Case-Based Teacher Training Environment for Pre-service Teachers' Noticing Skills about Students' Thinking in Statistical Concepts

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This study investigated how pre-service middle school mathematics teachers attend, interpret, and respond to the students' thinking about statistical concepts in a case-based teacher training environment. 25 pre-service middle school mathematics teachers participated in the study, and they were asked to make evaluations on the cases given. Based on these evaluations, classroom discussions were held on student thinking, the reasons for students' difficulties, and how to overcome these difficulties. The discussions and evaluation reports were analyzed within the framework of professional noticing of children's mathematical thinking. The results showed that although the pre-service teachers could mostly attend to student thinking correctly, these identifications remained superficial. The pre-service teachers were mostly able to interpret student thinking in such a way as to indicate the state of understanding or misunderstanding. When the pre-service teachers' responding skills were examined, it was revealed that they mostly suggested approaches based on partial conceptual understanding to overcome students' difficulties. In addition, the results obtained from the study revealed that the pre-service teachers' noticing skills differed from one case to another.
Keywords: noticing skills, pre-service middle school mathematics teachers, statistical concepts, case-based teacher training environment

Introduction

Cases in Teacher Education

To improve the teaching and learning process, case-based teaching has been used in the education literature since the early 1900s (Fasko, 2001). Using cases in teacher education can potentially support teacher educators in creating authentic learning environments (Koury et al., 2009). Through cases, pre-service teachers (PTs) can comment on situations they cannot see at school (Kolodner, 2006). In addition, PTs develop higher-order thinking and reflection skills by reading and discussing complex, real-life cases (Butler et al., 2006). Therefore, cases are effective tools used in teacher education so that PTs can understand the teaching process's complexity and their students (Butler et al., 2006; Mayo, 2004). Cases serve as a reflection of the theoretical knowledge into practice, imitating the real classroom environment, which enables PTs to think like teachers and to be active, as well as supporting their meaningful learning (Ertmer & Russell, 1995; L. Shulman, 1992; Sudzina, 1999). The positive effects of using cases in teacher education have led many researchers (e.g., Shulman, 1991) to this field. Another benefit of PTs working on cases is that they notice student thinking (Tekin-Sitrava et al., 2021).

Teacher Noticing about Student Thinking

Although researchers working in the field of mathematics education emphasize that many factors affect an effective teaching process, the foremost of these factors is that teachers should keep students' thinking at the center of teaching and notice this thinking (Mason, 2002; Jacobs et al., 2010; Jacobs & Spangler, 2017) because the teacher's ability to notice has the potential to affect the instructional decisions directly related to the student's learning (Mason, 2002). This indicates that the ability to notice is "a process rather than a static category of knowledge" (Sherin et al., 2011, p. 5). With the understanding of the importance of the teacher having this skill, many studies have been conducted and it has been tried to reveal what this skill includes (Jacobs et al., 2010; Star & Strickland, 2008; van Es, 2011). For example, Van Es and Sherin

(2002) evaluated noticing in terms of attending (identifying important situations in the classroom) and interpreting (associating these selected situations with learning and teaching principles and using what is known about context to reason about classroom interactions). Kaiser et al. (2015), on the other hand, defined teacher noticing within the framework of three main components; "(a) Perceiving particular events in an instructional setting, (b) Interpreting the perceived activities in the classroom, (c) Decision making, either as anticipating a response to students' activities or as proposing alternative instructional strategies" (p. 374). Stockero et al. (2017b) classified noticing as noticing within a case and noticing among cases. In noticing within a case, teachers/PTs are given a sample of a student's thinking and are asked to examine it.

In contrast, in noticing among cases, teachers/PTs choose important events in class videos and discuss what they have noticed. On the other hand, Jacobs et al. (2010) defined the noticing skill by focusing on students' mathematical thinking. Stating that this skill consists of three main components, they expressed these components as attending, interpreting, and deciding how to respond. While attending is defined as paying attention to student thinking, interpreting is expressed as evaluating this thinking. In the deciding how to respond component, the focus is on responding to these interpreted student thinking (Jacobs et al., 2010). In their study, Jacobs et al. (2010) asked teachers and PTs to study each video clip and written case containing student solutions. Although experienced teachers' noticing skills provide more robust evidence, it has been revealed that there are various difficulties in noticing the skills of both teachers and PTs. They also draw attention to how the components of noticing (attending, interpreting, and deciding how to respond) are related. In other studies (e.g., Sánchez-Matamoros et al., 2019; Schack et al., 2013), attention was drawn to the relationships between these components and the difficulties experienced by PTs.

Researchers also recommend that noticing be examined in a content-specific manner (e.g., Jacobs & Empson, 2016; Nickerson et al., 2017; Walkoe, 2015). One of these contents is statistics. The current study focuses on PTs' skills to notice student thinking in statistical concepts through as-within-cases. In this context, it was considered appropriate to adopt the framework proposed by Jacobs et al. (2010) because Jacobs et al. (2010) have focused on how teachers notice students' thinking rather than interpreting the complex classroom environment where multiple events co-occur (Sherin et al., 2011). This is the first of the points that the current study focuses on. Another point addressed was how PTs' classroom discussions are related to students' statistical thinking and how they changed according to the case. In this context, the framework proposed by Walkoe (2015) was adopted because Walkoe emphasized the social aspect of discussions and identified categories that focus on PTs' noticing skills in discussion environments. Another point emphasized is the importance of focusing on specific learning areas to understand better how teachers notice students' thinking (Walkoe, 2015).

Teacher Noticing about Students' Statistical Thinking

Encountering statistics in all areas of life has caused the knowledge and skills related to this field to be included in curricula (Rossman et al., 2006). At this point, statistics has found a separate place as a distinct learning area in the curriculum. Although mathematics and statistics share some common points, they also encompass significant differences. For instance, their methods of using numbers are different. While mathematics primarily deals with numbers, their operations, generalizations, and abstractions, statistics considers numbers as data associated with context (Gattuso & Ottaviani, 2011). In addition, numbers gain meaning in statistics within the relevant context (Groth, 2007; Rossman et al., 2006). It is emphasized that the teaching of statistics should be structured within the framework of statistical problem solving (Bargagliotti et al., 2020; Franklin et al., 2015). Statistical problem solving consists of four components. The first of these components, formulate statistical investigative questions, covers the process of clearly defining the problem and creating the question/questions that can be answered with the help of data. The second component, collect/consider the data, involves planning for the appropriate data collection and implementing this plan. Deciding on appropriate data collection methods and collecting data according to these methods are two important skills in this process. In the third component, analyze the data, the analysis is carried out by selecting the appropriate methods. Classification, using graphical representations or the measures of central tendency and dispersion are tools used to represent the data to be analyzed. In interpret the results, the last component of statistical problem solving, the results are interpreted and correlated with the initial research question based on the analyses made in the third stage (Bargagliotti et al., 2020).

The realization of this process in the targeted way depends on teachers' competencies in this regard (Langrall et al., 2017). One of these competencies is noticing (Llinares, 2013). However, when the literature is reviewed, it is seen that there are limited studies examining teachers/PTs' statistics-specific skills to notice student thinking (Nagle et al., 2020; Shin, 2020; 2021). For example, Nagle et al. (2020) gave mathematics teachers tasks that included students' answers to the fit line, allowing them to see the correlation and expected them to evaluate, interpret, and explain these tasks appropriately. The results revealed that the teachers generally interpreted student answers evaluatively. In addition, they generally tended to describe how students thought and why they thought this way. Shin (2021), on the other hand, examined the selective attention and knowledge-based reasoning of PTs watching statistics lesson videos. The results showed that, regarding the selective attention of the PTs, they paid more attention to the teacher pedagogy in the video than to the students' statistical thinking. Analysis of the PTs' knowledge-based reasoning showed that they used an interpretative rather than a descriptive or evaluative stance to reason about what they observed. However, the PTs used less statistical information they needed to teach students to reason about their statistical thinking. Instead, they used general pedagogical knowledge and knowledge unrelated to statistical thinking.

Rationale of the Study

The main reason for conducting this study is to investigate how PTs attend, interpret, and respond to students' thoughts on statistical concepts in the case-based teacher training environment (CBT) because successful teachers can analyze the events that occur in the classroom (e.g., student responses) (Leong, 2012). However, PTs often encounter theoretical knowledge in their undergraduate education and may not have the opportunity to gain sufficient experience in terms of how this theoretical knowledge is reflected in practice (Darling-Hammond, 2006). This may result in "practice shock" when PTs enter real classroom environments (Ball, 2000; L. Shulman, 1992). Another reason for this shock is that every situation in practice is unique, and there is no recipe for how a teacher should respond (Helleve et al., 2021). Therefore, PTs should encounter rich context-based situations that have been experienced/are likely to be experienced before starting the profession (Argyris & Schon, 1974). The fact that teachers should be prepared for possible student thinking that they may encounter in the classroom is directly related to their preparation before starting the profession. At this point, analyzing cases can be an alternative (Heitzmann, 2008; Helleve et al., 2021; Merseth, 1996). Korthagen (2011) argues that PTs may more readily accept the value of theory-based knowledge when faced with the practical dilemmas represented in cases and then engage with theoretical ideas. Thus, cases were used in the current study. The names case-based pedagogy /instruction/teaching/method are used in the literature (Gravett et al., 2017; Heitzmann, 2008; Mayo, 2002). The current study uses the term "case-based teacher training environment (CBT)."

The features of the cases used in the CBT are important for creating effective learning environments (Sowder, 2007). Using cases in teacher training is recognized as a promising instructional method for creating authentic learning environments (Koury et al., 2009; Levin, 2001). Heitzmann (2008) argues that cases should be a central component of teacher education programs because cases provide PTs with many opportunities and strategies to understand the events that occur in schools and classrooms (Helleve et al., 2021). Merseth (1996) emphasized that the use of cases in teacher training is supportive in terms of encountering complex teaching situations, seeing examples of real situations, and reflecting on real situations because the use of cases in teacher training provides an opportunity to understand the multifaceted nature of teaching and learning (Tippins et al., 1999). In this process, PTs develop higher-order thinking and reflection skills by examining the cases in depth (Butler et al., 2006). Furthermore, it facilitates a deeper understanding of the focused subject and supports lifelong learning (Kolodner et al., 2003). This makes creating meaningful connections between theoretical and practical knowledge easier (Stanley, 2019). It is stated that PTs gain the ability to think like a teacher while working on cases (Gravett et al., 2017; Shulman, 1992) because researchers state that the skill to notice can be learned through various supports (e.g., teaching experience, professional development) rather than being an innate ability (Jacobs et al., 2010; van Es & Sherin, 2008). Another advantage of PTs working on cases is that it allows noticing student thinking rather than focusing on unnecessary complexities in the classroom environment (Stockero et al., 2017a). Attention is drawn to the fact that PTs' noticing skills do not naturally develop (Stockero et al., 2017b). This underscores the significant importance of organizing teacher preparation programs to enable PTs to understand what and how students think in a mathematical situation (Stockero et al., 2017a).

Another reason for using cases is that PTs are required to focus directly on student thinking because, when various situations related to the classroom environment (e.g., through videos) are given, PTs may miss student thinking by focusing on other features such as classroom management and environment (Santagata et al., 2007; Star & Strickland, 2008). Studies emphasize that inexperienced teachers may consider students' behaviors, classroom management, and classroom environment rather than students' understanding (Berliner et al., 1988). When evaluated in this context, focusing on students' written solutions can enable PTs to focus directly on student thinking (Tekin-Sitrava et al., 2022).

The current study was carried out within the scope of the "probability and statistics teaching" course, a compulsory course in the Elementary Mathematics teaching undergraduate program at a state university in Turkey. One of the objectives of this course is to enable PTs to acquire the necessary knowledge and skills regarding student thinking about statistical concepts. Discussing what student thinking might be and ways to overcome students' difficulties based on cases can support the development of skills to attend, interpret, and respond to students' thinking by enabling PTs to reflect on their own and others' ideas, respond and ask questions to others (Sherin & Han, 2004; Sherin &van Es, 2009). Collaborative discussions at this point can support PTs' skill to notice (Walkoe, 2015).

Another focus of the current study is to examine PTs' skills to notice student thinking with statistical concepts. It has been emphasized by many researchers (Schlesinger et al., 2018; Walkoe, 2015) that content-specific noticing skills should be examined. In addition, it is noted that more studies have been conducted on teachers' noticing skills related to different learning areas, including algebra (e.g., Walkoe, 2013), patterns (e.g., Callejo & Zapatera, 2017), numbers (e.g., Jacobs et al., 2010) and proportional reasoning (e.g., Son, 2013). However, studies examining the noticing of teachers/PTs in statistics are limited. In addition, the results have shown that teachers/PTs' noticing skills are weak (Han et al., 2018; Nagle et al., 2020; Shin, 2020; 2021). Researchers' drawing attention to the differences between statistics and mathematics (delMas, 2004; Rossman et al., 2006) makes it an important content area to examine how teachers notice student thinking in statistics classrooms (Shin, 2020; 2021) because in an increasingly data-driven world, knowledge and skills related to statistics have become a core competence not only for researchers who conduct formal statistical analysis, but also for informed citizens who make decisions based on data (Franklin et al., 2015). The current study examined how PTs attend, interpret, and respond to the students' thinking about statistical concepts in a CBT.

Method

A qualitative research approach was employed as it was aimed to examine PTs' noticing of the students' thinking about statistical concepts through cases in the CBT and to make inferences about them because qualitative research allows researchers to observe the investigated situation in its natural environment and answer what it means (Merriam, 2009).

Participants and Context

The participants of this study are 25 third-year students (19 female, 6 male) taking the "Probability and Statistics Teaching" course in the sixth term of the Elementary Education Mathematics Teaching Undergraduate Program of a state university in Turkey. PTs who complete this program are granted the title of "middle school mathematics teacher" and can teach mathematics to middle school students (11-14 years old) in public and private schools. The study was carried out within this course because, in the content of the course, the main focus is on student thinking on statistical concepts and how these concepts are taught, which are directly related to this study. The course was conducted for 14 weeks with three class hours each week. The PTs were asked to get into groups of 3-4. Statistical concepts and how to teach these concepts were discussed in class weekly during the course.

Nature of Cases

In the planning process of the study, firstly, students' ways of thinking regarding statistical concepts were determined based on the literature and listed in Table 2. Then, based on students' thinking, cases were created by the researcher. Some student responses in the cases (cases 1, 4, 5, and 6) were based on actual student responses, while other student responses (cases 2, 3, 7, 8, and 9) were constructed using hypothetical student responses. The prepared cases were sent to two experts working in this field, and they were asked to evaluate them. In light of the feedback received, the cases were finalized. The finalized cases were given to the PTs during the lesson, and they were asked to evaluate them. Sample cases are presented (See Figures 2, 3, 4, and 5).

Data Collection

This study's data collection process included reading, evaluating, and interpreting the cases containing student thinking regarding statistical concepts (Figure 1). The PTs read each case, discussed it for 15-20 minutes as a group, and wrote a report. Afterward, the whole class discussed the cases. This process is shown in Figure 1.

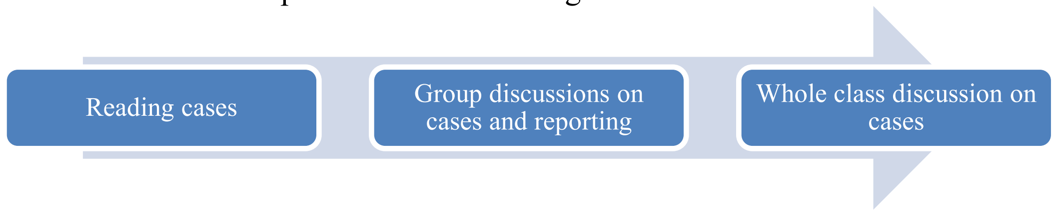


Figure 1. Data collection process

The researcher did not interfere with the explanations made by the PTs about their answers and just guided the discussion. The questions in the cases were shaped based on the framework of Jacobs et al. (2010) (Table 1).

Table 1

Questions Included in the Cases

Question number	What it is related to	Question
1st question	Student thinking	Evaluate the student's knowledge. Explain in detail how the student thinks.
2nd question	Reasons for student thinking/difficulty	Why do you think this student/these students answered/solved the question in this way?
3rd question	Suggestions to overcome student thinking/difficulty	What can be done to overcome this difficulty that your student is experiencing? How would you respond to such a student?

The first question explored whether PTs pay attention to students' thinking and, if so, how they attend to it. The component of "attend" proposed by Jacobs et al. (2010) also serves the same purpose. The second question aimed to uncover how PTs interpret students' thinking and whether they examine the underlying reasons for this thinking. The component of "interpret" in the framework proposed by Jacobs et al. (2010) was also associated with this component, as it aims to achieve a similar goal. The final question aimed to determine PTs' challenges regarding students' thinking and what solutions they suggested to overcome conceptual misconceptions. This was associated with the "how PTs respond" component based on students' thinking, as Jacobs et al. (2010) proposed. A similar process was carried out for all the cases. The reading of each case by the groups, the discussion and reporting of the groups on the case, and the class discussion on the case took between 2 and 3 class hours. The whole process was video-recorded. In addition, the discussions of each group were audio-recorded.

Table 2

Cases, Content, and Sources

Cases	Title	Content	Sources
Case 1	Research question	Features of a statistical research question	Arnold & Franklin, 2021; Arnold & Pfannkuch, 2018; 2019
Case 2	Data collection	Considering the representation of the population by the sample and the randomness when selecting a sample	Topan, 2019; Watson, 2006
Case 3	Data collection	Evaluation of the sample selection method and sample size together	Topan, 2019; Watson, 2006
Case 4	Data representation-graphs	Understanding the concept of frequency and associating it with a bar graph	Sorto, 2004
Case 5	Data representation-graphs	Constructing a graph based on more than one criterion	Tortop, 2011
Case 6	Data representation-median	The meaning of the concept of median	Barr, 1980; Mercimek, 2013
Case 7	Data interpretation	Comparison of the variability of datasets presented with	Bjornsdottir, 2012; DelMas et al., 2007; Gürel, 2016; Park, 2012

		histogram-datasets for different groups	
Case 8	Data interpretation	Comparison of the variability of datasets presented with histogram-datasets for different groups	Bjornsdottir, 2012; DelMas et al., 2007; Gürel, 2016; Lancaster, 2007; Sevimli, 2010
Case 9	Data interpretation	Comparison of the variability of datasets presented with histogram-dataset for a single group	Bjornsdottir, 2012; DelMas et al., 2007; Gürel, 2016

Data Analysis

Jacobs et al.'s (2010) framework was used to analyze PTs' skill of noticing student thinking. The unit of analysis in the current study was the discussion of the cases by the PTs divided into groups of 3-4 and their statements and explanations in their reports. In this framework, the components defined for noticing are defined with the level of evidence. The attend component is categorized as evidence and lack of evidence; the interpret and how to respond components are categorized as robust, limited, and lack of evidence. These pre-defined components and indicators led to deductive coding (Patton, 2002). While analyzing PTs' comments, explanations, and evaluations for each case, a more detailed categorization was required to consider how to respond to components' indicators. To the indicators of the attend component, alongside no evidence and robust evidence, partial evidence was also added because it was found that PTs could describe students' thinking accurately in some cases. However, their descriptions were either superficially associated with statistical concepts, or they identified without referencing students' thinking. Moreover, to the respond component, weak evidence was also added alongside no evidence, partial evidence, and robust evidence. Sometimes, PTs overlooked students' thinking and provided general responses, suggesting procedural understanding and memorization-based approaches. The weak evidence indicator was added to describe this situation. These changes were included in the data analysis, and deductive and inductive analyses were used (Patton, 2002). Each case the PTs worked on was analyzed in this direction and coded according to the noticing skills in Table 3.

In addition, Walkoe's (2015) categories were used to analyze the classroom discussions conducted by PTs. In this categorization, classroom discussions are defined as sustained and disjoint. In sustained discussions, participants listen and respond to each other, so the discussion focuses on a comment or suggestion. Thus, participants develop a new idea together by making a detailed analysis of the same topic. Participants share new ideas in disjoint discussions unrelated to previous comments or suggestions. In this case, comments from different participants do not focus on one topic in detail (Walkoe, 2015). In the classroom discussion on cases, the unit of analysis was a sentence, a paragraph, or a dialogue to capture the connection between the PTs' interpretations. A similar process was carried out in the classroom discussions of the PTs on each case, and the discussions were coded as sustained, disjoint, or both sustained and disjoint.

Triangulation was used to ensure the validity of the study. The two researchers coded the data independently, and then the inter-coder agreement percentage was calculated. The inter-coder agreement percentage was found to be 88%. The points of disagreement were discussed until a consensual coding was agreed. In addition, to ensure the reliability of the study, all the processes of the study (e.g., data collection, context) were explained in detail and direct quotations supported the results.

Table 3
Framework of Analysis on PTs' Noticing Skills

	No evidence		Partial Evidence	Robust Evidence
Attend	Identifies student thinking wrongly. Cannot identify student thinking		Can accurately identify student thinking but relates these identifications superficially to statistical concepts or identifies student thinking without reference.	Accurately identifies student thinking.
Interpret	Interprets student thinking wrongly. Does not interpret student thinking		Indicates the state of understanding or misunderstanding of student thinking.	Explains the state of understanding or misunderstanding student thinking with its reasons
	No evidence	Weak evidence	Partial Evidence	Robust Evidence
Respond	Cannot provide a suggestion for a solution.	Gives general answers by ignoring student thinking. Suggests approaches based on procedural understanding and/or memorization to address the student's difficulties.	Tries to expand student difficulties or helps students consider alternative approaches. Suggests approaches based in part on conceptual understanding to address student difficulties	Tries to expand student difficulties and helps students consider alternative approaches. Suggests approaches based on conceptual understanding to address student difficulties

Findings

The findings were arranged to reveal how the PTs attended to, interpreted and responded to student thinking in the cases. In addition, how the PTs discussed these cases as a class, and the depth of their discussions were examined. In addition to the similarities/differences, the difficulties experienced by the PTs regarding their noticing skills in different cases are presented. The findings regarding how the PTs noticed students' thinking in the cases are presented in Table 4.

It was observed that the PTs did not attend to student thinking eight times. However, they were expected to do so in all the cases, and although they could identify student thinking correctly 21 times, these identifications remained superficial. The PTs were able to identify students' thinking 16 times correctly. Although the PTs were able to identify students' thinking correctly, it can be said that these identifications remained superficial. While the PTs could not interpret students' thinking eight times, they could explain students' thinking or misunderstanding 25 times. They were able to explain students' thinking and misunderstanding 12 times. It can be said that the PTs mostly interpreted students' thinking or misunderstanding. It was observed that the PTs could not offer suggestions for overcoming student difficulties five times, and they either ignored students' thinking and gave general answers or suggested approaches based on procedural understanding and memorization 12 times. While the PTs proposed approaches based on partial conceptual understanding 17 times to overcome student difficulties, they were able to offer suggestions based on conceptual understanding to overcome student difficulties 11 times. In light of these findings, it can be argued that the PTs mostly suggested approaches based on partial conceptual understanding to overcome student difficulties.

The findings are presented under four headings. First, examples from the cases where the PTs mostly had robust and partial noticing skills are presented. Secondly, examples of the cases that presented mainly no or weak evidence and where the PTs had difficulties are presented. Thirdly, the findings regarding the differences in the noticing components of the PTs were discussed, although they were revealed based on similar content. Finally, although the PTs mainly had robust and partial evidence in attending and interpreting, examples of the cases that mainly gave no evidence and weak and partial evidence in responding are given.

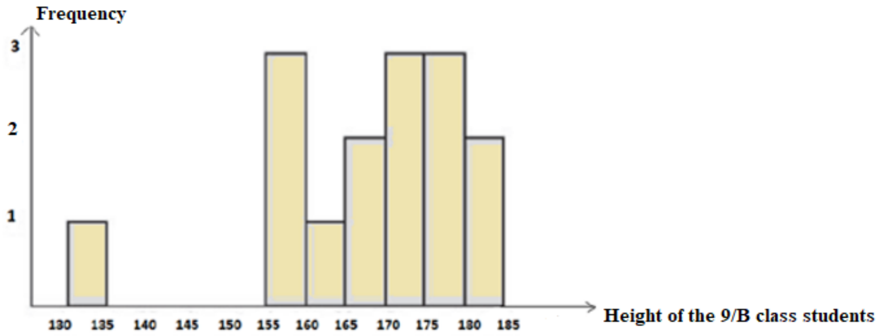
Cases where the PTs Mostly had Robust and Partial Noticing Skills

Cases where the PTs had more partial and robust evidence in their noticing skills for all three components were determined as cases 2, 3, and 4. These cases are related to data collection (Case 2 and 3), title, and data representation-graphs (Case 4). For example, a section from the classroom discussion of the PTs on Case 4 is given below.

Table 4
Frequencies of Evidence Regarding PTs' Noticing Skills

Cases	Subject	Attend			Interpret			Respond				Depth of the discussion
		No evidence	Partial evidence	Robust evidence	No evidence	Partial evidence	Robust evidence	No evidence	Weak evidence	Partial evidence	Robust evidence	
Case 1	Features of a statistical research question	1	4	0	1	4	0	2	0	3	0	Disjointed
Case 2	Considering the representation of the population by the sample and the randomness when selecting a sample	1	2	2	2	1	2	0	2	1	2	Sustained
Case 3	Evaluation of the sample selection method and sample size together	1	0	4	1	1	3	1	1	2	1	Sustained
Case 4	Understanding the concept of frequency and associating it with a bar graph	0	3	2	0	3	2	0	0	1	4	Sustained
Case 5	Constructing a graph based on more than one criterion	3	0	2	3	0	2	2	1	1	1	Disjointed
Case 6	The meaning of the concept of median	0	4	1	0	4	1	0	3	2	0	Sustained
Case 7	Comparison of the variability of datasets presented with histogram	0	3	2	0	4	1	0	1	3	1	Sustained-Disjointed
Case 8	Comparison of the variability of datasets presented with histogram	1	2	2	0	5	0	0	1	3	1	Sustained-Disjointed
Case 9	Commenting on the data set presented with the histogram	1	3	1	1	3	1	0	3	1	1	Sustained-Disjointed
Total		8	21	16	8	25	12	5	12	17	11	

The graphic below shows the information about the height of the middle school 9/B class students.



Suppose you ask your 8th grade students how many children are shown in this graph. And let's assume that one of your students answered "**7 children are shown**".

- Evaluate the student's answer to the question. What do you think the student knows and does not know? Explain in detail how the student thinks.
- Why do you think this student answered the question in this way?
- Suppose you are this student's teacher. What can be done to overcome this thinking that your student is experiencing? How would you respond to such a student?

Figure 2. Case 4

When the PTs worked on the cases, partial and robust evidence in the noticing components drew attention as seen in Table 4. Below is a section from the class discussion on the case, which deals with students' difficulty in understanding the concept of frequency and associating it with the bar graph.

Researcher: What do you think about this answer of the student? How would you evaluate it?

Group 2: The student does not know how to read the bar graph. He/she doesn't know that the values on the y-axis show the number of people. He/she thinks each bar in the graph represents one person.

Group 4: So, we can say that the student does not know how to read graphs (based on the interpretation of Group 2.

Group 3: Here, besides not knowing how to read graphs, the student may not know the meaning of the word "frequency". Since he/she did not know the meaning of the numbers represented by the bars, he/she answered the question by directly counting the bars themselves.

Group 6: We think that since he/she did not know the meaning of the word "frequency", he/she may have thought that he/she should count the bars one by one. So, he/she thinks that each bar represents a student.

Group 2: The student did not take into account what the values on the y-axis mean. Here, each bar shows not how many people there are, but how many people are at that height. Actually, the side that he/she should look at for the number of people is the y-axis.

Group 3: The student has misconceptions about the concept of frequency. He/she did not focus on the numbers on the y-axis, as he/she did not know that frequency refers to how much of a multiplicity or object is represented. Instead, he/she chose to count the bars one by one. At the same time, he/she may have thought that there could be one student of each height and therefore did not feel the need to look at the y-axis.

Researcher: So what can we do as teachers? How can we overcome this difficulty experienced by the student?

Group 6: The student read by ignoring the units on the axis. In order to overcome this difficulty, I tell my student to read the names of the axes of the given graph again. I say that while the vertical axis expresses the frequency, the horizontal axis expresses the height of the students. For example, by choosing one of the bars, I want the students to choose the range of those with a height between

155 and 160 cm from the axis that expresses their height and look at its correspondence on the vertical axis. When my student looks at the graph, he/she will see that it corresponds to 3. I want him/her to read the graph here. If he/she can't read, I repeat the sentence "There are 3 students with a height between 155 and 160 cm" by stating that the vertical axis represents the frequency. I then ask my student to do this reading for each given bar. Finally, my student will be able to count the number of students specified in each bar. When she/he completes this part correctly, I ask him/her again how many children are shown in the graph.

Group 3: We thought the same. In addition, we think it is useful to emphasize the concept of frequency at the beginning.

Group 1: We thought like this. First, we show the student the axes and ask their meaning. If he/she thinks that each bar represents one person, we ask why all the bars are not equal and thus create a contradiction. Then we explain the topic and the meanings of the axes and reinforce them with activities.

Group 2: For this, we can first give simple examples to the student. For example, we can create a graph consisting of one or two bars and have the student read this graph. Then we move on to this graph. Thus, the student will understand the importance of the values given in the x and y axes and will see depending on what the number of bars changes.

As seen in the opinions of the PTs above, the discussion started with interpreting the students' thoughts on the given Case 4 by Group 2. Here, Group 2 was able to attend to the student's difficulty and presented robust evidence of the student's state of understanding and misunderstanding. In the flow of the discussion, Group 4 provided partial evidence as they stated that the student thought was wrong and made partial comments on his/her thinking style. Group 3 provided robust evidence as they interpreted the student's thinking and reasons for his/her thinking wrongly. Groups 1 and 6 presented robust response evidence, although they partially addressed and interpreted the student's difficulty. Group 1 offered suggestions on how to overcome student difficulties by creating cognitive contradictions in students. At the same time, Group 6 suggested that besides the meaning of the axes in the given graph, the meaning of the bars and the data in the graph should be read one by one.

When the discussions conducted by the PTs were evaluated in terms of the depth of the discussion, it was seen that while attending to and interpreting student thoughts, they made statements that supported each other or made comments that added their thoughts to each other. Group 2 stated that the student did not know what the frequency values on the y-axis meant. In addition to the interpretations of this group, Group 4 expressed an opinion that he/she did not know how to read graphs. Group 3, agreeing with the interpretation of Group 4, made a comment that drew attention to the student's lack of knowledge about the meaning of "frequency." Groups 6, 2, and 3 stated that the lack of knowledge about this concept may be the reason for the student's difficulty. It was seen that the groups used expressions that supported each other to overcome the difficulty experienced by the student. Group 6 expressed what the graph axes represented and that he/she would focus on reading the graph. In addition to the interpretation of Group 6, Group 3 pointed out that the concept of frequency should be emphasized. To emphasize the importance of the concept of frequency, Group 1 drew attention to the fact that they might create a contradiction. Group 2 stated that by having students read graphs containing more straightforward examples, they could focus on what this graph meant. The depth of the discussions was coded as sustained since the PTs listened to each other in their interpretations, used expressions that supported their interpretations, and tended to analyze the focused student thoughts in detail.

Cases where the PTs mostly had Difficulties, and There was mainly No Evidence or Weak Evidence of Noticing Skills

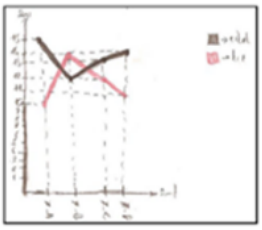
It was revealed that the PTs' noticing skills regarding some cases provided less evidence. Case 1 and Case 5 can be shown as examples of this. In Case 1, the PTs could not provide robust evidence for all three noticing components (attend, interpret, respond). In Case 5, it was revealed that the PTs presented no evidence in all the noticing components.

Table Male and female students in 7th grade classes

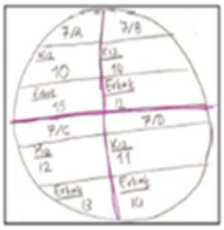
	Female	Male
7-A	10	15
7-B	14	12
7-C	12	13
7-D	11	14

In the table above, the number of male and female students in 7th grade classes in a school is given. Draw a graph using the information provided.

Beliz Teacher asked her students the above question. Below are the answers given by her two students, Bengi and Sude.



Bengi's answer



Sude's answer

- Evaluate the answers given by the students to the question (separately), what do you think the students know and do not know? Explain in detail how students think.
- Why do you think these students may have answered this question in this way?
- Suppose that you are the teacher of this student. What can be done to overcome this difficulty that your student is experiencing? How would you respond to such a student?

Figure 3. Case 5

For example, a section from the classroom discussion of the PTs regarding Case 5 is given below.

Researcher: What do you think about this answer of the student?

Group 3: Bengi actually drew a correct graph, but if she had drawn a bar graph, she would have made a better comparison between them. Sude, on the other hand, tried to draw a pie graph, but since the available data included two genders, they were not suitable for this graph.

Group 6: Bengi did not know that it would be more appropriate to draw a bar graph for this question. Bengi did not know that the line chart should be used when the change of data obtained from a single source at any time was questioned. Sude also did not draw the appropriate graph. He also drew the pie graph incorrectly. Sude summed up the numbers of the students in the classes and since there were 25 students in three of them and 26 in one of them, she thought that she could express each of them as a quarter circle. She did not know that the pie graph should be drawn in situations where the part-whole relationship needs to be questioned. Sude did not know that it would be more appropriate to draw a bar graph for this question.

Group 1: Bengi might have thought that these data could take half-values by confusing categorical data such as gender with continuous data. Sude, on the other hand, might have thought that since there were 4 classes in the data in her hand, she would first divide the pie graph into four equal *parts, and then, since there were two data sets in each class as females and males, they should be divided equally with a horizontal line.*

....

Researcher: So what can we do to overcome the student's difficulty?

Group 2: We would tell Bengi that there she wouldn't have to write up to 10 on the y-axis and that it would be more accurate to start with the starting number of the data and end with the last data.

Group 4: I would explain the usage areas of the graph on various examples and give a few examples for them to solve.

As can be seen in the opinions of the PTs above, Group 3 attended to the students' answers wrongly. Group 6, on the other hand, could attend to student thoughts correctly and provided robust evidence as they could explain the state of understanding and misunderstanding student thoughts with their reasons. Similarly, Group 1 presented robust evidence as they could understand the reasons for student difficulties and interpret the reasons for this understanding. However, when the interpretation made by Group 2 was examined, it was coded as no evidence because it could not offer a suggestion for overcoming student difficulties. Group 4, on the other hand, was coded as weak evidence because it ignored student thoughts and suggested approaches based on procedural understanding and memorization to overcome these difficulties.

