Emergence of Third-Grade Students' Perceptions of Fractions Through the Cover-Uncover Fraction Game

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This study explores the strategies employed by twelve third-grade students (aged 8) during the development and implementation of the Cover–Uncover Fraction Game and examines how these strategies relate to their perceptions of fractions. Structured, non-participant observation was used to record students' strategies. The game appeared to support students' understanding of fractions as parts of a whole, quotients, and measurements, while also eliciting ratio-based reasoning in some high-achieving students. The findings highlight varied levels of comprehension, particularly regarding equivalence, underscoring the need for instructional designs that foster both recognition and strategic use of fraction knowledge. Moreover, the study suggests that both the development and implementation of the game can offer insights into how students interpret different dimensions of fractions, providing teachers with valuable information for designing more effective instructional practices and offering a foundation for future research on the game's efficacy in supporting fraction learning or assessing students' fraction knowledge.

Introduction

Key findings reveal that fraction learning is a crucial yet challenging part of mathematics education, essential for broader math development and success (Syed Ismail, Maat & Khalid, 2024). Understanding fractions is crucial not only for academic success and career development but also for managing everyday tasks. Mastering fractions is fundamental for building knowledge of rational numbers and enhancing students' ability to generalize and work with unknown quantities (Hackenberg & Lee, 2015). Consequently, many studies emphasize the importance of understanding fractions as a prerequisite for students' progress in mathematics. Fractions are also an integral part of daily life, as they are directly related to a wide range of professional and everyday activities (Tian & Siegler, 2017).

However, fractions are among the most complex and multidimensional mathematical concepts that students encounter. They are connected to various mental schemas (Tsai & Li, 2016), and understanding them requires demanding cognitive processes, including analogical and spatial reasoning (Mamolo et al., 2011). The complexity of fractions often leads to both conceptual and procedural difficulties for students (Gould et al., 2006; Hansen et al., 2017).

Studies show a rise in research output over time, with influential contributions primarily from Western countries, particularly the USA. Despite advances, significant gaps remain, especially in long-term retention of fraction knowledge, effectiveness of adaptive and personalized learning strategies, and research in diverse cultural contexts (Syed Ismail et al., 2024).

Addressing these challenges is not easy, but presenting fractions through multiple representations is recommended for positive outcomes (Soni & Okamoto, 2020; Wilkie & Roche, 2023). Incorporating games is also recommended because they can yield positive learning results (Bennett et al., 1997; Bragg, 2012; Edo et al., 2009). These studies highlight the educational advantages of games, emphasizing their ability to promote alternative ways of thinking, develop various abilities and skills, actively engage students in understanding mathematical concepts, and assess students' mathematical knowledge.

Based on these findings, the present study aimed to assess third-grade students' (aged 8) perceptions of fractions by investigating their strategies employed during the development and implementation of the Cover-Uncover Fraction Game. The research questions guiding this study were: What strategies do third-grade students use during the development and implementation of the Cover-Uncover Fraction Game, and how are their strategies related to various interpretations of fraction concepts?

Literature Review

The Five Key Constructs of Fractions

Fractions are a fundamental component of mathematics curricula worldwide (CCSSM, 2021; Greek Primary School Curriculum, 2003). Research indicates that children as young as preschool age begin to develop an intuitive understanding of fractions (Marshall, 1993). This early understanding often emerges naturally as children encounter real-life situations involving the fair division of quantities, such as splitting items into halves. Despite this early exposure, teaching and learning of fractions present significant challenges for both educators and students (Clarke & Roche, 2009). The complexity of fractions arises from the multiple mental schemas they involve (Fronger et al., 2015), which are often insufficiently developed in traditional educational settings (Naik & Subramanian, 2008).

