A Comparative Analysis of Tasks of the 6th Grade Mathematics Textbooks in the USA and Türkiye

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Textbooks are the most tangible resource of learning opportunities offered to students. This study investigated the mathematical task concerning the three dimensions: representation, contextual features, and cognitive demands of the 6th-grade textbook between the USA and Turkiye. The tasks in the learning domains of numbers and operations, algebra, and geometry in the countries' mathematics textbooks were considered. Using content analysis methodology, the Chi-square test results revealed significant differences between the textbooks of the two countries in terms of specific categories in three dimensions. In the three learning domains analyzed, the total number of tasks in the US textbook is more than in the TR textbook. Whereas verbal representations are used intensively in the TR textbook, the tasks in the US textbook show a more balanced distribution of representation types. Although intra-mathematical and low cognitive tasks were more intensively used in the tasks in the textbooks of the two countries, differences according to learning domains were remarkable. The study contributes to the discussions on improving the tasks in mathematics textbooks.
Keywords: Mathematics textbooks, representations, contextual features, cognitive demand

Introduction

Mathematics textbooks are among the most important resources for teaching and learning mathematics worldwide (Fan et al., 2013; Wijaya et al., 2015). The quality of the mathematics textbook directly affects the teaching and learning of mathematics in the classroom (Stein et al., 2007). Mathematics textbooks play a crucial role in bridging between the intended and implemented curriculum (Valverde et al., 2002). The mathematics education community has a consensus that textbooks play a vital role in the teaching and learning of mathematics (Author, 2022). Furthermore, several studies emphasize mathematics textbooks as the main source of knowledge and learning for teachers and students (San Giovanni et al., 2020; Fan et al., 2013; Wijaya et al., 2015). Hwang et al. (2021) stated that students cannot independently acquire mathematical knowledge, skills, and ideas if not presented in textbooks. The close relationship between different learning opportunities offered to students and the role of textbooks in this context has increased the interest in studying mathematics textbooks (Van den Ham & Heinze, 2018). At the same time, analyzing and comparing textbooks can reveal the mathematics learning opportunities countries offer or aim to offer their students (Choi & Park, 2013).

Countries have different education systems as a natural consequence of different factors such as social, cultural, or historical dynamics. Such various factors certainly influence the education system of each country. Each country has unique educational strengths, which can be shared to improve the educational system for any country worldwide. Mathematics is considered one of the most challenging subjects to teach and learn; it is important to share knowledge by comparing and analyzing textbooks among various countries. Further analysis and comparison of textbooks from different countries can provide strong indications about their general educational philosophy and subject-based pedagogical approaches. For example, O'Keeffe (2014) suggested that mathematics textbooks are one of the most important indicators for comparing the quality of countries' education systems. Comparing textbooks among the countries also enables analyses of what students are expected to learn in mathematics. This is because mathematics textbooks are written texts embody the objectives of the mathematics curriculum in the country (Alajmi, 2012; Zhu & Fan, 2006).

The results from analyzing textbooks can play vital roles in the effectiveness of teaching processes, especially in courses that are difficult for students, such as mathematics. Studies (Fan, 2013; Howson, 1995; Stylianide, 2014; Valverde, 2003) also emphasize that textbooks significantly impact mathematics courses' in-class and/or out-of-class practices and comparing and contrasting textbooks between countries likely provide valuable insights. In this respect, analysis of mathematics textbooks likely provides important ideas and insights for both students and teachers. The perception and understanding of mathematics textbooks could be different among the countries. For example, Mullis et al. (2012) stated that almost half of the students in the USA consider the mathematics textbook as the basis of mathematics education; in some countries, this rate exceeds 90%. Textbooks are also recognized as one of the most important criteria for comparing countries' education systems. Since research has shown a link between earning opportunities in mathematics textbooks and learning outcomes (Schmidt et al., 2001; Törnroos, 2005), comparing and analyzing textbooks, likely provide better ideas and suggestions regarding students' mathematics development.

Designing and developing good textbooks requires various elements. It is important to understand that the various aspects of textbooks, such as color, structure, pictures, representations, cognitive demand, contextual features, language, task selection, terminology, etc., can play a crucial role in enhancing the quality of mathematics textbooks (Mainali, 2022). In this study, however, we aim to focus on three aspects of the textbook: representation, contextual features, and cognitive demand. Furthermore, we primarily aim to investigate the mathematical task in conjunction with these three dimensions of the textbook**.** Mathematical tasks are central to the teaching and learning of mathematics (Chapman, 2013). Well-designed mathematical tasks will enhance both the teaching and learning of mathematics. For example, a good task "provide the stimulus for students to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts" (NCTM 1991: 24). The way we design mathematical tasks, taking various elements such as symbol, figure, real-life context, cognitive load, etc., likely changes the dynamics of teaching and learning in the classroom. The nature of the tasks to which students are exposed determines what students learn (NCTM 1991).

Theoretical Framework

The theoretical framework for the study revolves around three main themes: (i) multiple representation, (ii) contextual features, and (iii) cognitive demand. The study focuses on mathematical tasks, one of the most important components of mathematics textbooks worldwide. Although mathematical tasks have a wide range of meanings, Stein & Smith (1998) define a task as "a part of a classroom activity designed to develop a mathematical idea" (p. 269). Tasks in textbooks can be used in both teaching and assessment processes. Furthermore, mathematical tasks require finding answers without focusing on the solution process.

Representation

Representation is one of the important aspects of textbooks that has been attracting growing interest in mathematics education. Representations are fundamental to mathematical thinking, and most textbooks today use various representations to promote mathematical understanding (Janvier, 1987). The meaning and interpretation of representation are not uniform (Mainali, 2021). Typically, representation is a sign or combination of signs, symbols, characters, objects, diagrams, or graphs (Goldin & Shteingold, 2001).

The abstract nature of mathematics makes it necessary to access mathematical ideas only through representations (Pape & Tchoshanov, 2001). Goldin (1987) defines mathematical representations as tangible products embodying mathematical ideas. Representation as a core competence in mathematics has been the focus of research in mathematics education. Many international educational organizations and educators use it to indicate students' mathematical abilities (Author 2021). More than any other subject, mathematics is based on the representation system. Mathematics teachers barely think of teaching mathematics without using some representation as a pedagogical strategy. In 2000, The role of representation in mathematics is supported by the National Council of Teachers of Mathematics (NCTM), which includes representation as one of the process standards. Representation acts as a tool for manipulation and communication and for conceptual understanding of mathematical ideas (Zazkis & Liljedahl, 2004). Representations also play an essential role in teaching and learning mathematics because they help teachers and students grasp the abstract notion of mathematics (Roubicek, 2006). Various researchers contend that representation is important, and its use is fundamental in teaching and learning mathematics (Arcavi, 2003; Janvier, 1987; Goldin, 1987).

Contextual Features

In the literature, although there is no consensus about the contextual characteristics of tasks in textbooks, they are generally categorized as non-application (intra-mathematical) and application context. A non-application task is related to the mathematical world rather than the real world. In contrast, an application task is related to or designed based on the context of a real-life world situation (Zhu & Fan, 2006). The analysis in our study is based on the classification of Gracin (2018), who further divides application tasks into Realistic and Authentic. The data in realistic contexts are for a specific purpose. On the other hand, authentic contexts are created from student data based on real-life situations or their daily lives (Zhu & Fan, 2006, p. 614). Although such contexts are generally considered problematic for students, they attract more attention and contribute to their understanding of the real world.

Cognitive Demand Levels

Smith & Stein (1998) developed a taxonomy called Levels of Cognitive Demand, which categorizes mathematical tasks based on the level of student thinking required. This taxonomy divides tasks into four types: Memorization, procedures without connections, procedures with connections, and doing mathematics. Memorization requires students to recall previously memorized facts, formulas, rules, and definitions without following procedures or establishing connections. These types of tasks are direct. Procedures without connections require students to produce solutions by following instructions or recalling procedures. They are unambiguous, with some conceptual connections, but without requiring explanations. Procedures with connections emphasize a deeper understanding of mathematical concepts. They can be represented in various ways, such as through pictures, graphs, or equations. While there may be specific procedures to follow, these procedures require making connections between multiple concepts (McCormick, 2016). Doing mathematics involves non-algorithmic approaches to investigate and understand mathematical relationships. These mathematical tasks demand significant cognitive effort as they are neither predictable nor well-rehearsed. Memorization and procedures with connections are considered lower-level cognitive demands as they emphasize students' ability to remember knowledge or their ability to follow procedures, whereas procedures with connections and doing mathematics are considered higher-level cognitive demands (Stein et al., 2009); they focus on students' conceptual understanding and establishing connections between mathematical concepts.

Statement of Problem

This study compared tasks in mathematics textbooks in the United States of America (USA) and Türkiye (TR). International assessments such as the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) show that the mathematics performances of students from these two countries with different education systems are not close to each other and are not at a high level (Mullis et al., 2020; OECD, 2023). While acknowledging that these low-performance levels may be due to many factors, Li (2000) points out that the mathematical problems presented to students in textbooks play an important role in these performances. Since textbooks are a potentially implemented curriculum, this study on the analysis of mathematical tasks provides evidence for the teaching opportunities offered to students in these two countries.

In parallel with the studies on educational systems, studies on the comparative analysis of mathematics textbooks are increasing. In the design of problems or tasks in mathematics textbooks, the dimensions of representation types, contextual features, and cognitive demand levels are particularly prominent (Charalambous et al., 2010; Fan & Zhu, 2000; Fuson et al., 1988; Glasnovic Gracin, 2018; Hwang et al., 2021; Li et al., 2009; Sianturi et al., 2021; Yang & Lin, 2015; Yang & Sianturi, 2017; Zhu & Fan, 2006). Studies comparing the USA and Turkish education systems from a mathematics education perspective mainly focus on comparing mathematics curricula or textbooks (Avcu, 2019; Erbas et al., 2012; İncikabı & Tjoe, 2012; Kar et al., 2018; Keskin, 2018; Özer & Sezer, 2014; Özer & Yaman, 2021; Reçber, 2012). While these studies on textbooks focus on content and problems or tasks, there is a limited study on a holistic analysis of representation types, contextual features, and cognitive demand levels.

Designing the tasks in the textbooks according to the types of representations, contextual features, and levels of cognitive demand will allow the mathematics textbook to be used more effectively. Although it is widely consensus that textbooks should be designed according to the dimensions, there is a gap in the literature on the extent to which these features are included and compared in mathematics textbooks. This study provides a framework for comparing mathematics textbooks in terms of multiple representations, contextual features, and cognitive demands. In this context, the following research questions are addressed.

i. Which types of representation are used in the tasks in the USA and TR mathematics textbooks?
ii. Which contextual features are used in the tasks in the USA and TR mathematics textbooks?
iii. What are the cognitive demand levels of the tasks in the USA and TR mathematics textbooks?

Method

This study used a content analysis method to compare the mathematical tasks in 6th-grade mathematics textbooks between the USA and TR. We focused on analyzing three aspects of mathematical tasks: types of representations, contextual features, and levels of cognitive demand. All the items considered in this study are characterized as tasks. The word 'task' includes all situations that need to be answered regardless of the solution in the textbook. This includes exercises, activity questions, problems, etc. The study included all tasks on familiar topics in the textbook between the USA and TR.

Grade and the Textbook Selection

While TR's education system is centralized, there is a decentralized education system in the USA. Since TR has a mathematics program developed and implemented at the national level, mathematics textbooks are inspected by the Ministry of National Education, while in the USA, states are independent in developing their standards and curriculum. However, most states have used Common Core State Standards (CCSS) for mathematics. Each school district can choose a mathematics textbook for the curriculum based on the state's standards. Thus, the usage of mathematics textbooks varies from district to district. However, some mathematics textbooks have been used more than that of others. An attempt was made to select mathematics textbooks that best represent these two countries. Thus, the selection of textbooks in TR is straightforward and clear since one or two types of textbooks are primarily available. However, there are varieties of textbooks available in the USA. For the research, we have selected a textbook published by EdGems. The textbooks are based on Common Core State Standards of Mathematics, a newer version of mathematics textbook available in the market. Another main reason for selecting this textbook is that it primarily focuses on middle school mathematics.

Sixth grade is one of the most important grades in mathematics. Students learn various mathematical concepts important for high school mathematics. The content of sixth-grade mathematics is not as simple as elementary level but not too complex. The sequence and progression of sixth-grade mathematics are more consistent worldwide than those of high school mathematics. However, the content and mathematics courses taught in high school in the USA and TR are not similar. The details of the selected textbooks are provided in Table 1.

Table1

Textbooks analyzed in the study.

Country	Publisher	Textbook	Chapter Title	Pages
USA	EdGems	Maths 6th grade textbook	Ratios	2-19
			Multiplying and Dividing Fractions	69-84
			Variables and Expressions	94-106
			Area of Polygons	177-201
TR	Ministry National Education	Maths 6th grade textbook	Ratios	117-128
			Multiplying and Dividing Fractions	70-90
			Variables and Expressions	130-138
			Area of Polygons	170-198

Data Analysis

The common learning domains and topics of the 6th-grade level mathematics textbook in the USA and TR were identified. Furthermore, the detailed content domain of each textbook commonly used in these two countries was assessed, and common mathematics topics were identified. Both textbooks identified numbers and operations, algebra, and geometry as common learning domains. Next, more specific content in both textbooks was identified. For example, multiplication-division, ratio and proportion, fractions, algebraic expressions, and area measurement were identified as common topics in both textbooks.

To gain a deeper insight into mathematical tasks in the different learning domains, we focused on the characteristics of the mathematical tasks, including representation types, contextual features, and cognitive demands. These three dimensions are labeled micro by Li et al. (2009) and vertical by Charalambous et al. (2010). The micro-level analysis focuses on how the textbook presents mathematical content for teaching and learning. In contrast, the vertical analysis concerns how the textbook conveys mathematical knowledge to students, its requirements for students, and the connections it provides. We used Zhu & Fan's (2006) framework for representation types and Glasnovic Gracin's (2018) framework for contextual features. The classification of the cognitive demand levels of mathematical tasks in textbooks was taken from Stein et al. (1996). 2005 Ferrini-Mundy & Schmidt suggest using this framework in international comparison studies.

Each unit examined in the mathematics textbooks is called a unit task in this study. Although it has many meanings, Stein & Smith (1998: 269) define a task as "a part of a classroom activity designed to develop a mathematical idea." Mathematical tasks are treated as processes that require answers in textbooks without focusing on the solution process. Mathematical tasks are not only used to enhance students' understanding but also can be used to assess students' mathematical understanding. Based on the theoretical framework, a list of the criteria we used to code the textbooks of the two countries is given in Table 2. Table 3 shows sample examples of tasks and their categories, as listed in Table 2.

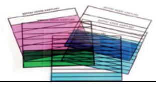
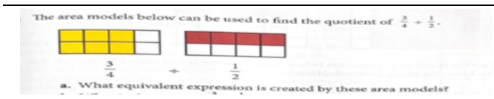
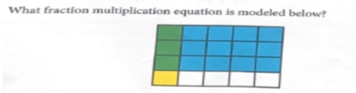
Table 2

Categories in math textbooks

Component of Analysis	Category
Mathematical content	Numbers & operations Geometry Algebra
Representation	Symbolic (Sym) Verbal (Ver) Visual (Vis) Combined (Com)
Contextual Features	Intra-mathematical context (IMC) Realistic context (RC) Authentic context (AC)
Cognitive Demands	Memorization (M) Procedure without connection (PC_-) Procedure with connection (PC_+) Doing mathematics (DM)

Table 3

Sample textbook tasks and categories

Examples	Category
Transparent fraction cards can be used in math courses to understand simple fractions, compare fractions with each other and learn how to multiply fractions. Let's model the multiplication of the fraction $\frac{4}{10}\cdot\frac{4}{5}$ using transparent fraction cards and find the result. 	Com IMC PC+
The ratio of boys to girls in soccer league is 3:4. What is the ratio of girls to all players in the soccer league?	Ver RC PC-
	M PC+ DM
Consider and explain the importance of the units that multiplicities have. For example, is there a difference between $\frac{100km}{5\,km}$ and $\frac{100km}{5h}$?	Ver AC DM
Match pairs of like terms 6x 2y $9y^2$ x 9 $4y^2$ 5 14y	Sym IMC M
	Vis IMC PC+

Coding Reliability

A total of 967 mathematical tasks were selected from the task book based on the three common learning domains. Identifying the given tasks was straightforward and clear since both textbooks gave mathematical tasks under “exercise or to do the task” for students to do. Firstly, both researchers independently categorized the selected task based on the three domains: representation, contextual features, and cognitive level. After the initial categorization of the tasks, both researchers discussed the mathematical task where they disagreed with classifying them based on the three domains. Secondly, both research studies focus on the classification of tasks, where they disagree. Thirdly, both researchers discussed tasks on which they disagreed for further classification. Finally, the agreement between the researchers in the categorizations according to learning domains in each textbook was over 90%. During this process, intercoder agreement exceeded 90% for simple percent agreement, and kappa statistics, which exceeded 90%, indicate high consistency between coders (Lombard et al., 2002). The researchers met occasionally to discuss different coding, and the process was concluded after complete agreement was reached on all of the tasks/codes.

Findings and Results

The data analysis revealed that there are more tasks in the US textbook (n=573) than in the TR textbooks (n=394). Although this finding is more evident in the numbers and operations and algebraic domains, it is valid for the rest of the learning domains. Table 4 shows the distribution of the types of representations utilized in the tasks in USA and TR textbooks. The chi-square test was used to analyze whether the types of representation showed any differences between the USA and TR mathematics textbooks. The results of the chi-square test showed that there was a significant relationship between the types of representation utilized in the domains of numbers and operations and geometry (X^2 $_{(df=6)=}34.98$, p< .01; X^2 $_{(df=6)}=20.07$, p< .01). In other words, the percentages of the types of representation usage in these learning domains in the textbooks of these two countries differ significantly from each other.

In the TR textbook, verbal representation is used intensively in numbers and operations and geometry tasks. In contrast, the tasks in the USA textbooks show a more balanced distribution of multiple representations in all three learning domains. However, the chi-square test result (X^2 $_{(df=6)}$= 2,36) indicated no statistically significant difference in the algebra domains between the two textbooks. There is also no significant relationship between the multiple representations used and the distribution of the four representation types in the textbooks between the countries. Verbal representation is the TR textbook's most dominant type of representation (Table 4). Verbal representation was utilized in more than two out of every three tasks in the learning domains of numbers& operations. In the US math textbook, verbal representation is also predominant, but symbolic representation is also used extensively, especially in the learning domain of numbers and operations and algebra. The least preferred representation in the math textbooks of both countries is a visual representation, which was used minimally in both countries' textbooks.

Table 4

Frequency of Representation Types in Math Textbook

Types of Representation	Numbers & Operations			Algebra			Geometry		
	USA	TR	X^2(df)	USA	TR	X^2(df)	USA	TR	X^2(df)
Symbolic	109 (37.2%)	32 (18%)		53 (39.6%)	29 (33.7%)		15 (10.3%)	25 (19.2%)	
Verbal	119 (40.6%)	122 (68.5%)	34.98	49 (36.6%)	38 (44.2%)	2.35	51 (34.9%)	49 (37.7%)	20.07
Visual	8 (2.7%)	3 (1.7%)	(6)*	7 (5.2%)	2 (2.3%)	(6)	48 (32.9%)	15 (11.5%)	(6)*
Combined	57 (19.5%)	21 (11.8%)	(p=.000)	25 (18.7%)	17 (19.8%)	(p=.501)	32 (21.9%)	41 (31.5%)	(p=.000)
Total	293	178		134	86		146	130	967

* *p < .01*

Table 5 shows the distribution of contextual features used in the tasks in TR and US textbooks according to learning domains. The chi-square test was used to examine whether the contextual features between the textbooks were significantly different. The chi-square test (X^2 $_{(df=4)}$ = 11.19) revealed a statistically significant difference in the domains of numbers and operations. The analysis further reveals a significant relationship between the contextual features of the tasks in the domain (i.e., there are significant differences in the contextual features of the tasks in the textbooks of both countries). However, the results of the chi-square test for the tasks in algebra and geometry learning domains ($X^2_{(df=4)}$= 1.17; $X^2_{(df=4)}$= 3.63) are not statistically significant. There is no significant relationship between the contextual features of the tasks used in the domains of algebra and geometry in both textbooks, i.e., the distribution of contextual features is similar between the textbooks of both countries.

Especially in the algebra domain, the intra-mathematical content is relatively high in both countries (Table 5). While this rate is 77.9% in the TR mathematics textbook, it is as high as 82.1% in the USA textbook. In the other learning domains, the intra-mathematical content is relatively low in both countries. In particular, the analysis revealed that authentic content remains very low in both countries' textbooks. This situation further suggested that in practical tasks, the data are presented ready-made by the textbook authors rather than the original productions of the students based on their daily lives and experiences.

Table 5
Frequency of contextual features in math textbooks

Contextual features	Numbers & Operations			Algebra			Geometry		
	USA	TR	X^2(df)	USA	TR	X^2(df)	USA	TR	X^2(df)
IMC	147 (50.1%)	62 (34.8%)	11,19 (2)* p=.003	110 (82.1%)	67 (77.9%)	1,17 (2) p=.55	91 (62.3%)	89 (68.5%)	3,63 (2) p=.162
RC	138 (47.1%)	112 (62.9%)		22 (16.4%)	16 (18.6%)		53 (36.3%)	36 (27.7%)	
AC	8(2.7%)	4(2.3%)		2(1.5%)	3(3.5%)		2(1.4%)	5(3.8%)	
Total	293	178		134	86		146	130	967

*p < .01

Table 6 shows the distribution of the cognitive demand levels required to solve the tasks in USA and TR mathematics textbooks. In general, the two textbooks have different emphases regarding cognitive demand. A chi-square test was used to examine whether the cognitive demand levels between the textbooks differed significantly. The results of the chi-square test in the learning domains of numbers and operations and geometry ($X^2_{(df=6)}$=11.60; $X^2_{(df=6)}$= 20.77) were statistically significant. The findings reveal a significant relationship between the cognitive demand levels of the tasks in the domain of numbers and operations and geometry between the textbooks; that is, the distribution of the cognitive demand levels task between the textbooks of both countries is not similar. In other words, the frequency and percentage of cognitive demands in the tasks in the US and TR textbooks vary as a function of the textbooks. In the TR textbook, tasks at the procedure with connection level are quite dominant in the domains of numbers, operations, and geometry (62.3% and 61.5%). However, in these domains, the distribution of cognitive demand levels of tasks in US textbooks is more balanced. The chi-square test results for the algebraic domain tasks ($X^2_{(df=6)}$=5.76) are not statistically significant. The distribution of cognitive demand levels of tasks in the algebra domain in both countries' textbooks is similar. It is noteworthy that the tasks at the level of doing mathematics are at the lowest level in all three learning domains in the textbooks of both countries. As seen in Table 6, according to learning domains, the rates in US textbooks are 9.9%, 3.7%, and 4.1%, and in TR, they are 2.8%, 4.7%, and 1.5%. The data reveal that both USA and TR students experience complex, non-algorithmic, and unclear solution processes of tasks in mathematics classes at very low rates.

Table 6
Frequency of cognitive demands levels in math textbooks

Cognitive demands levels	Numbers & Operations			Algebra			Geometry		
	USA	TR	X^2(df)	USA	TR	X^2(df)	USA	TR	X^2(df)
M	21 (7.2%)	16 (9%)		12 (9%)	9 (10.5%)		6 (4.1%)	17(13.1%)	
PC_-	80(27.3%)	45(25.3%)	11.60 (6)* p=.008	67(50%)	29(33.7%)	5.76 (6) p= .123	67(45.9%)	31(23.8%)	20.77 (6)* p= .000
PC_+	163(55.6%)	112 (62.9%)		50(37.3%)	44(51.2%)		67(45.9%)	80(61.5%)	
DM	29(9.9%)	5(2.8%)		5 (3.7%)	4 (4.7%)		6 (4.1%)	2 (1.5%)	
Total	293	178		134	86		146	130	967

Discussion

This study primarily compares the representation types, contextual features, and cognitive demand levels of the tasks in the common topics in three domains: numbers and operations, algebra, and geometry in the 6^{th}-grade mathematics textbooks between the USA and TR. The result of this study, in which a total of 967 tasks were analyzed, contends that the total number of tasks in the USA textbooks is higher than that of the TR textbook. The findings further reveal that the differences in the number of tasks are more pronounced especially in the domain of numbers and operations and algebra.

Mathematical textbook content directly impacts learning opportunities and achievement (Van den Ham & Heinze, 2018). Grouws and Cebulla (2000) suggested that students should be exposed to as many opportunities as possible to solve mathematical tasks to develop their problem-solving competence. The high number of tasks in the USA book likely provided more opportunities for students to learn mathematical concepts. Studies conducted in different grade levels and subjects show that the number of problems (or tasks) in TR mathematics textbooks is lower than in the textbooks of countries such as the USA, Canada, or Singapore (Özer & Sezer, 2014; Özgür Şen, 2022). The findings of this study advocate that having a higher number of tasks likely has positive effects on learning mathematics (Valverde et al., 2002). The fact that the textbook contains more tasks probably provides more opportunities for students in the USA to develop problem-solving skills than that of TR.

The study showed statistically significant differences between mathematics tasks in three domains: representation, contextual features, and cognitive demand levels. Related literature (Wijaya et al., 2015; Zhu & Fan, 2006) also suggests that textbooks can affect the intended curriculum and tasks. The study's findings support that textbooks in the USA and TR are similar and different.

The tasks in the three learning domains in the textbooks of the two countries showed differences in all dimensions of representation types ($X^2 = 34.98$; $p = .000$), contextual features ($X^2 = 11.19$; $p = .003$), and cognitive levels ($X^2 = 11.60$; $p = .008$). The tasks in the TR textbooks are concentrated in specific categories, whereas they show a more homogenous distribution in the USA textbooks.

Incorporating multiple types of representations in designing mathematical tasks contributes to students' understanding of mathematical concepts and developing problem-solving skills (Christine, 2012). The more tasks (or problems) in the textbooks that require making connections between multiple representations, the more opportunities students are likely to develop deep mathematical understanding. The study's findings suggested that verbal representations were used extensively in both countries' textbooks. This is especially evident in the learning domain of numbers and operations. In this learning domain, 68.5% of the 178 tasks in the TR textbook and 40.6% of the 293 tasks in the US textbook utilized this type of representation. Verbal representations, including written and spoken language, detail the task (or problem). In this context, this type of representation contributes to a clearer understanding of the tasks in the textbooks. This suggests that the mathematical tasks in the textbooks of both countries focus on students' understanding. However, visual representations were found to be the least used representation type in the tasks in the mathematics textbooks. While the rates of using visual representations in total tasks in the TR textbook were 1.7%, 2.3%, and 11.5%, these were 2.7%, 5.2%, and 1.4% in the US textbook, respectively, according to learning domains. Considering the effect of visual representations in developing students' conceptual understanding (Zhu & Fan, 2006), mathematical tasks in both textbooks should be enriched with visual representations. It is important to include both visual and verbal representations equally in teaching and learning mathematics (Mainali, 2014). The percentages of the representations used in the textbooks differed significantly in the learning domains of numbers and operations and geometry, except for the algebraic domain. Although there is no balance between the representation types in the textbooks, symbolic forms are emphasized more in TR textbooks. However, The US textbooks do not focus on a particular type of representation, indicating a relatively more balanced distribution of representation types.

