kuo, Y.R. and Wu, J.Y. (2020). Features and trends of teaching strategies for scientific practices from a review of 2008–2017 articles. *International Journal of Science Education*, 42(7), 1183-1206. https://doi.org./10.1080/0

Heuett, W. J. (2017). Flipping the math classroom for non-math majors to enrich their learning experience. *Primus*, 27(10), 889-907. https://doi.org./10.1080/10511970.2016.1256925

Hu, G., & Duan, Y. (2019). Questioning and responding in the classroom: a cross-disciplinary study of the effects of instructional mediums in academic subjects at a Chinese university. *International Journal of Bilingual Education and Bilingualism*, 22(3), 303-321. https://doi.org./10.1080/13670050.2018.1493084

Intepe, G., Hacinliyan, I., & Mizrahi, E. (2021). Towards learning approaches of engineering students studying mathematics and the validity of approaches and study skill inventory for students. *International Journal of Mathematical Education in Science and Technology*, 1-15. https://doi.org./10.1080/0020739X.2021.1980127

Johnson, D. W., Johnson, R. T., & Smith, K. A. (2014). Cooperative learning: Improving university instruction by basing practice on validated theory. *Journal on Excellence in University Teaching*, 25(4), 1-26.

Keiler, L. S. (2018). Teachers' roles and identities in student-centered classrooms. *International journal of STEM education*, 5(1), 1-20. https://doi.org./10.1186/s40594-018-0131-6

Kizito, R., Munyakazi, J., & Basuayi, C. (2016). Factors affecting student success in a first-year mathematics course: a South African experience. *International Journal of Mathematical Education in Science and Technology*, 47(1), 100-119. https://doi.org./10.1080/00207

Kogan, M., & Laursen, S. L. (2014). Assessing long-term effects of inquiry-based learning: A case study from college mathematics. *Innovative higher education*, 39(3), 183-199. http://doi.org./10.1007/s10755-013-9269-9

Lahdenperä, J., Postareff, L., & Rämö, J. (2019). Supporting quality of learning in university mathematics: A comparison of two instructional designs. *International Journal of Research in Undergraduate Mathematics Education*, 5(1), 75-96. https://doi.org./10.1186/s40594-018-0131-6

Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, 45(4), 406-418. https://doi.org./10.5951/jresematheduc.45.4.0406

Lessani, A., Yunus, A., & Bakar, K. (2017). Comparison of new mathematics teaching methods with traditional method. People: *International Journal of Social Sciences*, 3(2), 1285-1297. https://doi.org./10.20319/pijss.2017.32.12851297

Lesseig, K., & Krouss, P. (2017). Implementing a flipped instructional model in college algebra: profiles of student activity. *International Journal of Mathematical Education in Science and Technology* 48(2), 202–2014. https://doi.org./10.1080/0020739X.2016.1233586.

Lew, K., Fukawa-Connelly, T. P., Mejía-Ramos, J. P., & Weber, K. (2016). Lectures in advanced mathematics: Why students might not understand what the mathematics professor is trying to convey. *Journal for Research in Mathematics Education*, 47(2), 162-198. https://doi.org./10.5951/jresematheduc.47.2.0162

Lin, Y. W., Tseng, C. L., & Chiang, P. J. (2016). The effect of blended learning in mathematics course. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(3), 741-770. https://doi.org./10.12973/eurasia.2017.00641a

Lugosi, E., & Uribe, G. (2020). Active learning strategies with positive effects on students' achievements in undergraduate mathematics education. *International Journal of Mathematical Education in Science and Technology*. https://doi.or.g/10.1080/0020739X.2

McCabe, A., & O'Connor, U. (2014). Student-centred learning: the role and responsibility of the lecturer. *Teaching in Higher Education*, 19(4), 350-359. https://doi.org./10.1080/13562517.2013.860111

Mazana, M. Y., Montero, C. S., & Casmir, R. O. (2019). Investigating Students' Attitude towards Learning Mathematics. *International Electronic Journal of Mathematics Education, 14(*1), 207-231. https://doi.org./10.29333/iejme/399

Mazana, M. Y., Montero, C. S., & Casmir, R. O. (2020a). Assessing Students' Performance in Mathematics in Tanzania: The Teacher's Perspective. *International Electronic Journal of Mathematics Education, 15(*3), em0589. https://doi.org./10.29333/iejme/7994

Mazana, M. Y., Njotto, L. L., & Oyelere, S. S. (2020b). Flipping college mathematics classroom. *In 2020 15th Iberian* Conference on Information Systems and Technologies (CISTI) (pp. 1-6). IEEE.

Mazana, M.Y., Montero, C.S., Njotto,L.L., (2023). Exploring Mathematics Teaching Approaches in Tanzanian Higher Education Institutions: Lecturers' Perspectives. *The International Journal of Research in Undergraduate Mathematics Education*.

Mesa, V., Celis, S., & Lande., E. (2014). Teaching approaches of community college mathematics faculty: Do they relate to classroom practices? *American Educational Research Journal*, 51(1), 117-151. https://doi.org./10.3102/0002831213505759

Moate, R. M., & Cox, J. A. (2015). Learner-Centered Pedagogy: Considerations for application in a Didactic Course. *Professional Counselor*, 5 (3), 379-389.

Nanes, K. M. (2014). A modified approach to team-based learning in linear algebra courses. *International Journal of Mathematical Education in Science and Technology*, 45(8), 1208-1219. Htps://doi.org./10.1080/0020739X.2014.920558

Olukayode, O. J. (2012). Inquiry method, teacher guided discussion method and student's attitude and performance in social studies. *Global Journal of Management and Business Research*, *12*(15), 54-59

Paoletti, T., Krupnik, V., Papadopoulos, D., Olsen, J., Fukawa-Connelly, T., & Weber, K. (2018). Teacher questioning and invitations to participate in advanced mathematics lectures. *Educational Studies in Mathematics*, 98(1), 1-17. https://doi.org./10.1007/s10649-018-9807-6

Phungphol, Y. (2005). "Learner-centered teaching approach: A paradigm shift in Thai education." *ABAC Journal* 25 (2).5-16

Ross, S. M., & Morrison, G. R. (2013). *Experimental research methods*. In Handbook of research on educational communications and technology (pp. 1007-1029). Routledge.

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, 196(2), 1-38. https://doi.org./10.1177/002205741619600202

Saka, A. H. M. E. T. (2011). The effect of the guided discussion method on student success in the teaching of physics. *Energy Education Science and Technology Part B-Social and Educational Studies*, *3*(4).

Sharma, S. (2015). Teaching probability: a socio-constructivist perspective. *Teaching Statistics*, 37(3), 78-84. https://doi.org./10.1111/test.12075

Stanberry, M. L. (2018). Active learning: a case study of student engagement in college Calculus. *International Journal of Mathematical Education in Science and Technology*, 49(6), 959-969. https://doi.org./10.1080/0020739X.2018.1440328

Stols, G., Ono, Y., & Rogan, J. (2015). What constitutes effective mathematics teaching? Perceptions of teachers. *African Journal of Research in Mathematics, Science and Technology Education*, 19(3), 225-236. https://hdl.handle.net/10520/EJC181280

Ting, F. S., Lam, W. H., & Shroff, R. H. (2019). Active learning via problem-based collaborative games in a large mathematics university course in Hong Kong. *Education Sciences*, 9(3), 172. https://doi.org./10.3390/educsci9030172

Trigwell, K., Prosser, M., & Taylor, P. (1994). Qualitative differences in approaches to teaching first year university science. *Higher education*, 27(1), 75-84. https://doi.org./10.1007/BF01383761