Students' understanding of fractions develops progressively, encompassing five key constructs: fractions as part-whole relationships, fractions as measures, fractions as quotients, fractions as ratios, and fractions as operators (Behr et al., 1992; Marshall, 1993). Fractions as part-whole relationships are a foundational concept that involves recognizing a fraction as representing a part of a whole object or set and is often introduced in the context of dividing or sharing objects (e.g., dividing a pizza into equal slices or shading 3 out of 4 equal parts of a circle illustrates the fraction 3/4). Fractions as measures are understood as numbers that can represent quantities on a number line, indicating a specific magnitude or distance from zero (measure). This concept helps children grasp that fractions are numbers with precise locations and values (Resnick et al., 2016). Fractions as quotients are understood as the result of division (quotients), where the numerator is divided by the denominator (e.g., 3 divided by 4 yields the fraction 3/4, representing equal sharing or partitioning) (Getenet & Callingham, 2017). Fractions as ratios express a relationship between two quantities, highlighting how many times one quantity is contained within another. (e.g., ratio of 3 boys to 4 girls can be represented by the fraction 3/4) (Gabriel et al., 2013). Recognizing that different fractions can represent the same quantity (e.g., 1/2 = 2/4) develops as children begin to compare fractions with visual aids and manipulatives. By upper elementary school, students should understand how to generate equivalent fractions by multiplying or dividing the numerator and denominator by the same number (Empson & Levi, 2011; Lamon, 2001). Fractions as operators represented an advanced understanding that treats fractions as functions that can scale or transform quantities [e.g., applying fraction 3/4 to a number involves multiplying that number by 3/4, effectively resizing it ($3/4 \times 12 = 9$)] (Getenet & Callingham, 2017). (Behr et al., 1992; Marshall, 1993). Research indicates that children typically first grasp the part-whole concept, as it is more concrete and directly observable. As they progress, they develop an understanding of fractions as measures and quotients, which require more abstract thinking. The concepts of fractions as ratios and operators are usually comprehended later, as they involve more complex relational reasoning (Wilkins & Norton, 2018). Understanding these developmental trajectories is crucial for educators to design effective instructional strategies that align with children's cognitive development stages.

Research has highlighted the difficulties students face with fractions. For example, only 59% of 11-year-olds could correctly order ten fractions, and just 40% of students aged 10 to 12 could accurately place a fraction on a number line between 0 and 1 (Mazzocco & Devlin, 2008). These difficulties are further compounded by the structural differences between fractions and natural numbers (Van Dooren et al., 2015). Students often fail to grasp the relationship between the numerator and denominator, treating them as independent entities. This misconception is evident in fraction addition, where students mistakenly add numerators and denominators separately (Cramer & Wyberg, 2009; Post et al., 1985), and in fraction valuation. For instance, when primary students (ages 6-11) were asked to represent fractions like 1/2, 1/3, and 1/6 using circular diagrams, most could correctly divide a circle into halves with a vertical or horizontal line. However, they struggled with dividing the circle into thirds and sixths, often producing unequal parts and treating the components of the fraction as separate variables (Gould et al., 2006). Many children struggle to understand that the size of a fraction depends on both the numerator and the denominator, not just one or the other (Kieren, 1980).

Students often approach fractions procedurally rather than conceptually (Behr et al., 1992). This procedural focus is primarily attributed to classroom teaching practices that prioritize algorithms over conceptual understanding (Post et al., 1991; Moss, 2005). As a result, even when students can correctly execute procedures, they may be unable to explain the underlying reasoning, as documented by several studies (Shin & Bryant, 2015; Şahin et al., 2015).

Given these challenges, the difficulty in teaching and understanding fractions presents a significant obstacle to the development of students' mathematical thinking (Behr et al., 1992). Consequently, fractions remain a key focus for mathematics education researchers, who seek effective educational practices that integrate both procedural and conceptual understanding (Van de Walle, 2007). One promising approach is the use of educational games, which offer a playful yet meaningful context for exploring mathematical concepts.

Games in Teaching Practice

The use of games to support the teaching and learning of mathematics has gained significant attention in recent decades (Brogström, 2017; Edo et al., 2009; Fleer, 2010; Park et al., 2008). Games provide an integrated learning experience, establishing a new educational framework that fosters a deep conceptual understanding of mathematics. Research suggests that game-based learning is a dynamic process through which students develop new skills and competencies needed as future citizens (Eskicioglu et al., 2016). This approach supports students' mathematical development in both informal and formal contexts and can serve as a meaningful activity (Sancar-Tokmak, 2014; Bragg, 2012; De Holton et al., 2001). Teaching through games enables educators to achieve specific learning objectives, skills, and competencies, with knowledge construction emerging naturally from gameplay (Perrotta et al., 2013). This approach actively engages students in the educational process, encouraging learning through trial and error, role-playing, and navigating rules, processes, choices, and outcomes. As a result, games facilitate knowledge generation and construction, supporting student-centered learning. They also serve as potential tools for assessing students' mathematical perceptions (Skoumpourdi, 2016).

Regarding fractions, Martin et al. (2015) investigated how the digital game Refraction supports students' understanding of fractions through the concept of splitting (dividing wholes into equal parts). They found that the game is an effective tool for fostering understanding of fractions because it promotes engagement, experimentation, and strategy development. In other words, game-based learning can significantly enhance students' understanding of fractions.

While studies such as Martin et al. (2015) demonstrate the effectiveness of digital games in supporting fraction learning, there remains a lack of research specifically examining elementary school students' perceptions of fractions through the development and play of a physical game.

The Cover-Uncover Fraction Game

The Cover-Uncover Fraction Game is designed to engage the player with fraction concepts. The player is given a rectangular strip representing the whole, along with five additional strips of the same length, divided into fractional parts (e.g., halves, quarters, etc.) (Figure 1). A die with corresponding fractional parts is also included in the game (Figure 2).

The game consists of two variations: the Cover Fraction Game and the Uncover Fraction Game. In the Cover Fraction Game, the objective is to completely cover the whole strip. The player rolls the die, selects the fractional piece that matches the result, and places it on the whole strip. The game continues until the entire strip is covered. In the Uncover Fraction Game, the goal is to remove all pieces from the entire strip. The player begins with their whole strip fully covered. (S)He rolls the die, selects the fractional piece matching the result, and removes it from the strip. The game continues until the entire strip is uncovered. The player may also replace pieces with equivalent ones if needed.

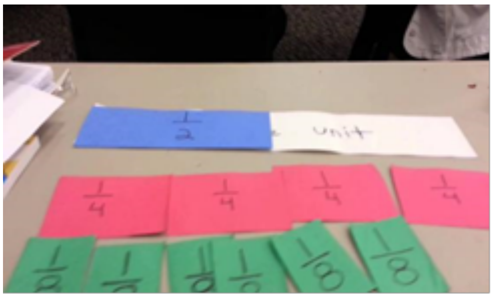

Figure 1. Pieces of the game

Figure 2. Pieces and die of the game

The Cover-Uncover Fraction Game embodies multiple interpretations of fractions through the five key concepts. Fractions as parts of wholes: The whole strip is divided into equal parts, represented by the game's pieces, reinforcing the idea of partitioning a continuous quantity into equal sections. Fractions as measures: Each fractional piece can be viewed as a unit fraction that measures a distance on an imaginary number line, starting from zero and extending to one—the whole strip. Fractions as quotients: Each fractional piece represents the division of a whole into equal parts, with the numerator indicating the number of parts and the denominator representing the division. Fractions as ratios: The game introduces the concept of equivalent fractions when players exchange pieces to continue gameplay. For example, a 1/2 piece can be replaced by two 1/4 pieces, or four 1/8 pieces, etc. Fractions as operators: A fraction divides the whole into equal parts, and when applied as an operator, scales the whole to the specified portion. Conversely, multiplying the fractional part by the reciprocal of the fraction reconstructs the original whole (e.g., $4\times1/4 = 1$).

A previous study on this game has focused on the nature and quality of teacher-student interactions during its use (Heshmati et al., 2017). The present study has focused on using that game as a tool for assessing students' perceptions of fractions.

Methodology

Sample

The sample for this study consisted of 12 third-grade students (aged 8). To capture a range of mathematical abilities, the students were selected based on their mathematics performance, as assessed by their teacher, who also served as the researcher. The sample included high-achieving students: two boys and two girls (B: Boy, G: Girl, H: High-achieving, BH1, BH2, GH1, GH2), moderate-achieving students: two boys and two girls (M: Moderate-achieving, BM1, BM2, GM1, GM2), and low-achieving students: two boys and two girls (L: Low-achieving, BL1, BL2, GL1, GL2). The class was selected for convenience, and the participation of the specific students depended on parental consent. The students had not been taught the fractions unit before the research was conducted.

According to the Greek Primary School Mathematics Curriculum (2003), fractions are introduced in the third grade, with an emphasis on developing an understanding of basic fractional units (e.g., 1/2, 1/4, 1/3, 1/8, 1/16, 1/5, 1/10, etc.) and learning to compare them using appropriate representations.

Research Process and Data Collection Tools

The data collection process utilized the structured, non-participant observation method. During one-on-one sessions with each student, the teacher-researcher explained that the student would develop and play a game. The prospect of playing the game could serve as a key motivating factor, encouraging the children to focus on creating it in the best possible way. Initially, verbal instructions were provided to guide the construction of the game. Each student received a 60cm × 60cm white cardboard and was instructed to construct the game by creating six strips of identical length: one strip representing the whole and the remaining five divided into fractional parts (1/2, 1/3, 1/4, 1/5, and 1/6, respectively). Students had access to various materials, including scissors, pencils, a 10 cm ruler, crayons, and markers.

