Mathematical and Pedagogical Knowledge amongst First- and Second-Grade In-service and Pre-service Mathematics Teachers

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Abstract

The study investigates the mathematical and the pedagogical content knowledge among in-service and pre-service first- and second-grade mathematics teachers. The sample of 300 subjects consisted of 150 first- and second-grade in-service teachers and 150 pre-service teachers studying in a college of education, 75 of whom were first-year students and 75 third- and fourth-year students. The data was collected using two tools - a mathematical content knowledge (MCK) test and a mathematics pedagogical content knowledge (MPCK) test, the items on each of these representing the four sub-domains studied in first- and second-grade: numbers, arithmetic operations, geometry/measurements, and word problems. The principal findings of the study indicate that all three groups possess limited knowledge of both MCK and MPCK in all sub-domains. The in-service teachers achieved the highest mean in the MCK and MPCK and the differences between the in-service teachers and the two groups of pre-service teachers were statistically significant in general MCK and MPCK and in all the domains. Moreover, the lowest MCK mean was in the geometry/measurements domain, and the lowest MPCK mean was in the word-problems domain.

Keywords: Mathematical content knowledge, pedagogical content knowledge, pre-service teachers, mathematics teacher.

Introduction

The responsibility for introducing seven year old children to mathematics has traditionally rested in the hands of first- and second-grade mathematics teachers. It has been shown that, in the first grade, teachers' background qualifications have a greater effect on students' achievements in mathematics (Boonen, Van Damme, & Onghena, 2014). The first grade of primary school is critical for providing all students with a basic understanding of mathematics and thus preventing future difficulties and learning disabilities (Clarke, Doabler, Nelson, & Shanley, 2015). Despite the importance of the role of mathematics teachers in the first and second grade, in Israel these teachers have generally been trained through the Department of Early Childhood Education that provides training both for pre-schooler teachers and first- and second grade teachers of primary school. This option is still provided by continuing education courses given at the end of the pre-school teacher training program. This track resembles the training in various other countries that allows first- and second grade teachers to teach language, science, and mathematics (Akerson, 2004; Cheang et al., 2007). The majority of education colleges in Israel offer two or three mathematics and mathematical education courses at most in their Early Childhood training programs. As a comprehensive study conducted in the United States has indicated, first- and second-grade mathematics teachers frequently lack mathematics qualifications (Malzahn, 2002).

There are, in addition, very few studies in the literature on the mathematical and pedagogical content knowledge required by first- and second-grade teachers which reflect the assumption that teachers understand foundational topics such as addition and subtraction, whole numbers, and other primary subjects (Mewborn, 2001) and how they teach these topics. In light of this gap in the literature, the present study examines the mathematical and pedagogical content knowledge of first- and second-grade teachers in Israel and questions whether their training within the framework of the Early Childhood track provides them with an adequate level of mathematical and pedagogical content knowledge. It is important to note that the current study examines the participants' mathematical and pedagogical content knowledge using items that relate to first and second grade curricula.

Theoretical background

Mathematical Content Knowledge (MCK)

Content knowledge includes the structure of knowledge, facts, theories and principles in the field (Shulman, 1986), constituting a foundational base of factual knowledge and an understanding of facts and ideas within a conceptual framework that facilitates the organisation, implementation, and recall of knowledge (National Research Council, 2010). The depth of content knowledge is expressed in insights regarding the historical development of central mathematical concepts and the relationship between the ideas, analogies, and images relating to the various principles (Davis & Simmt, 2006). Ball, Thame and Phelps (2008) refine mathematical content knowledge by separating it into two subcategories: common content knowledge and specialised content knowledge. The former relates to the content of the curriculum and includes recognition of the terms and correct notions and the ability to read and write these notions correctly, thus perceiving when the textbook gives a wrong definition. This knowledge is not unique to teachers alone and is understood by other educated non-teachers as well (Delaney, Ball, Hill, Schilling, & Zopf, 2008; Hill & Ball, 2004). Specialised content knowledge refers to the knowledge and skills of teachers, which surpasses the mathematical knowledge of educated non-teachers (Delaney et al., 2008); in other words, content knowledge needed for the teaching of mathematics and is beyond the common knowledge need by others (Markworth, Goodwin, & Glisson, 2009). It includes an understanding of mathematical structures which enables the handling of tasks that require significant mathematical resources (Ball et al., 2008) and thus allows teachers to follow students' mathematical thinking, evaluate the validity of student-generated strategies, and make sense of a range of student-generated solution paths (Hill, