Analysis of Grade Ten Students' Errors in Solving Trigonometry Problems

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This study aimed to investigate the types of errors grade 10 students make in solving selected trigonometry problems and to understand the origins of these errors that students make when solving trigonometry problems. Another central purpose of the study was to provide instructional recommendations for addressing the issue. Forty-one Grade 10 students were selected using multistage sampling. The Trigonometry Achievement Test (TAT) was used as an instrument for the measurement process. Descriptive statistics was used for the analysis. To categorize errors, a modified New Man's Error Hierarchy was used to give stages for the mistakes, and an independent samples *t*-test was used to compare errors by gender and Family Total Income per Month (FTIPM). This study found that female students made significantly more errors than males. Students with lower FTIPM committed more errors than those with higher FTIPM, with a significance that can be shown numerically. We recommended using *step-by-step controlled practice* (*SSCP*). Some targeted actions should be taken to elevate the trigonometry achievement of females and students with lower FTIPM.

Keywords: grade ten, trigonometry errors, Newman hierarchical model, mathematical errors, Family Total Income per Month (FTIPM), Gender

Introduction

Mathematical competence is essential for future technologies and other advancements (Geraniou & Misfeldt, 2023; Larbi et al., 2024). This mathematical competence is developed from an early stage. However, many students encounter challenges in learning mathematics (Irfan et al., 2020; Moschkovich, 2010). The factors influencing these challenges are students' backgrounds regarding their academic life and culture, the teacher's teaching method, and classroom resources (Oleson & Hora, 2014). As a solution strategy, using manipulative materials helps an individual understand trigonometric concepts (Ulyani & Qohar, 2021). There are also challenges specific to trigonometry, which will be explained as follows. Since trigonometry integrates algebraic, geometric, and pictorial reasoning, challenges arise specific to trigonometry (Hanifah et al., 2014).

Students usually perceive trigonometry as challenging because it requires them to form relationships between triangle diagrams and numerical givens (Gur, 2009). They struggle with fundamental trigonometric concepts because of teacher-centered approaches that try to make students memorize without understanding (Orhun, 2002).

Researchers have noted that error analysis of student work plays two key roles (Radatz, 1980). The first is to identify learning issues and then define instructional standards. The second is to enhance the field of mathematics education by providing more findings that can improve the teaching and learning process.

Additionally, researchers identified different types of student errors. For example, Dewanto et al. (2018) and others identified three types of errors. Error Type I represents difficulty in comprehending the question. Error Type II recognizes difficulty using certain trigonometric concepts or formulas. Error Type III notes a computational error if students attempt to apply the correct formula but miscalculate it. Sari and Wutsqa (2019) categorized seven types of errors: Type R (Reading), Type C (Comprehension), Type T (Transformation), Type P (Process Skill), Type E (Encoding), Type L (Language), and Type N (Carelessness). Karthikeyan (2017) classified errors into three categories: errors that occur in students' use of concepts and symbols; errors that occur in students' use of operations, which are processes needed to get an answer; and errors that combine the above two errors.

This research focuses on gender differences as a factor and Family Total Income per Month (FTIPM) differences as a factor. Some studies in different countries show contradictory results regarding gender differences and mathematics performance, often favoring males in higher-level mathematics courses. Other researchers found that the causes of females' lower performance are due to school-related factors such as lack of female role model teachers, sociocultural factors such as societal perception of female students' academic abilities, and socio-economic factors such as parents' financial status (Tiruneh & Petros, 2014). When differences in FTIPM and academic achievement were explored, a positive correlation was found between FTIPM and academic achievement (Gobena, 2018). Furthermore, the relationship between students' GPA and family income was weak and positive, implying that higher family income slightly improved students' academic performance. External factors such as teacher quality affect students' achievement (Darling-Hammond, 2000) because high-quality teacher education creates highly skilled and knowledgeable teachers who will produce highly achieving students in their subject (Figure 1).

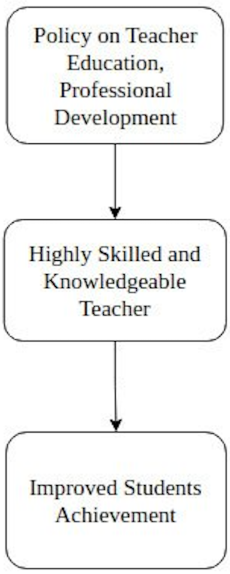

Figure 1. Cause and effect diagram of teacher quality and student achievement

Several research gaps inform this study's objectives. First is the inconsistency of findings regarding gender differences and problem-solving skills in trigonometry. There has also been limited investigation regarding angles of elevation, depression, and bearings.

Research Questions

The primary purpose of the research is to answer the following questions:

1. What types of errors (or mistakes) do grade ten students make when solving trigonometry problems?
2. Is the difference in error levels between males and females significant?

3. Is the difference in error levels between students with higher FTIPM and students with lower FTIPM significant?

The following are the hypotheses for questions 2 and 3:

- *Ho1*: No significant variations or differences exist between the mean scores of male and female students in solving trigonometry problems.
- *Ho2*: No significant variations or differences exist between the mean scores of students with higher and lower FTIPM in solving trigonometry problems.

Operational Definitions

- This study interchangeably uses mistakes and errors, referring to the different error types according to the modified Newman's Error Hierarchical Model.
- SSCP (step-by-step controlled practice): teaching students using steps and helping them practice again using those steps

Theoretical Review

Theories and models guide researchers by explaining and helping to predict a particular phenomenon (Prakitipong & Nakamura, 2006). This study's theoretical framework was based on a well-supported theory (Grant & Osaloo, 2014). We used the Modified Newman's Error Hierarchical Model to categorize students' difficulties in solving the selected trigonometry problems (Newman, 1977). According to this model, there are five types of errors: (1) Reading errors happen when students cannot detect keywords or symbols central to laying a solid initial solution to the problem. (2) Comprehension error is related to the meaning of the problem. As a result, when students do not understand the meaning of the problem, they then use an incorrect problem-solving approach. (3) Transformation error is related to a student's ability to convert a given meaning of the problem into the correct mathematical expression or a diagram, which can simplify the problem-solving process for the student. (4) Process Skill error is related to students' knowledge of correct steps or operations to solve the problem accurately. (5) Encoding error is related to a student's ability to communicate the correct answer using the correct mathematical notation, even after solving the problem correctly. Furthermore, the errors are hierarchically arranged, which means failure to read and understand the meaning of the problem hinders an individual from correctly solving the problem (Arhin & Hokor, 2021).

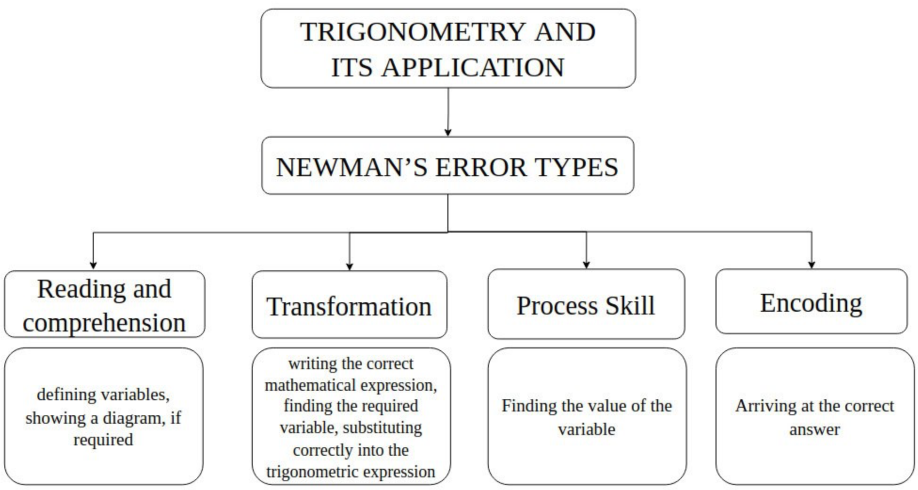

Figure 2. Modified Newman's Error Hierarchical Model

Newman's model provides a sequential diagnosing tool by categorizing the cognitive processes of solving word problems into five stages. The model's hierarchical nature reflects its simplicity, as an error in reading leads a student to make an error in the processing, transformation, and encoding stages. Furthermore, the practical model enables teachers to design suitable instructional strategies to help their students. In conclusion, we found the model relevant to analyzing the typical steps in solving the trigonometry achievement test questions.

Methodology

This research used a quantitative approach to identify students' errors in solving trigonometry problems related to the subtopics of distance bearing, angles of elevation, and angles of depression. It was conducted at Menelik II Secondary School in the Arat Kilo area of Addis Ababa, Ethiopia. The school was selected based on two criteria: proximity and accessibility.

The target population was Grade 10 students. The research population comprised 246 students from six Grade 10 classes, each containing 41 students. All students in Grade 10 mathematics had completed the trigonometry chapter and begun studying circle geometry. The sample size, representing 16.67% of the total school population, is within the acceptable range of 10% to 20% (Gorard, 2021).

A purposive sampling method was used to select the school because it was convenient and relevant to the study. Stratified sampling was used to create groups (strata) to ensure a balanced representation, as each group, strata, or section contained 41 students. Random sampling was then conducted by randomly selecting students' names from different sections using a computer to form the main group to which the research instrument would be applied.

Data Collection

The Trigonometry Achievement Test (TAT) was designed to assess and evaluate students' proper understanding of fundamental trigonometric ratios, the extent to which they misunderstood them, and their ability to solve problems related to bearings, angles of elevation, and angles of depression. The TAT asked students to write their gender (male or female), their Family Total Income per Month (FTIPM), and their name (which would not be used for reporting purposes). Students were assured anonymity. Before the test day, students were told to ask their parents about their FTIPM. Students were told about social desirability bias to prepare them to bring the correct data.

Furthermore, students were told about the disadvantages of dishonesty and to be honest. Additionally, we collected FTIPM data from the school, checked it against the data we obtained from students, and confirmed the data to be true. On the other hand, the test contained ten open-ended questions that analyzed students' problem-solving mistakes. The first five questions measured general trigonometry to evaluate mastery, while the last five questions aimed to measure specific applications such as distance bearing and angles of depression.

Scoring and Test Construction

The scoring was based on the complexity of each question, with marks allocated out of 4 or 5 accordingly. Three key criteria guided the test construction process. The first criterion was difficulty level, balancing easy and difficult questions. The second criterion was question variety, ensuring coverage of all essential subtopics of the trigonometry chapter.

The third criterion was expert consultation, involving experienced teachers in constructing the test items to create high-quality and relevant test items.

The study used a multi-stage sampling approach and a properly designed test instrument to identify common errors in trigonometry problem-solving among Grade 10 students.

Population

Table 1. Population of six sections of Grade 10 students at Menelik II secondary school

Sections	Male(M)	Female(F)	Total (M+F)
Section A	20	21	41
Section B	21	20	41
Section C	20	21	41
Section D	19	22	41
Section E	20	21	41
Section F	20	21	41

[1]Source: Menelik II secondary school, 2024

Table 2. Family Total Income per Month (FTIPM) population of six sections of Grade 10 students at Menelik II secondary school

Sections	Higher FTIPM	Lower FTIPM	Total
Section A	21	20	41
Section B	21	20	41
Section C	21	20	41
Section D	19	22	41
Section E	21	20	41
Section F	21	20	41

[2]Source: Menelik II secondary school, 2024

Research Processes

To ensure the validity of the TAT, experts in mathematics education and assessment from Addis Ababa University were consulted to verify that the test items accurately evaluate the intended sub-areas. A pilot test was also conducted at a preparatory school to assess the instrument's consistency. The scores were analyzed using Cronbach's alpha reliability measure, yielding a reliability coefficient of 0.707, indicating high reliability (Fraenkel & Wallen, 2005).

The school heads gave permission, and participants were briefed on the study's objectives. Confidentiality and anonymity were protected. Additionally, all participants completed the test under standardized conditions at the same time. Test items were collected, coded, and prepared for data storage and analysis.

Using content analysis, we identified specific student errors in solving the trigonometric achievement test instrument questions. We also used descriptive statistics and the modernized Newman method to categorize and interpret the errors. Furthermore, we used inferential statistics (Python) to generalize the findings.

Assumptions for Independent Sample *T*-Test

Three assumptions were considered. (1) Because test scores appeared normally distributed (See Figure 2 and Figure 3) with slight deviations from a straight line, normality was verified using Q-Q Plots. Students' test scores (dependent variable) were normally distributed within the two categories (male and female). Similarly, students' test scores (dependent variable) were normally distributed within the two groups (higher FTIPM and lower FTIPM). (2) Homogeneity of variance was tested using Levene's test, and *p*-values above 0.05 indicated no significant differences in variances between groups. (3) The scores of male and female participants were found to be independent, and the scores of higher and lower FTIPM students were found to be independent.

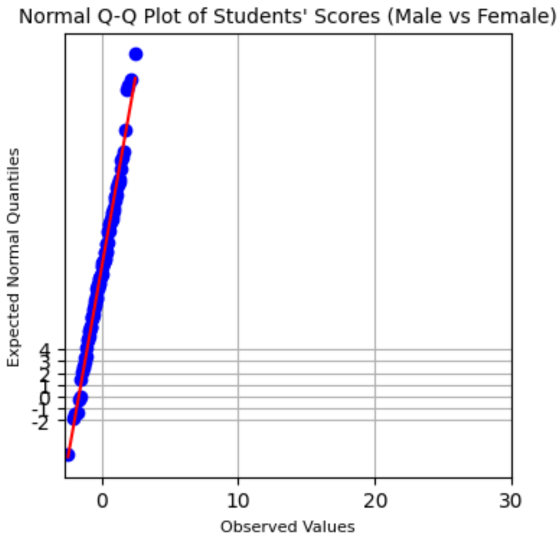

Figure 3. Normal Q-Q plot of students'(Male vs Female)

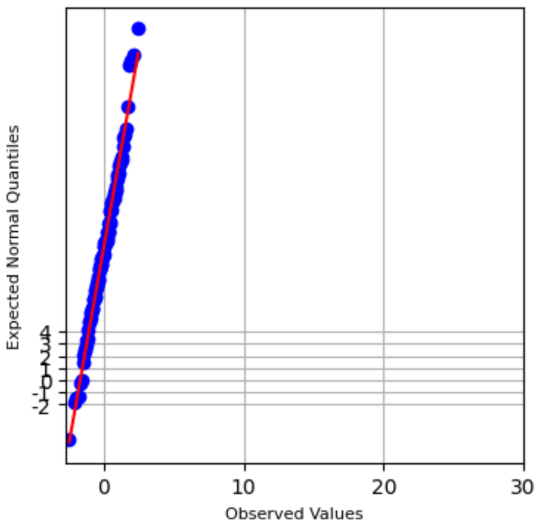

Figure 4. Normal Q-Q plot of students' (Higher FTIPM Vs. Lower FTIPM)

We applied Eta squared to measure the magnitude of differences in means. As a result, we applied the following interpretation: 0.2 corresponds to a small effect, 0.5 corresponds to a moderate effect, and 0.8 corresponds to a large effect (Cohen, 1988).

Table 3. Results Summary: Grade Ten Students' Errors in Solving Trigonometry Problems

Item	Level	Attempted	Attempted %	Error	Error %
Item 1	Reading and Comprehension	33	80.4878	17	41.46341
	Transformation	33	80.4878	20	48.78049
	Processing skills	31	75.60976	19	46.34146
	Encoding	26	63.41463	16	39.02439
Item 2	Reading and Comprehension	37	90.2439	27	65.85366
	Transformation	36	87.80488	27	65.85366
	Processing skills	35	85.36585	26	63.41463
	Encoding	35	85.36585	25	60.97561
Item 3	Reading and Comprehension	35	85.36585	7	17.07317
	Transformation	33	80.4878	14	34.14634
	Processing skills	32	78.04878	25	60.97561
	Encoding	31	75.60976	30	73.17073
Item 4	Reading and Comprehension	32	78.04878	20	48.78049
	Transformation	32	78.04878	23	56.09756
	Processing skills	30	73.17073	23	56.09756
	Encoding	24	58.53659	18	43.90244
Item 5	Reading and Comprehension	36	87.80488	17	41.46341
	Transformation	36	87.80488	20	48.78049
	Processing skills	35	85.36585	19	46.34146
	Encoding	32	78.04878	20	48.78049
Item 6	Reading and Comprehension	29	70.73171	19	46.34146
	Transformation	26	63.41463	19	46.34146
	Processing skills	21	51.21951	19	46.34146
	Encoding	20	48.78049	19	46.34146
Item 7	Reading and Comprehension	26	63.41463	17	41.46341
	Transformation	23	56.09756	18	43.90244
	Processing skills	21	51.21951	17	41.46341
	Encoding	21	51.21951	18	43.90244
Item 8	Reading and Comprehension	21	51.21951	17	41.46341
	Transformation	11	26.82927	10	24.39024
	Processing skills	7	17.07317	7	17.07317
	Encoding	6	14.63415	6	14.63415
Item 9	Reading and Comprehension	20	48.78049	7	17.07317
	Transformation	14	34.14634	13	31.70732
	Processing skills	13	31.70732	13	31.70732
	Encoding	13	31.70732	13	31.70732
Item 10	Reading and Comprehension	14	34.14634	6	14.63415
	Transformation	10	24.39024	5	12.19512
	Processing skills	9	21.95122	5	12.19512
	Encoding	9	21.95122	6	14.63415

[3]Note: Distribution of Students' Error in Solving Trigonometry Problems

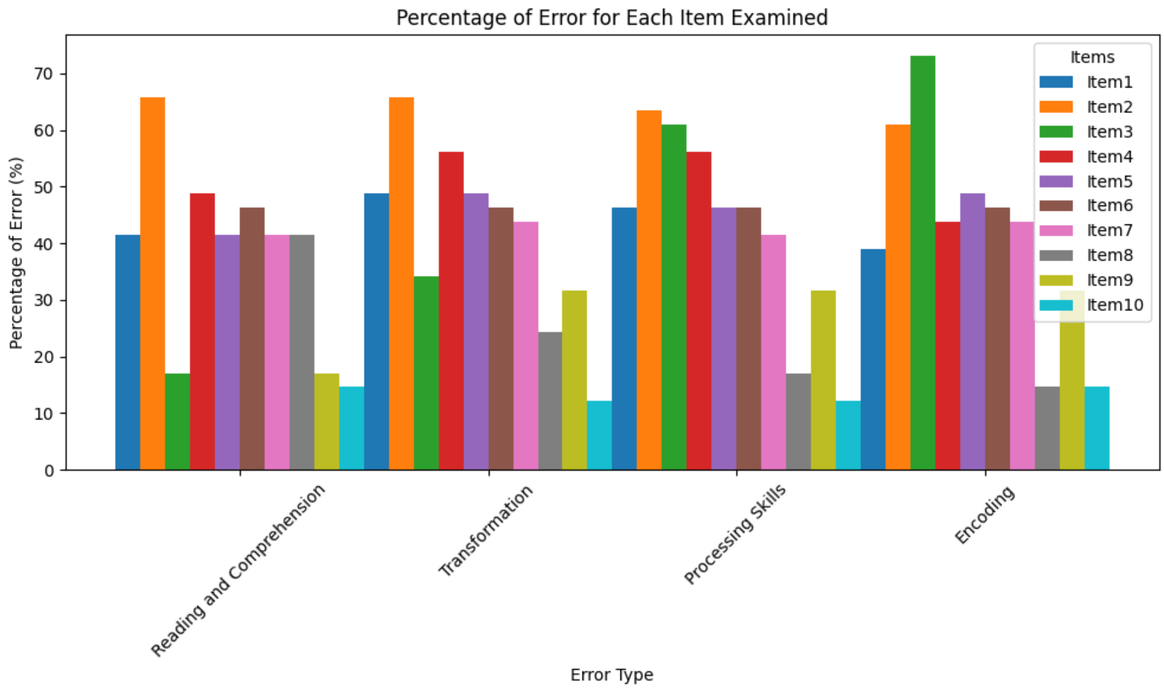

Figure 5. Percentage of error for each item examined

According to our theoretical review, we used the modified Newman's Error Hierarchical Model (NEHM) to analyze the errors at four levels. The first one was reading and comprehension, and according to the result, most students attempted to read and comprehend, but many struggled to understand the problems thoroughly. For example, while 80.49% attempted Question 1, only 39.03% understood it.

Table 3 demonstrates an attempt rate above 50% in reading and comprehension levels. In contrast, the error rate ranges from moderate to high. For example, it is 65.85% on Question 2. Consistency describes the error nature of transformation and processing skills, often exceeding 50% (Figure 4), implying significant difficulty in these areas. Additionally, encoding has the lowest attempt percentage and the highest error rate, which implies that students struggle in the final stage of the problem-solving process, which requires them to write the answer in the correct format. For example, the attempt rate for Question 8 was 14.3%. For Question 2, many students made errors (i.e., transformation error rate of 65.85% and encoding error rate of 60.98%). The encoding and processing skills level of Question 3 showed error rates of 73.17% and 60.98%, respectively (see Figure 5), which implies that many students demonstrated a significant struggle in executing and expressing their answers in the correct format. There is consistency in the transformation and processing skills on Questions 4 and 5 (over 50%). Students demonstrated minimal processing and encoding skills for Questions 8 and 9, with 17.7% and 14.63% attempt percentages, respectively. Conversely, students did not make many errors in item 10, which implies that it was the most straightforward problem.

In general, most students struggled with the encoding stage, and their errors in the transformation and processing skills demonstrated their misunderstanding about the applications of those trigonometric ideas tested. Therefore, efforts should focus on helping students develop the proper understanding needed to succeed in the processing and transformation stages. Finally, efforts should also focus on helping the students develop their ability to write answers using the correct format or encode them properly.

Analysis of Students' Errors in Exhibit 1 (Figure 6)

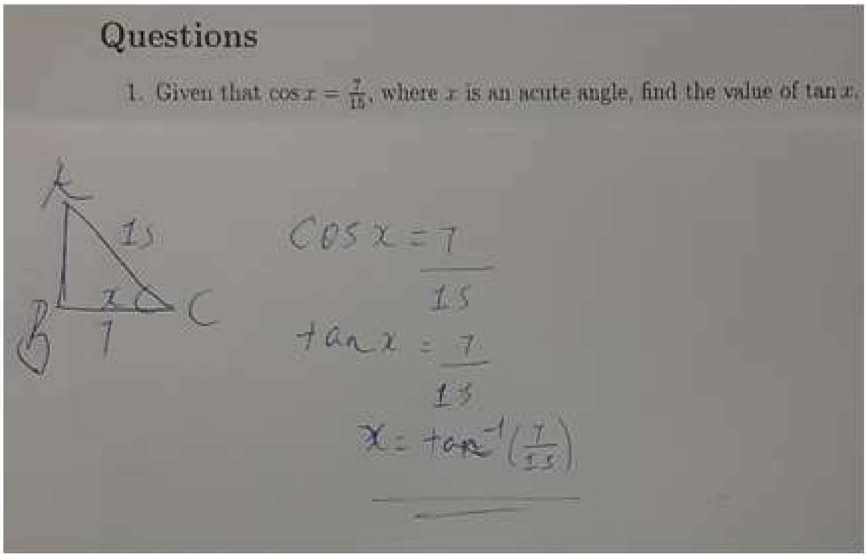

Figure 6. Analysis of students' errors in Exhibit 1

Table 3 denotes that most students successfully passed the reading and comprehension level stage by identifying the sides of the triangle from the questions given, Exhibit 1 reveals that students struggle to correctly write the necessary mathematical statements using the Pythagorean Theorem's mathematical principle. Since this transformation error is caused by a gap in students' conceptual understanding, targeted instructions that aim to help students go beyond comprehension and reach the level of correctly applying the correct mathematics are needed. One form of targeted instruction may be to teach students how to use steps and help them practice using them again, which is called *step-by-step controlled practice (SSCP)*. We may also use visual aids to help students see how mathematics can be transformed through the support of visual aids. Additionally, teachers should bring real-world scenarios containing word problems to the class so that students can practice how word problems can be converted to mathematical expressions, which can be used to solve these word problems.