After the game was constructed, the researcher provided the student with a die marked with fractions (1/2, 1/3, 1/4, 1/5, 1/6) and a wildcard (*) that allowed the student to select any fractional value. The researcher gave verbal gameplay instructions, presented a video demonstrating how to play the game, and provided written rules. Once the researcher was confident that the student understood the gameplay, the student began playing independently.

The researcher observed and recorded qualitative data throughout both the game development and gameplay phases (Appendix). Observations from the game development focused on how the student created the strips and divided them into the appropriate fractional parts. Gameplay observations emphasized how the student manipulated the fractional pieces to cover or uncover the whole strip, their decision-making when using the wildcard (*), and their reasoning processes.

Each student was given two instructional hours (2 × 45 minutes): one hour for constructing the game and one for playing it.

Results

Data from game development

The development of the Cover-Uncover Fraction Game revealed varied strategies for creating equal-sized strips and fractional pieces. Some students focused on making the strips uniform in size to match the intended fractional value, while others did not prioritize uniformity.

The strategies employed by the students can be categorized as follows:

- Creation of Six Isometric Strips Using the Ruler: One student (BH1) used the ruler to measure the cardboard width (60 cm), mentally divided it by 6, and created six equal strips, each 60 cm × 10 cm. He explained that this approach ensured all strips were equal without leaving excess cardboard.
- Creation of Six Isometric Strips Using the Ruler as a Template: Another student (BM2) used the ruler's width as a template to draw and cut the first strip, then measured and cut subsequent strips by direct comparison with the first.
- Creation of Six Isometric Strips Without Measuring a Specific Width: Six students (BH2, GH1, GH2, BM1, GM1, and BL1) cut the first strip using the ruler to ensure straight edges, but did not specify a precise width. They used the first strip as a reference for cutting the others. Minor deviations in width did not affect the game's progress.
- Creation of Random Width Strips Without Measurement: Four students (GM2, GL1, GL2, and BL2) produced strips of unequal width but of the same length (the cardboard’s length). While some claimed they were trying to make equal strips, they either used the ruler only to draw straight lines (GM2 & GL1) or divided the strips arbitrarily without any measurement tools (GL2 & BL2).

After creating and colouring the strips, the students divided them into fractional parts (1/2, 1/3, 1/4, 1/5, and 1/6). The strategies they employed for this task were as follows:

- Creation of Isometric Pieces Using Ruler and Division Strategies: Six students (BH1, BH2, GH1, GH2, BM1, and GM1) used the ruler to measure the length of the strips and divided them into fractional segments. For example, BH1 mentally divided the 60 cm strip into two, three, four, five, and six equal segments, measuring each segment with the ruler. For instance, when creating 1/4, he said: Since 10 times 4 is 40, another 5 times will give half of it. Similarly, GH1 initially struggled to divide 60 cm by 4, eventually resorting to a finger-counting method to arrive at the correct result. GH2, when dividing 60 by 2, 3, and 6, used a shortcut by covering the '0' in 60 and quickly finding the quotient, saying: I hid it to divide faster, then I put it back. However, for dividing by 4 and 5, she counted by 4s and 5s, starting from 40 and 50, respectively. BM1 encountered difficulties when dividing the strip; he measured 61 cm instead of 60 cm, requiring additional guidance and re-measurement. GM1 also faced trouble with the divisions, counting with her fingers or adding one unit at a time. Despite these challenges, she successfully completed the divisions. When the researcher asked about her method for dividing 1/3 into 20 cm segments, she explained: I added three fingers each time until I reached 60. BH2 folded the strip in half for the 1/2 division and applied similar folding techniques for the other fractional values.

- Creation of Approximate Isometric Pieces by Folding: One student (BM2) folded the strips into halves, thirds, quarters, fifths, and sixths. However, this approach resulted in inaccuracies, particularly with the 1/5 and 1/6 pieces, which were nearly identical in size.
- Creation of Isometric and Non-Isometric Pieces Using Mixed Methods: One student (BL2) used a ruler to measure the length of the strip and divided it into fractional segments using repeated addition. However, for the 1/4 and 1/5 divisions, the student deviated from this method, resulting in creating unequal parts.
- Creation of Non-Isometric Pieces, Except for 1/2 Fraction: Four students (GM2, BL1, GL1, and GL2) divided the strips into fractional parts without precise measurements, comparisons, or arithmetic calculations, except for 1/2, and folded the strips in half, resulting in equal parts.

In summary, the strategies students used to develop the Cover-Uncover Fraction Game varied significantly, ranging from precise measurements to arbitrary divisions. Most students successfully created the game despite some deviations from the intended fractional values.

Data from gameplay

During the gameplay, two distinct patterns emerged in how students manipulated the pieces. 1. Students who covered and uncovered the strip accurately: These students (BH1, BH2, GH1, and BM2) successfully covered the entire strip using individual fractional parts and uncovered it correctly. Some of these students (BH1 and BH2) also demonstrated a perception of equivalence by substituting fractional parts (e.g., replacing 1/2 with two 1/4 pieces). 2. Students who exceeded the length coverage of the strip of cardboard: These students (GH2, BM1, GM1, BL1, BL2, GL1, and GL2) completed the Cover Fraction Game exceeding the length of the strip, due to inaccuracies in their pieces or a wrong combination of pieces. In the Uncover Fraction Game, they rolled the die until they obtained the required values, but did not make substitutions.

BH1 completed the Cover Fraction Game in three rolls of the die, obtaining 1/4, a wildcard (*), in which he selected 1/4, and 1/2. When asked why he chose 1/4, he explained that it would complete half of the strip, leaving only one piece needed to win. During the Uncover Fraction Game, BH1 initially covered the whole strip with two 1/2 pieces. He then rolled 1/6, 1/3, and a wildcard (*), where he removed one 1/2 piece. Next, he rolled 1/4 and substituted the remaining 1/2 piece with two 1/4 pieces, removing one of them. The game ended with a final roll of 1/4.

BH2 completed the Cover Fraction Game in three moves with rolls of 1/6, 1/2, and 1/3. In the Uncover Fraction Game, BH2 initially used three pieces (1/2, 1/4, and 1/4) to cover the strip. The first roll indicated 1/5, followed by 1/4, leading BH2 to remove one 1/4 piece. After rolling the wildcard (*), BH2 removed the second 1/4 piece. Then, after rolling 1/4 again, BH2 decided to substitute the 1/2 piece with two 1/4 pieces, removing one. The student completed the Uncover Fraction Game after two more rolls of 1/4.

GH1 completed the Cover Fraction Game in three rolls, obtaining 1/6, 1/2, and 1/3 (Figure 3). After the first two rolls, the student reasoned that only one yellow piece (1/3) was needed to cover the strip, and if 1/6 were rolled, he would need more rolls to finish the game. In the Uncover Fraction Game, the student selected the same pieces—1/6, 1/3, and 1/2—to cover the strip. During the game, the die initially landed on the wildcard (*), prompting the student to remove the 1/6 piece. On the subsequent roll, the die indicated 1/4, but the student removed the 1/3 piece. When the die showed 1/4 again, the student had an opportunity to substitute but opted not to. After two more rolls, the student completed the Uncover Fraction Game when the die landed on 1/2.

BM2 completed the Cover Fraction Game in three rolls with results of 1/6, 1/2, and a wildcard (*), choosing 1/3. For the Uncover Fraction Game, he covered the strip with two 1/2 pieces. The first roll indicated 1/4, followed by 1/5, but the student did not substitute any pieces. After rolling the wildcard (*), BM2 removed one 1/2 piece and completed the Uncover Fraction Game in four rolls. When asked why he did not make any substitutions, BM2 explained that "avoiding substitutions would expedite the game, as only one piece remained to be removed".

GH2 completed the Cover Fraction Game in five moves, rolling 1/5, 1/6, 1/4, 1/5, and a wildcard (*), selecting 1/3, exceeding the length of the strip (Figure 4). In the Uncover Fraction Game, GH2 chose the two 1/2 pieces to cover her strip and removed them after rolling 1/2 twice within three rolls.

BM1, during the Cover Fraction Game, rolled 1/2, 1/4, and 1/3. When asked about alternative fractions, BM1 struggled to combine fractional values, suggesting a limited understanding of equivalence. In the Uncover Fraction Game, the student chose two 1/2 pieces to cover his strip. Despite rolling 1/6 and 1/3, he opted not to substitute. He finished the game after rolling 1/2 twice.

GM1 completed the Cover Fraction Game in three rolls, selecting 1/2 when the die showed wildcard (*), with the other rolls being 1/3 and 1/5. For the Uncover Fraction Game, she chose three 1/3 pieces to cover her strip, which, although unequal, still covered the strip accurately. She completed the Uncover Fraction Game by rolling 1/5, 1/3, 1/2, 1/6, 1/2, 1/3, 1/5, 1/3.

GM2, during the Cover Fraction Game, rolled the die and got 1/5, 1/4, 1/4, and 1/2, placing the pieces without commenting on exceeding the strip's length. In the Uncover Fraction Game, GM2 selected three 1/3 pieces and completed the game after five rolls of the die, with no substitutions, indicating a focus on straightforward gameplay.

BL1 completed the Cover Fraction Game without realizing that the fractional parts he had developed were incorrect in size. The game was completed with rolls of 1/6, 1/3, 1/3, and 1/2. In the Uncover Fraction Game, after several attempts, BL1 covered the strip with two unequal 1/2 pieces (Figure 5) and finished after rolling 1/2 twice.

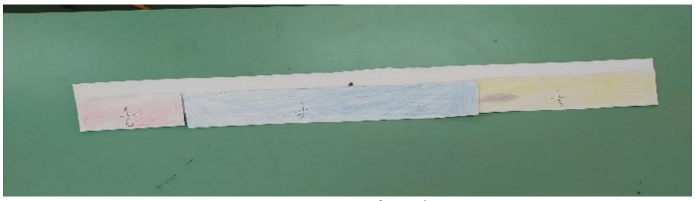

Figure 3. GH1's cover fraction game

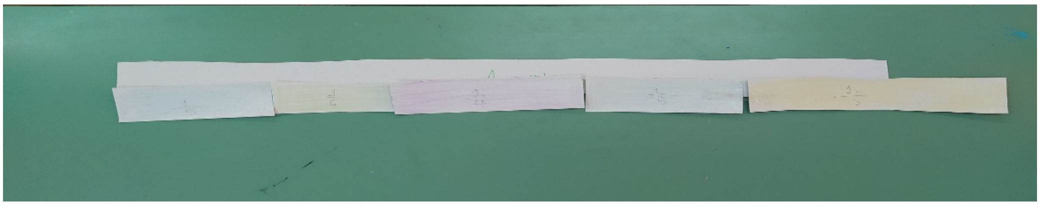

Figure 4. GH2's cover fraction game

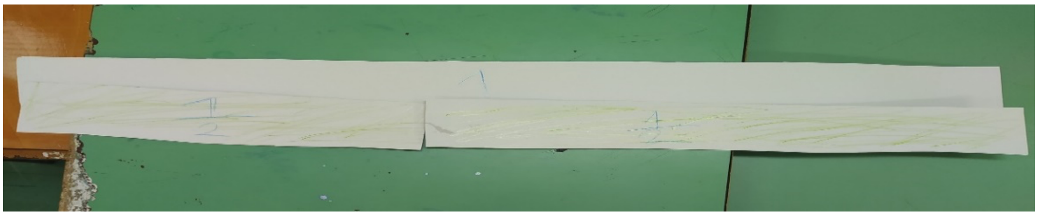

Figure 5. BL1's strip for the uncover fraction game (2 unequal parts of ½)

BL2 completed the Cover Fraction Game with rolls of 1/2, 1/3, and 1/5, without recognizing the incorrect sizes of the pieces or the excess length of the strip. In the Uncover Fraction Game, BL2 selected two 1/2 pieces to cover his strip, thinking this choice would expedite the game. The game was completed without substitution after four rolls, as the die showed a wildcard (*) twice.

GL1 struggled with fractional parts and completed both games without recognizing issues with her piece sizes. In the Cover Fraction Game, she rolled 1/2, 1/3, 1/4, and 1/6. Due to the incorrect dimensions of the pieces, her strip was covered. In the Uncover Fraction Game, GL1, like BL1, used unequal 1/2 pieces and finished the game after rolling 1/2 twice.

Finally, GL2 completed the Cover Fraction Game with rolls of 1/4, 1/3, and 1/2, unaware that the pieces were incorrectly sized. In the Uncover Fraction Game, GL2 chose four 1/4 pieces to cover her strip. This selection prevented her from recognizing the incorrect creation of the fractional parts. The Uncover Fraction Game was completed after ten rolls, during which the die showed 1/4 four times.

Discussion

The strategies students used to develop and implement the Cover-Uncover Fraction Game provided insights into their perceptions of the various interpretations of fractions. It appeared that most students were able to perceive fractions as parts of a whole, as suggested by their efforts to divide the strips into fractional parts and cover the whole strip with the corresponding pieces (which could alternatively be interpreted as measurements).