Regarding contextual features, the density of non-contextualized tasks in both USA and TR textbooks is noteworthy. In the TR textbook, 34.8%, 77.9%, and 68.5% are noncontextualized tasks for the domain of numbers and operations, algebra, and geometry. In contrast, these values are 50.1%, 82.1%, and 62.3% for the US textbook. Hwang et al. (2021) state that mathematics textbooks frequently use non-contextual situations. Research findings reveal that non-contextualized tasks are more intense, especially in algebra and geometry. Gu et al. (2004) state that providing students with more opportunities to solve context-based problems in mathematics teaching and learning provides a learning environment based on a higher level of mathematical thinking and understanding, while Yang et al. (2010) point out that the trend in mathematics education is towards learning mathematics in real life situations. Contextualized problems help to teach and learn mathematics in a more meaningful way. The study's finding suggests that including more contextualized problems in both textbooks is important.

The results of this study showed differences between textbooks in terms of the use of mathematical tasks with different cognitive domains. The differences are especially evident in the learning domains of numbers and operations and geometry. In the TR textbook, procedures with connection-level tasks are prominent in all three learning domains. In contrast, in the USA textbook, procedure with connection and procedure without connection levels tasks are more evenly utilized. In this context, both textbooks must align more with Stein et al. (2000) framework concerning developing and designing mathematical tasks. The procedure of connecting and doing mathematics involves high cognitive level tasks that provide challenging learning opportunities for students (Smith & Stein, 1998). Balanced types of mathematical tasks in the textbook based on different cognitive levels will likely enhance mathematics teaching and learning (Gracin, 2018).

Conclusion and Implication

This study reveals that US and Turkish 6th-grade mathematics textbooks emphasize different dimensions of mathematical tasks (representation types, contextual features, and levels of cognitive demand). The results suggest that students in these two countries have different opportunities to develop mathematical knowledge and skills, especially in numbers and operations.

Verbal representation, non-contextual features, and lower cognitive demand dominate the tasks in the textbooks of both countries. It is, however, important to include more balanced types of mathematical tasks in the textbook to enhance teaching and learning mathematics. This study's results also help explain the differences between the USA and the TR in international student assessments such as PISA or TIMSS. Cultural differences influence the development and design of textbooks in countries (Ka & Leung, 2012). In this regard, improving the balance and quality of mathematical tasks in mathematics textbooks is important by considering various aspects of mathematical tasks.

This study will contribute to the textbook analysis and comparison literature. Firstly, this study's methodological, analytical framework can be used to analyze possible learning opportunities for students in mathematics textbooks. This multidimensional framework is a powerful analytical tool for identifying students' learning opportunities. Balancing different tasks in textbooks and other curriculum materials may interest curriculum developers and researchers.

Teacher training needs to be developed in this context so that textbooks used or used in countries can be utilized effectively and focus on the intended curriculum (Stein et al., 2007). Nevertheless, the study presents the design of various mathematical tasks based on different dimensions in three learning domains. This will help teachers and textbook designers understand the advantages and disadvantages of the textbooks they will use and improve their teaching and textbooks in the future. The results of this study will likely help investigate further research about mathematics textbooks and task design, such as the effect of textbook tasks on students' mathematical competencies and teachers' use of textbook tasks in mathematics classroom practice.

Acknowledgments

This study was supported by Türkiye Bilimsel ve Teknolojik Araştirma Kurumu (TUBITAK) [grant number 1059B192300741].

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