Analysis of Students' Errors in Exhibit 2 (Figure 7)

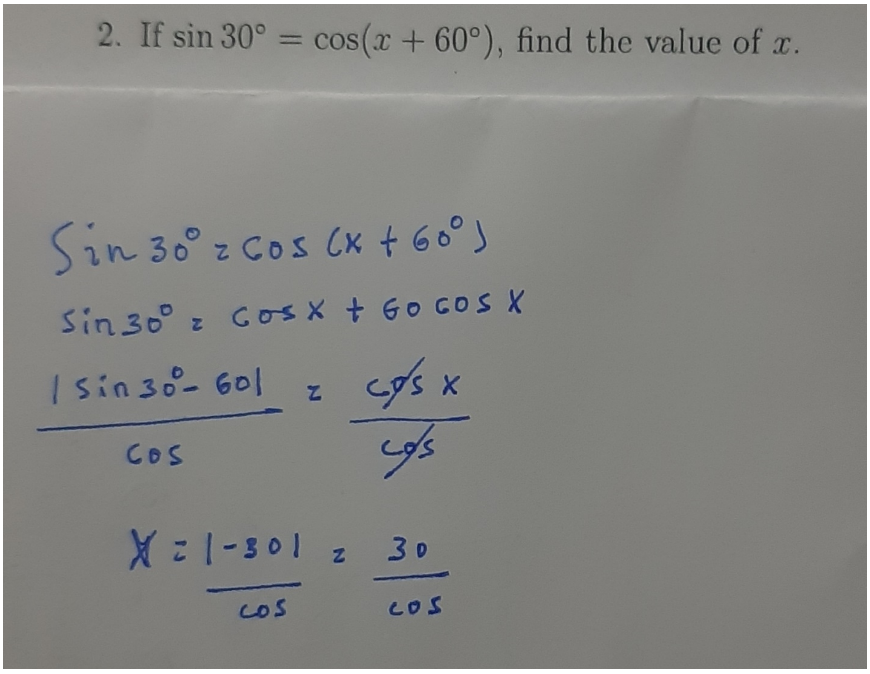

Figure 7. Analysis of Students' Errors in Exhibit 2

The principle that should be applied in Exhibit 2 is the complementary angle relationship. As a result, if $\alpha+\beta=90^0$, then $\sin\alpha=\cos\beta$. Students could not remember the fundamental trigonometric identity and thus simply applied an incorrect way of solving called distributive property to the trigonometric expression. Students did not understand $\cos(x+60^0)$ as a single trigonometric function, which incorrectly led them to apply the distributive property. Some demonstrated an incorrect application of distributive property: $\cos(x+60^0)=\cos x+60\cos x$ rather than $\cos(A+B)=\cos A\bullet\cos B-\sin A\bullet\sin B$, based on $\cos(x+60^0)=\cos x\bullet\cos 60^0-\sin x\bullet\sin 60^0$. Students' misconceptions caused this process error.

Process errors have many learning implications. Teachers should help students internalize trigonometric identities using angle sum and difference formulas. Teachers should help students understand the difference between algebraic and trigonometric expressions and when to use the distributive property. Teachers should instruct students about derivations of common trigonometric identities, which can help them internalize the formulas and the correct mathematical ways they can use and are not allowed to use. Students should also be taught about real-world applications of trigonometric formulas to help them grasp the formulas and their common associated methods.

Analysis of Students' Errors in Exhibit 3 (Figure 8)

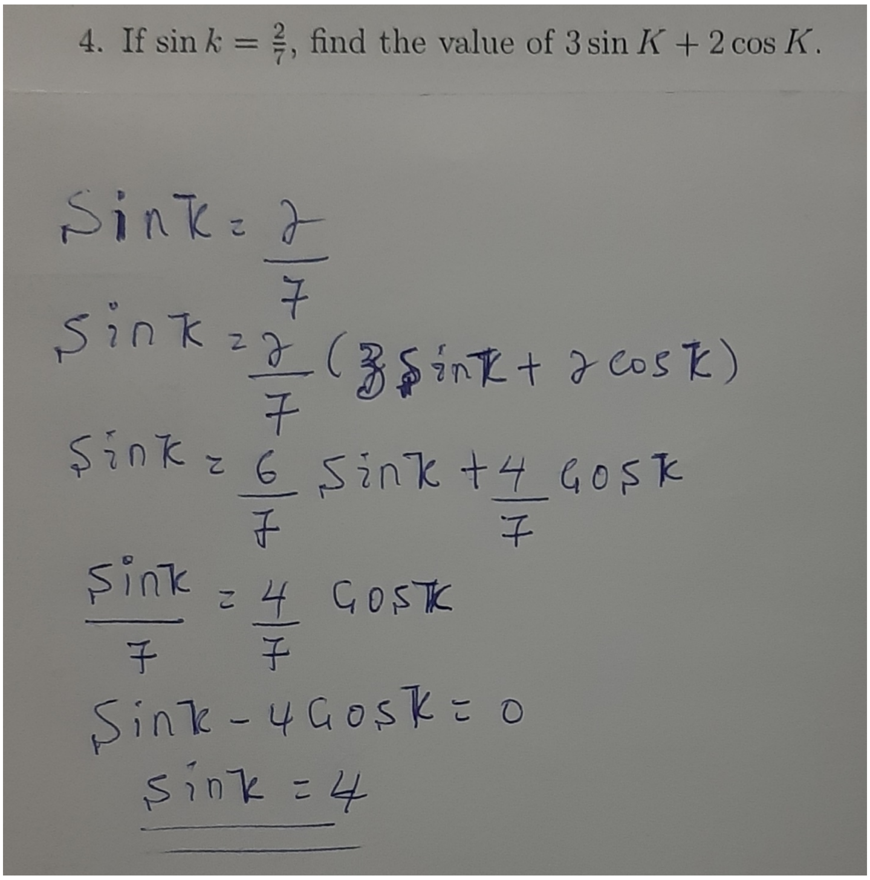

Figure 8. Analysis of Students' Errors in Exhibit 3

Exhibit 3 demonstrated students' inability to transform from sin*k* to cos(90^{o}-K) (process error). The first observed error in Exhibit 3 is a misapplication of the formula, as students did not relate sin*k* with cos(90^{0}-K). They also made a substitution error (process skill). Substitution errors imply a lack of fluency skills in performing mathematical operations. Therefore, the student was not able to reach the encoding level.