This aligns with the Greek Primary School Mathematics Curriculum (2003), which emphasizes this interpretation of fractions as a key educational objective. Related research (Kaiafa, 2020) suggests that third-grade students tend to perceive a fraction as part of a whole, even when it is represented as a measurement on a graduated number line.

High-achieving students (e.g., BH1, BH2, GH1, and GH2) seemed to demonstrate a more sophisticated perception of fractions, treating them as quotients. They often divided strips into equal parts using mental strategies, paper-based techniques, or a combination of both. However, some moderate and low-achieving students, while attempting to interpret fractions as quotients, occasionally produced unequal pieces either due to difficulty folding the strip into specific parts (e.g., folding into five parts as seen with BM2) or challenges in making precise divisions (e.g., dividing by five, as seen with BL2). Research by Gould et al. (2006) highlights that students may encounter difficulties dividing a whole into an odd number of parts or when the number of divisions increases. Other moderate and low-achieving students (GM2, BL1, GL1, GL2) did not seem to grasp fractions as quotients and proceeded to arbitrary partitioning of the strips without measurement. Creating strips of ½ generally posed little difficulty, except for a few low-achieving students (BL1, GL1, GL2), most of whom made them by folding the strip in the middle.

Students' recognition of fractions as measurements was suggested by their actions when placing pieces on the whole strip. Only high and moderate-achieving students (BH1, BH2, GH1, and BM2) were able to cover the entire strip from end to end during the Cover Fraction Game. Other students either extended pieces beyond the strip or placed pieces unevenly along it, which may indicate a partial understanding of fractions as measurements or challenges in measuring length using fractional units.

During the Uncover Fraction Game, students generally demonstrated an emerging perception of fractions as measurements, placing pieces on the strip to cover it from end to end (which could alternatively be interpreted as parts of a whole). When constructions were accurate, using pieces of the same value (e.g., 1/2+1/2 or 1/3+1/3+1/3) suggested repetition of the unit fraction to measure or cover the whole. When pieces of different values were used (e.g., 1/2+1/4+1/4 or 1/6+1/2+1/3), students appeared to employ composition skills or, in some cases, to rely on trial and error without a clear conceptual understanding. Even when constructions were not precise, such as when pieces of the same fractional value varied in length, students often managed to cover the whole strip due to the nature of their constructions.

Perceptions of fractions as ratios were most evident among high-achieving students (BH1 & BH2), who not only manipulated pieces mechanically but also verbalized equivalences (e.g., recognizing that 1/2 can be represented by two pieces of 1/4 or three pieces of 1/6). This indicates an emerging understanding of fraction equivalency, even if not systematically applied across all situations. One moderate-achieving student (GM1) seemed to recognize the equivalence 1/3 = 1/6 + 1/6 during interaction with the researcher, but did not apply it in gameplay. Other students did not make substitutions during the game or when prompted by the researcher. Notably, a closer look reveals two distinct patterns: some students demonstrated awareness of equivalences but strategically chose not to implement substitutions, explaining that substitutions would have slowed down their play or that they preferred to rely on luck. For these students, the issue was not a lack of conceptual understanding but rather a problem with their strategic decision-making process. In contrast, other students appeared unable to identify equivalent fractions at all, consistent with previous findings that only a small percentage of primary school students can recognize equivalent fractions (McIntosh, Reys, Reys, Bana & Farrell, 1997) and that difficulties and errors are common in this area (Pinilla, 2007). Finally, the students' actions did not indicate that they perceived fractions as operators.

This distinction between students who understand equivalence but choose not to act on it and those who struggle to identify equivalence altogether is crucial: it highlights different layers of fraction knowledge, namely the conceptual recognition of equivalence versus the strategic use of equivalence in problem-solving contexts. Instructionally, this suggests that teaching should not only aim to strengthen students' ability to detect equivalent fractions but also encourage them to recognize the value of applying such knowledge strategically, for instance, by designing tasks in which substitutions demonstrably simplify rather than complicate gameplay.