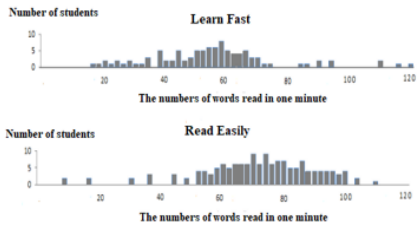
In the interpretations made by the PTs, they mainly shared thoughts that they produced independently from each other. Group 3 made an interpretation that wrongly attended to students' thoughts. Group 6, on the other hand, shared their own thoughts instead of a statement confirming or falsifying the opinion expressed by Group 3. Group 1, on the other hand, did not express an opinion as to whether they agreed or disagreed with the thoughts of Group 6, although there were attempts to interpret student difficulties. Group 2, on the other hand, used expressions independent of the opinions of the other groups and could not offer a solution to students' difficulties. Group 4, on the other hand, attempted to offer solutions by ignoring student thoughts independently of the opinions of the other groups. The discussion that PTs made was coded as a disjoint discussion since they expressed their opinions by sharing new ideas that were not related to previous comments or suggestions.

Cases where the Noticing Components of the PTs Differed although They were given within Similar Contents

Although they were given within similar contents, it was revealed that there were differences in the noticing components of the PTs in Cases 7, 8, and 9. In these three cases, the noticing skills of the PTs regarding student thoughts on the comparison of the variability of the data sets presented with the histogram were evaluated.

Ethem Teacher asked 8th grade students the following question

The numbers of words read in one minute by 100 students taking a learn-fast course with and by 150 students attending a read-easily course are shown below. According to these results, which course is more effective?



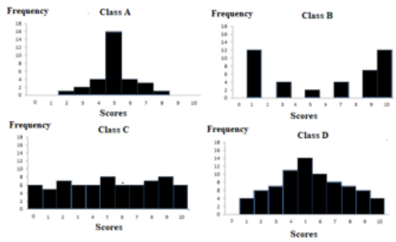
Student comments on these courses are given below.

Buse: The learn-fast course may be preferred to the read-easily course because the students who read the fastest (the students who read the most words per minute) attended the learn-fast course.
Orhan: The read-easily course may be preferred to the learn-fast course because the average word count is higher and the variability in the obtained data set is high for both groups.
Özlem: The two courses are likely to be equal in effectiveness because the range of the data obtained in the two courses is approximately equal.
Nur: We cannot draw conclusions from these data because the number of students participating in the read-easily course is greater than the number of students participating in the learn-fast course.

- ➢ Evaluate each student's answer to the question. What do you think the students know and do not know? Explain in detail how students think.
- ➢ Why do you think these students might have answered the question in this way? What could be the difficulties, misconceptions, and mistakes that the students experienced based on these answers?
- ➢ Suppose you are the teacher of these students. What can be done to help students overcome their difficulties? How would you respond to your students?

Case 7

Hakan Teacher asks the 8th grade students the following question in a lesson on the subject of variability.



Each histogram above shows the scores of students in five different classes on a test where a score of 0 to 10 can be obtained.

Question: Which class would you expect the data from to have a lower variability? Why?

Five students in the class answered this question as follows.
Erkan: Class A because most of the data are close to the mean
Okan: Class B because it has the smallest number of different scores (there are only 6 different scores)
Seçkin: Class C because there is no variation in the scores.
Başak: Class D because it most resembles the normal distribution

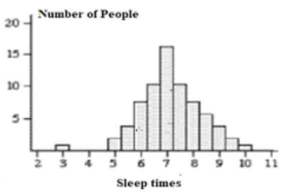
Question: Which class would you expect the data from to have a higher variability? Why?

Five students in the class answered this question as follows.
Serkan: Class A because the differences between the heights of the bars are the smallest.
Furkan: Class B because most of the scores are far from the mean.
Ecem: Class C because it has the highest number of different scores.
Esra: Class D because it has the broadest gap and does not resemble a normal distribution.

- ➢ Evaluate each student's answer to the question. What do you think the students know and do not know? Explain in detail how students think.
- ➢ Why do you think these students might have answered the question in this way? What could be the difficulties, misconceptions, and mistakes that the students experienced based on these answers?
- ➢ Suppose you are the teacher of these students. What can be done to help students overcome their difficulties? How would you respond to your students?

Case 8

In a lesson on graphical representations, Mustafa Teacher asked the 7th grade students the following question:



The graph on the right shows the distribution of sleep times last night for a group of middle school students.

Answers given by four students are as follows:

Ahmet: Bars are from 3 to 10, reaching the highest value at 7 and the lowest value at 10. The highest bar is at 7. There is a gap between 3 and 5
Ezgi: The data set shows a normal distribution with an arithmetic mean close to 7 and standard deviation close to 1.
Burcu: Most of the students did not sleep enough last night but some students slept a lot. However, one student stayed up late and slept very little.
Bahar: The distribution of sleep times has an extreme value at point 3 and is symmetrical. The typical sleep time is 7 hours and sleep time range is 7.

- ➢ Evaluate each student's answer to the question. What do you think the students know and do not know? Explain in detail how students think.
- ➢ Why do you think these students might have answered the question in this way? What could be the difficulties, misconceptions, and mistakes that the students experienced based on these answers?
- ➢ Suppose you are the teacher of these students. What can be done to help students overcome their difficulties? How would you respond to your students?

Case 9

Figure 4. Cases 7,8 and 9

While comparison and interpretation of the distributions of two or more data sets were requested in Cases 7 and 8, it was requested to interpret the distribution for a single data set in Case 9. As can be seen in Table 4, it was observed that the PTs had more no evidence and weak evidence for the attend, interpret, and respond components in Case 9 compared to

Cases 7 and 8. For example, below is the section from the classroom discussion of the PTs regarding student thoughts in Case 9.

Researcher: What do you think about the students' answers? How can we interpret them?

Group 3: Ezgi knows that there is a normal distribution, but she does not know that in a normal distribution, the arithmetic mean and standard deviation must be equal.

Group 6: We don't think the same. In a normal distribution, the standard deviation should not necessarily be equal to the arithmetic mean. We think that Ezgi's interpretation is correct. When we look at the distribution, its arithmetic mean is 7. We can say that its standard deviation is close to 1 and quite close to a normal distribution.

Group 1: We agree with Group 6. In our opinion, Ezgi can comment on the distribution of data and measures of central tendency by looking at the graph.

……

Group 4: The students do not know which measures of central tendency to use for the given data set. Some students do not know how to interpret the data shown in the graph and see the graph just as an arrangement of the bars.

Researcher: What would you do to overcome student difficulties? How would you respond to these students?

Group 3: Since we generally get correct answers, we just re-explain the points they had difficulty understanding and ask them to comment further with a few graphic questions.

Group 2: We have the students read the graph again.

Group 4: We think the same way; we have the students read the graph again.

Group 6: In our opinion, we should make students question their knowledge of what the concept of distribution means. We should also focus on how this concept relates to the concepts of arithmetic mean and standard deviation. For example, we can have students engage in these concepts over a smaller data set. We also discuss the difficulties they faced here. Then we ask them to comment on this distribution.

As seen in the group discussion, Group 3 was not able to attend to student thoughts. It can be said that PTs' over-generalizations about their knowledge (their generalization of a feature of the standard normal distribution to the normal distribution) caused difficulties in attending to student thoughts. Group 6 stated that they did not agree with the opinion expressed by Group 3 and presented robust evidence by stating that Ezgi's thought was correct. Group 1, based on the opinion expressed by Group 6, stated that Ezgi's thought was correct but they only commented on the measure of central tendency in their interpretations. They did not include a statement indicating that the standard deviation is a measure of dispersion. Therefore, it was evaluated as partial evidence. Group 4, on the other hand, interpreted student thoughts with a general point of view and presented partial evidence as they did not attempt to elaborate. It was observed that the PTs mostly offered memorization-based approaches to overcome student difficulties and presented weak evidence, while only one group (Group 6) offered conceptual understanding-based approaches to overcome student difficulties, in other words, presented robust evidence.

As the discussions that the PTs had in attending to student thoughts were in a way that supported or refuted each other's thoughts, they can be defined as sustained. For example, Group 3 attended Ezgi's thought wrongly and Group 6 was able to correctly interpret Ezgi's thought by expressing that they did not agree with the opinion stated by Group 3. Although Group 1 stated that they agreed with the opinion of Group 6, they related their comments superficially to statistical concepts. Although the PTs made interpretations that supported or refuted each other regarding attending to student thoughts, it was observed that the interpretations made by the PTs in interpreting and responding were mostly opinions independent from each other. For example, Groups 2 and 3 presented suggestions grounded on a memorization-based approach regarding student difficulties and made their explanations independently of each other's opinions. However, Group 4 again made a non-conceptual suggestion by referring to Groups 2 and 3. Group 6, unlike the other groups, did not make any reference to the ideas of the previous groups, although it offered conceptual suggestions for overcoming student difficulties. When the discussion was evaluated as a whole, some parts of the discussion were carried out with statements that supported or refuted each other, while some parts continued in the form of sharing independent ideas. Therefore, the depth of the discussion was coded as both sustained and disjoint.

Although the PTs mainly had Robust and Partial Evidence in Attending and Interpreting, There were Some Cases which were dominated by No Evidence, Weak and Partial Evidence in Responding

Although the PTs mainly had robust and partial evidence in attending and interpreting in Cases 3, 6, and 9, it was revealed that they mainly had no evidence, weak and partial evidence in responding. For example, below is the section from the classroom discussion of the PTs about student thoughts in Case 6.

İlke Teacher gave the following question about the concept of median to the 7th grade students as homework.

HOMEWORK

The table below shows the number of cracked eggs in each pack for a total of 21 packs. Calculate the median for this data set.

The number of cracked eggs	Frequency
2	*5*
4	*6*
6	*7*
9	*2*
11	*1*
Toplam	***21***

One day later, she collected the answers of the students. She observed that many students gave the answers 5 or 6.

- Evaluate each student's answer to the question. What do you think the students know and do not know? Explain in detail how students think.
- What could be the difficulties, misconceptions, and mistakes that the students experienced based on these answers?
- Suppose you are the teacher of these students. How would you respond to your students?

Figure 5. Case 6

Researcher: What do you think about the students' answers?

Group 6: When the answer given by the student who gave the answer b)5 is evaluated, it can be thought that the student ignored the number of cracked eggs and focused only on the frequencies and that he/she found the median of the frequency values of 1, 2, 5, 6, 7. Or he/she found the weighted average of these data instead of the median and considered this value as the median because the weighted average of the data is 5. He/she did not know what the data meant. When the answer of the student who gave the answer 6 is evaluated, it can be thought that he/she confused the *median with the mode. He/she may have considered the number of cracked eggs with the highest peak (6) as the answer. He/she did not know what the data meant.*

Group 3: Students gave wrong answers. In addition to the opinions of Group 6, they may have miscounted the data. They may have taken the mean of 6 and 4, or they may have said 6 because 6 is more than 4.

Group 2: Yes, we think like Group 3.

Group 1: In fact, when evaluated generally, students may have given such an answer because they did not know what the concept of median meant.

...

Researcher: So what can we do to overcome student difficulties?

Group 6: Instead of examining what peak value, arithmetic mean and median meant, we can deduce that students only looked at these concepts operationally. In order to overcome these difficulties, we can explain them again.

Group 4: We thought the same thing, and we can explain them again to overcome these difficulties of students.

Group 1: We think that we can explain them again, but while explaining, we need to say that it is important for students to find the medium value by opening the data we have in order for them to find the median.

As seen in the opinions of the PTs above, Group 6 identified student thoughts correctly and explained the state of understanding and misunderstanding student thoughts with reasons. Therefore, it is expressed as robust evidence. Groups 2 and 3, on the other hand, were able to partially justify their misunderstanding in interpreting student thoughts, although they correctly identified student thoughts. Therefore, it is coded as partial evidence. Group 1, on the other hand, presented partial evidence because they made more superficial interpretations of the reasons, although they identified the state of the students' misunderstanding while interpreting students' answers. Group 6, conversely, made a more detailed interpretation of both students' thoughts and the state of misunderstanding. Therefore, they presented robust evidence. However, they suggested an approach based on procedural understanding and memorization to overcome student difficulties. Group 4 adopted a similar responding approach. Group 1, on the other hand, proposed approaches based on conceptual understanding to overcome student difficulties, presenting partial evidence.

When the PTs were evaluated in terms of the depth of their discussions, it was seen that they made statements that supported each other or made interpretations that added each other's opinions while attending, interpreting, and responding. Group 6 interpreted student thoughts in detail and stated the reasons for their misunderstanding. Group 3 interpreted student thoughts more superficially, with expressions supporting the opinions of Group 6. It was observed that Groups 2 and 1 also presented ideas supporting other groups. It was observed that their inferences about overcoming their difficulties progressed similarly. Although their interpretations provided weak and partial evidence in general, it can be said that the groups listened to each other's opinions and supported their interpretations. When the discussion was evaluated holistically, the depth of their discussion was coded as sustained, as they tended to analyze the focused student's thoughts in detail.

Discussion and Conclusion

The current study aimed to examine and understand how the PTs attended, interpreted, and responded to student difficulties related to statistical concepts by allowing them to work on cases related to statistical contents. The study results showed that the PTs made partial evidence about student difficulties regarding all three components (attend, interpret, respond). This was followed by robust evidence for attending and interpreting and weak evidence for responding.

The findings showed that the PTs presented more robust evidence in attending and less robust evidence in interpreting. This result shows that the PTs had difficulties interpreting even though they could attend to students' thoughts. Interpreting skills are the ability to examine what students understand. The teacher needs to identify these understandings in detail to explain student understanding based on student solutions (Jacobs et al., 2010; Sherin & Van Es, 2009). Researchers agree that it is easier to interpret student solutions after attending to these solutions (Tekin-Sitrava et al., 2022). As noted in other studies (e.g., Tekin-Sitrava et al., 2022), more than attending to student solutions is need to interpret their thoughts.