Suggested teaching strategies to avoid these issues include helping students understand the relationship between sine and cosine using complementary angles, which can be achieved using visual aids such as unit circles and right-angled triangles. Focus can be placed on helping students develop strong writing skills, using these expressions to substitute for the givens and obtain results. Teachers can also demonstrate efficient problem-solving strategies, helping students divide one problem into many diminutive problems so that they can solve each problem to reach the complete solution of the problem.

Analysis of Exhibit 4 (Figure 9)

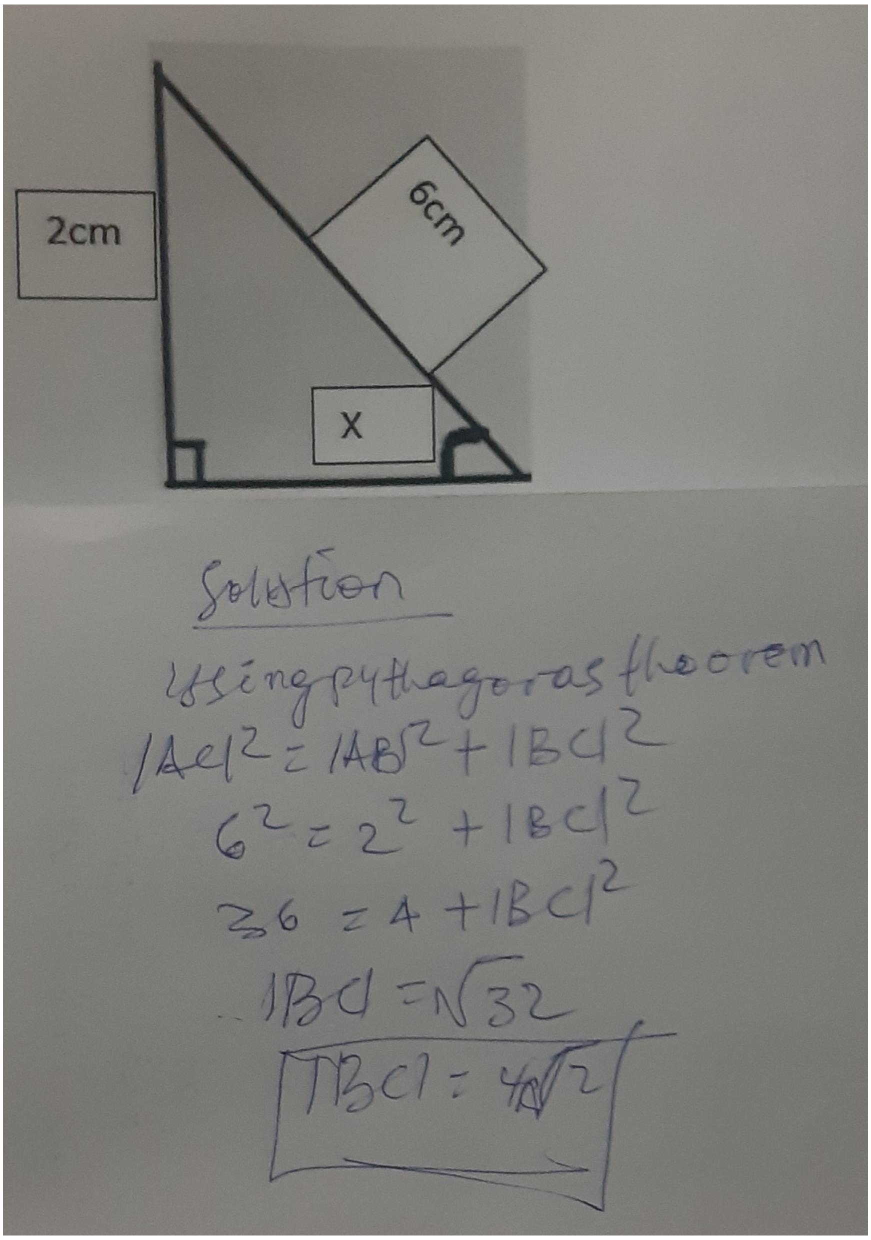

Figure 9. Analysis of Students’ Errors in Exhibit 4

In this exhibit, instead of using the arcsine (inverse sine) function to isolate x, the student mistakenly used the Pythagorean theorem, which was not required for solving this problem. Three key observations can be derived from this exhibit: The student misinterpreted the question because the interpretation was to simply use sin$x$=(2c)/(6c) and then take the inverse sin to obtain the value of x. Unlike the previous exhibits, the student struggled to comprehend the problem, demonstrating transformation and process skill errors. Students also demonstrated conceptual confusion between when and how Pythagoras' theorem and trigonometric ratios can be applied. The students applied Pythagoras' theorem to find a side, but the question asked the students to find an angle.

Specific teaching strategies to avoid these issues include: Teachers can encourage students to read and understand problems carefully before starting the problem-solving process. Students should practice correctly identifying the correct method, either trigonometric function or Pythagorean Theorem, to solve the problem. Students should be told to check whether or not the method they chose works to produce an accurate result by cross-checking their choice with other examples similar to the type of question they are now trying to solve.

Analysis of Exhibit 5 (Figure 10)

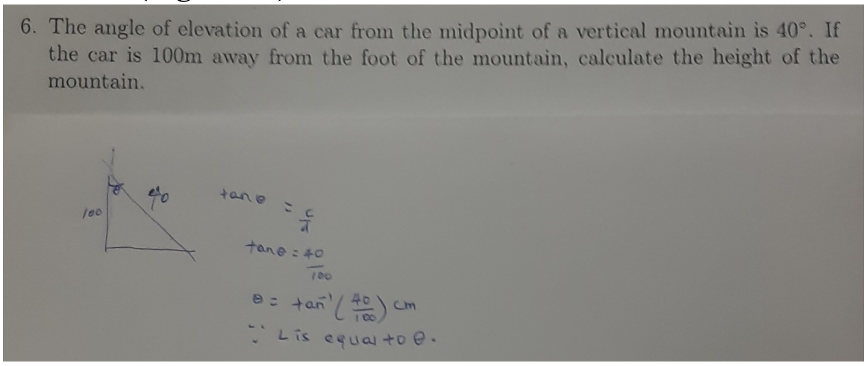

Figure 10. Analysis of Students' Errors in Exhibit 5

This exhibit demonstrated that many students struggle with applying trigonometric concepts. This exhibit also contained different mistakes at different stages of the problem-solving process, which implied a weak understanding and proficiency. Students struggled to define variables and write correct mathematical equations, suggesting a diminutive understanding of the trigonometric ratios (e.g., sin, cos, and tan).