In summary, the construction of the Cover–Uncover Fraction Game appeared to support the development of students' understanding of fractions as parts of a whole and, for several students, as quotients, as they were required to divide the strips into equal parts and create corresponding fractional pieces. The gameplay phase seemed to provide additional opportunities to engage with fractions as measurements, through repetition and composition of unit fractions to cover the whole strip, and, for high-achieving students, to explore fractions as ratios through the recognition and application of equivalences. Notably, the analysis revealed two ways in which equivalence knowledge manifested: some students recognized equivalences but chose not to act on them for strategic reasons. In contrast, others appeared unable to identify any equivalences. This distinction underscores the need to differentiate between conceptual recognition of equivalence and the strategic use of equivalence in game-based contexts, both of which should be explicitly nurtured in instruction. Inaccurate fractional pieces did not appear to lead to incorrect reasoning; instead, they may have offered opportunities for reflection and discussion, reinforcing students' understanding. At the same time, some students completed the cover game through trial and error without fully grasping equivalence, measurement, or ratio concepts, highlighting the need to support conceptual understanding alongside gameplay.

Conclusions

Overall, the construction of the Cover–Uncover Fraction Game seems to have facilitated the development of students' understanding of fractions as parts of a whole and, to a considerable extent, as quotients. Gameplay provided further opportunities to engage with fractions as measurements, particularly through repetition and composition of unit fractions. For some high-achieving students, it also appeared to elicit understanding of fractions as ratios. Interpretation of fractions as operators was not observed. The game could potentially highlight multiple interpretations of fractions—as parts of a whole, quotients, ratios, measurements, and operators—while possibly linking procedural skills with conceptual understanding.

The development and implementation of the game offered insights into students' reasoning and highlighted diverse levels of comprehension. While most students engaged with fractions as parts of a whole, fewer interpreted them as quotients or measurements, and only a small proportion seemed to recognize fractions as ratios. Importantly, within ratio-based reasoning, two distinct patterns were identified: some students conceptually recognized equivalences but chose not to apply them strategically, whereas others struggled to identify equivalences altogether. This distinction emphasizes the need for instruction to address both the recognition and the purposeful application of equivalence knowledge. These varied outcomes underscore the importance of instructional designs that acknowledge students' current conceptions while supporting the progression toward more sophisticated understandings. The study provides tentative insights not only into students' strategies and difficulties but also into how teachers can use game-based learning to design more effective, targeted practices that support the development of a robust, multifaceted conception of fractions.

Limitations and Proposals

The findings of this research, due to the small and convenience sample size, offer only preliminary insights into the potential benefits of the Cover-Uncover Fraction Game for assisting teachers and students in approaching the multidimensional concept of fractions.

The use of convenience sampling and grouping students by teacher-assessed achievement levels may have introduced bias into both observations and interpretations, while also limiting the generalizability of the findings beyond this particular classroom and curricular context.

Moreover, the cultural and curricular context itself deserves careful consideration: Greek third-graders, who had not yet received formal instruction on fractions, may approach this task very differently than students in other educational systems or those with prior fraction instruction. This contextual factor further constrains the transferability of the findings and underscores the need for cross-cultural or cross-curricular studies.

Another limitation is that the game was developed and implemented individually, which is time-consuming and does not involve competition. We acknowledge that the assessment was time-intensive, but it served an exploratory purpose, revealing insights unlikely to emerge from shorter tasks. Future work should streamline the approach to preserve its diagnostic value while improving classroom feasibility. If the game were implemented in a classroom setting, either individually or in small groups, the outcomes might differ. In a more competitive environment, students might be more motivated to outperform their peers, potentially leading them to use more advanced and complex strategies.

While we aimed to capture students' reasoning and justifications during the activities, this was not always feasible in practice. The participating students were not accustomed to articulating or explaining their mathematical thinking, which limited the extent to which we could systematically record their reasoning. This represents a genuine constraint of the study, and we acknowledge that more structured probing could have provided richer insights. Future research could therefore incorporate interview protocols or guided prompts to elicit students' reasoning alongside their observable strategies more effectively.

Additionally, designing games with different fractional units and engaging students in various discussions and interpretations could provide deeper insights into their conceptual understanding and strategic thinking regarding fractions.

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Appendix

Framework for recording qualitative data during gameplay

During the construction of the Cover-Uncover Fraction Game:

- How does the student construct the six strips?
- How does the student divide the strips to create the fractional parts (1/2, 1/3, 1/4, 1/5, and 1/6)?

During the Cover Fraction Game:

- Does the student cover the whole strip using only the fractional parts indicated by the die, or do they make substitutions with equivalent fractions?
- What does the student choose when the die lands on a wildcard (*)?

During the Uncover Fraction Game:

- Which fractional parts cover the whole at the start of the game?
- Does the student uncover the whole using only the fractional parts indicated by the die, or do they make substitutions with equivalent fractions?
- What does the student choose when the die lands on a wildcard (*)?