Although the PTs presented more robust evidence in attending than in interpreting, the fact that there was not much difference allows the interpretation that they were more successful in interpreting than in responding. PTs' interpreting skills can be related to the content of their undergraduate courses and their understanding of statistical/mathematical ideas from these courses (Casey et al., 2018; Fernandez et al., 2013; Tekin-Sitrava et al., 2022).

It was observed that the PTs were more successful in attending and interpreting than responding and presented more partial and robust evidence in attending and interpreting. Studies in the literature have shown that PTs' skill to attend is better than their skills to interpret and respond (Gonzalez & Skultety, 2018; Jacobs et al., 2010; Sanchez-Matamoros et al., 2015; Tekin-Sitrava et al., 2022). This is emphasized as a gap between attending, interpreting, and responding skills (Gonzalez & Skultety, 2018). In this sense, it can be said that the findings obtained in the current study differ from the literature. The PTs' ability to present partial and robust evidence in attending and interpreting can be related to their pedagogical content knowledge (Ball et al., 2008; Ballock et al., 2018; Casey et al., 2018). It is emphasized that teachers' ability to notice is directly related to their content, students, and teaching knowledge (Ballock et al., 2018; Casey et al., 2018). In addition, it can be said that PTs' previous experiences and knowledge affect their skill to attend and interpret (Casey et al., 2018; Star & Strickland, 2008). The skill of how teachers respond to student thoughts requires them to make decisions about the following teaching moves based on their interpretation of student thoughts (Jacobs et al., 2010). It was noted that the PTs had more significant difficulties in responding to student difficulties than in the other two skills. It was seen that the PTs mainly offered partial conceptual suggestions, followed by conceptual suggestions and only operational suggestions, and their numbers were very close to each other. It can be thought that PTs taking various undergraduate courses on the importance of conceptual learning during their education and their beliefs about teaching mathematics support this result (Anderson et al., 2005; Russ & Luna, 2013). However, their offering only procedural suggestions or no suggestions brings to mind the lack of knowledge on the part of the PTs or their beliefs about teaching mathematics (Ballock et al., 2018; Casey et al., 2018; Russ & Luna, 2013). Researchers also stated that teachers had difficulty deciding how to respond in a way that would support and expand their students' current understanding (Jacobs et al., 2011).

In addition, the results obtained from the study revealed that PTs' noticing skills differ from one case to another. For example, in some cases (e.g., Cases 2, 3, and 4), the PTs presented more partial and robust evidence for all three components. In some cases (Cases 1, 5), it was observed that they could not mainly provide or present partial evidence. Another remarkable finding is the differences in noticing in different cases, although they are on similar subjects (Cases 7, 8, 9). In Case 9, it was observed that there was more no evidence and partial and weak evidence than in Cases 7 and 8. Finally, although in some cases, the PTs mainly had robust and partial evidence in attending and interpreting, mainly no evidence, weak and partial evidence in responding (Cases 3, 6, 9). In other studies, it was pointed out that there was no pattern in PTs' noticing skills (Kılıç, 2018). The differences between PTs' noticing skills from one case to another can be evaluated under several headings. PTs seeing student difficulties beforehand and having the necessary knowledge about them may affect their noticing skills. Moreover, it can be thought that the difficulty levels of the cases may have triggered the differences in the PTs' noticing skills. In various studies (e.g., Warsauer et al., 2021), it has been emphasized that the difficulty levels of the cases can affect PTs' ability to notice.

In the classroom discussions about the cases, it was noted that the PTs made interpretations supporting or refuting each other in some cases. In some cases, they made interpretations independent of each other and could not focus on the subject. The results allow the interpretation that the depth of the discussions on the cases in which the partial and robust evidence of the PTs' noticing skills can be sustained more. The depth of the discussions on the cases in which there is no evidence and weak evidence of the PTs' noticing skill can be said to be disjoint. It is thought that the fact that the discussions held by PTs have the potential to allow the formation of new ideas (e.g., Girit-Yıldız et al., 2022) is important in terms of their noticing skill. Therefore, creating such learning environments is important for PTs to notice student thoughts because, given that PTs have not started active teaching yet, providing such a learning environment for PTs who are in the teaching profession and who are not in direct contact with students is effective in their preparation for the profession (Alsawaie & Alghazo, 2010).

Suggestions and Limitations

The noticing skill of a teacher is a very important component in effective mathematics teaching, and it is important to support PTs who will be future teachers in learning to notice student thoughts (Star & Strickland, 2008; Van Es, 2011). The current study examined how PTs noticed students' thoughts about statistical concepts. The attention given to the importance of teachers in the effectiveness of statistics education (e.g., Martínez-Castro et al., 2023) is believed to provide significant insights for researchers and educators in uncovering how teachers' noticing skills are related to statistical concepts. The fact that PT's noticing varies in cases related to similar statistical concepts brings to mind the suggestion that these differences should be examined in more depth in future studies. Another result of this study is that although PTs mainly had robust and partial evidence in attending and interpreting, they mainly gave no evidence, weak and partial evidence in responding. All these results can guide researchers to use cases in the teaching process in teacher training programs and to organize them according to these points. In addition, teacher trainers can organize undergraduate courses according to these results. When the results were evaluated, PTs were more successful in attending and interpreting than responding. However, they still have some shortcomings. These findings emphasize the importance of knowledge-based mathematics education courses and practical courses in teacher education programs in revealing and developing PTs' noticing skills (Star & Strickland, 2008; Star et al., 2011). In research on statistics teacher education (Loya, 2008; Souza et al., 2015; Zapata-Cardona & González-Gómez, 2017), one of the main criticisms is the predominant focus on technical aspects and procedural components. In this context, such arrangements to be made in teacher training have the potential to be supportive in guiding PTs' training. Attention is drawn to the importance of arranging teacher training programs in a way that allows PTs to evaluate the statistical situations critically and the reflections of these situations in socio-cultural environments (Ernest, 2015). The results show that the development of PTs' noticing skills should be supported. In future studies, investigations can be carried out to support the development of this skill of PTs.

It should be noted that this study also has some limitations. First, the data obtained from this study were collected within the scope of a course. It should be known that addressing student difficulties within the scope of a single course, PTs noticing these difficulties and their thoughts on interpreting and responding will not solve all students' problems. In addition, it is known that the 9 cases in this study do not contain all the difficulties to be experienced by students on statistical concepts. However, it is still thought that addressing student difficulties is essential when structuring PTs' learning environments related to statistics.

The current study aimed to reveal how the PTs' noticing skills are through the given cases. In future studies, more research enriched with interviews can be done to learn more about the development process of PTs' noticing skills. In addition, it can be said that there is a need for studies on how teacher education programs can be designed and implemented in a way that will allow the emergence and development of PTs' noticing skills because mathematics educators have an ongoing responsibility to design their lessons in a way that will improve PTs' noticing skills (Roth McDufe et al., 2014).

The findings of this study were obtained from PTs attending a teacher education program at a single Turkish university. For this reason, it can be suggested that the study be repeated in different universities in Turkey and across cultures to investigate the effect of mathematics teaching courses on PTs' noticing skills.

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