Nearly half of the students demonstrated reading and comprehension difficulty, which blocked them from adequately interpreting the problem; out of 70.73% of students who attempted this problem, 46.34% made an error. Conversely, even students who passed the reading and comprehension level struggled to transform the word problem into the correct mathematical equation correctly. Students also failed in the calculation and encoding stage due to a lack of fluency in trigonometric calculations.

Suggested teaching strategies include: teaching students how to understand problems and convert the given problem into mathematical equations; using visual aids or diagrams and interactive models whenever teachers teach about sin, cos, and tan; and teaching structured problem-solving steps, ranging from reading and understanding the problem to double-checking their work.

Table 4. TAT scores among male and female students

Group	N	Minimum	Maximum	Mean	Std. Deviation
Male	20	0	28	14	8.08
Female	21	0	19	9.5	5.48

Table 5. Independent samples *t*-test on gender differences in errors in trigonometry

Group	N	Mean	Std. Deviation	*t*-Value	Df	*p*-Value	Eta Squared
Male	20	14	8.08	2.0963	39	0.0426	0.1013
Female	21	9.50	5.48				

Table 6. TAT scores among those with higher FTIPM and lower FTIPM

Group	N	Minimum	Maximum	Mean	Std. Deviation
Higher FTIPM	20	0	28	14.00	8.08
Lower FTIPM	21	0	19	9.5	5.48

Table 7. Independent samples *t*-test on FTIPM difference in errors on trigonometry.

Group	N	Mean	Std. Deviation	*t*-Value	Df	*p*-Value	Eta Squared
Higher FTIPM	20	14	8.08	2.0963	39	0.0426	0.1013
Lower FTIPM	21	9.50	5.48				

In Table 4, male students had a greater mean score (mean=14, SD=8.08) than females (mean=9.50, SD=5.48). Therefore, male students' made fewer errors than female students.' As reported in Table 5, an independent samples *t*-test conducted to test the significance level of the difference revealed to us that the difference in errors between males and females was statistically significant at the 95% confidence interval ($t(39)=2.0963$, $p<0.05$). Therefore, female students made significantly more errors than male students. Furthermore, the eta-squared revealed to us that the difference in errors was small based on Cohen's (1988) criteria because of the eta-squared value ($\eta^2=0.1013$) (see Table 5).

In this study, the mean score of students with higher FTIPM was 14 (SD=8.08) (see Table 6). In contrast, the mean score of students with lower FTIPM was 9.50 (SD=5.48). Therefore, a higher performance rate was observed among students with higher FTIPM, and an independent samples *t*-test revealed that the difference in these scores (higher versus lower FTIPM) was statistically significant ($t(39)=2.0963$, $p<0.05$) (see Table 7).

Discussion

Students often find trigonometric problems challenging and error-full is a skill most students have, but solving them is challenging and error-full, with most errors occurring in the comprehension, transformation, and numerical calculation stages (Marinas & Clements, 1990; Mensah, 2017; Singhata, 1991). Among the error types discovered in this study's student work include: misunderstanding formulas (e.g., incorrectly expanding $\sin(x+30^0)$); applying the Pythagorean Theorem to both right-angled and non-right-angled triangles; improperly converting the given word problem into the correct trigonometric equation (Norasiah, 2002); and incorrectly applying the distributive (Skane & Graeber, 1993).

In our study, we found that female students' made more errors in the trigonometric achievement test than male students, and this result aligns with previous researchers' findings (Brown & Kanyongo, 2010), demonstrating that boys generally perform better in mathematical tasks than girls.

To overcome this gender discrepancy, we propose some pedagogical recommendations: Provide female students with specialized tutorial classes that aim to make them excellent in specific weaknesses in their ability to solve trigonometric problems. Invite female role models in mathematics and science to classes to inspire female students and provide encouragement when needs arise. Use gender-sensitive teaching, lesson planning, and annual planning to fulfill the academic as well as emotional needs of girls. Apply the 5E Model (Tuna et al., 2013; Cepni et al., 2012), which improves students' trigonometric thinking and problem-solving by engaging students, making them explore, providing explanations and elaborations, and evaluating their progress to modify strategies for the best way of learning and teaching.

Previous research showed a positive correlation between FTIPM and academic achievement (Gobena, 2018). Our research found that students with lower FTIPM committed more trigonometric errors on the trigonometric achievement test (TAT) than students with higher FTIPM. For all students, we recommend using manipulative materials (Ulyani & Qohar, 2021) and visual representations to increase student understanding. Additionally, we recommend providing free reference books, computer access for educational sites to learn the ideas of trigonometry at their own pace, and free tutorial support and counseling services to help students with lower FTIPM.

Students generally experience difficulty learning trigonometry, especially understanding and applying trigonometric ideas. Female students and those with lower FTIPM need extra, targeted, and altruistic support. Furthermore, teachers should adjust their strategies using action research and other findings, focus on teaching students to understand the big picture, and provide focused support to anyone who needs it.

Conclusion and Implications

This study found that most students performed transformation, processing, and encoding errors. The lowest attempt and highest error rates were also seen at the encoding level. Furthermore, female students made more errors than males, with a statistically significant result, and students with higher FTIPM made fewer errors than students with lower FTIPM counterparts. Unfortunately, the methodology of this study disallows us from determining the sources of these errors. However, based on the findings, we recommend teachers use the 5E model (Tuna et al., 2013; Cepni et al., 2012) constructivist approach as it contains evaluation, which can help teachers know their learners and provide the proper instruction. Additionally, teachers should use SSCP (step-by-step controlled practice), which means teaching students how to use the steps and helping them practice again using those steps. As a result, students will begin to see how they can develop their transformation, processing, and encoding skills using properly written steps of the solution processes of any given problem. The other recommendation is introducing manipulative materials to the class (Ulyani & Qohar, 2021). These materials are scissors, markers, and cardboard because they can make the teaching-learning process more practical and thus engaging. The other recommendation is to incorporate a game-based learning and technology environment into the teaching-learning platform (Prabowo et al., 2018) because it can make students understand the topics more clearly and can also make them more engaged in the learning process.

Additionally, GeoGebra is one example of technology that can help students understand trigonometric topics better. The other recommendation is to help students understand trigonometric functions before they start solving problems, which requires them to apply trigonometric functions. Our other recommendation is to provide targeted support to improve students' ability to encode, transform, and process. Finally, targeted support that aims to improve and bridge the gap between boys and girls should be given to girls, such as remedial support and counseling support to improve their mental well-being and academic ability. Additionally, targeted support should also be given to students with lower FTIPM. Furthermore, these supports can include monetary help, reference book support, and academic counseling support.

Limitations of the Study and Suggestions for Further Research

One limitation of the study is that the findings are based on data obtained from Menelik II secondary school and thus cannot be generalized to all schools worldwide. Additionally, the Ethiopian instructional culture differs from other countries' school cultures, so it cannot be reproduced. Furthermore, curriculum-specific factors, such as textbooks used, vary from country to country.

Several types of research should be conducted across multiple schools in grade 10 sections to understand better the students' errors in solving trigonometry achievement test problems. Future research on different schools should also focus on the relationship between FTIPM and trigonometry achievement. Furthermore, the effectiveness of the targeted solutions we suggested for females and students with lower FTIPM should be studied more to produce modified methods for helping these two groups, females and students with lower FTIPM.

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Appendix A: Trigonometric Achievement Test

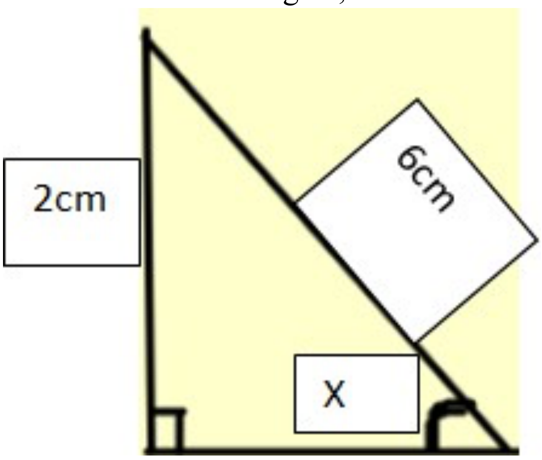
INSTRUCTIONS

1. Your performance in this test is not going to be added to your continuous assessment results.
2. This test is for research purposes aimed at improving the teaching and learning of trigonometry.
3. You are politely required to give your honest answers to all the questions on the paper provided.

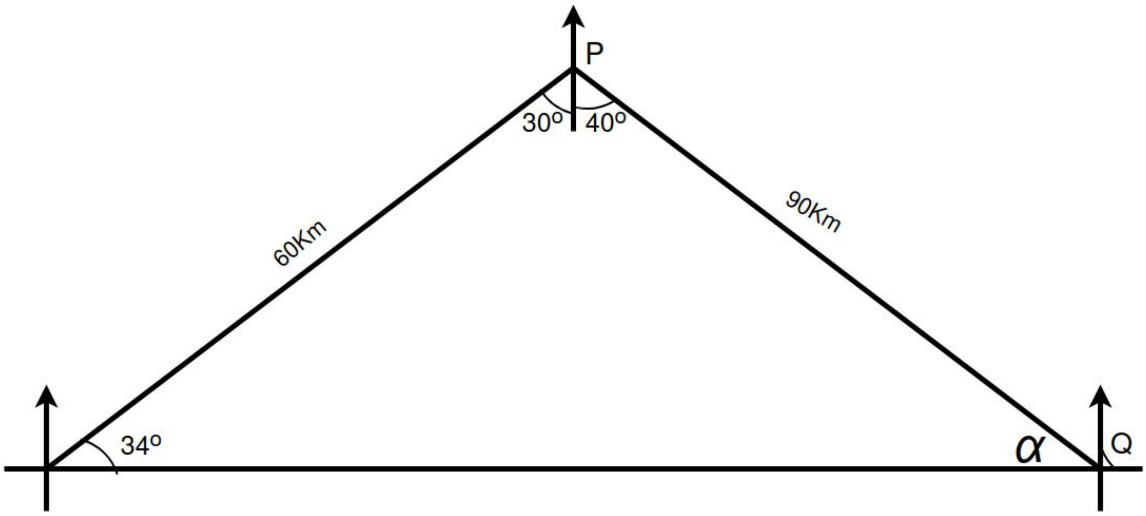
Student's registered number: ____ **Time allowed:** 1:30 hours
Sex: __ **Age:** __ **Class:** _______________
FTIPM (Family Total Income per Month): _________

Questions

1. Given that cosx=7/15, where x is an acute angle, find the value of tanx.
2. If $\sin 30^0 = \cos(x+60^0)$, find the value of x.
3. Given that $A = 30^\circ$ and $B = 60^\circ$, evaluate $\sin B + 2 \cos A \cos B$. (Express your answer in surd form). _____________
4. If sin k=2/7, find the value of 3 sin k + 2 cos k.
5. Calculate and round to the nearest degree, the value of x in the diagram below.



6. The angle of elevation of a car from the midpoint of a vertical mountain is 40°. If the car is 100m away from the foot of the mountain, calculate the height of the mountain.
7. The angle of elevation of the top of a flagpole is 30° from point *A*, which is 100 meters from the foot *B* of the flagpole. *C* is a point on the same horizontal line *AB* such that $|AC|$ = 30 meters. Calculate, rounding to one decimal place, the angle of elevation of the flagpole from *C*.
8. The bearing of point *P* from point *S*, 8 km away, is 300°. Another point *Q* is 4 km from *S* and on a bearing of 140°. Calculate:
   a. The distance *PQ*.
   b. The bearing of *P* from *Q*.
9. Using the diagram below, find and round to one decimal place:
   a. The distance between *R* and *Q*.
   b. The bearing of *R* from *Q*.



10. Two points A and C, on opposite sides of a vertical pole, are on the same level ground as the foot of the pole, B. The angles of elevation of pole D from A and C are 20° and 50° respectively. If the distance between A and C is 40 km, find $|BD|$, the height of the pole.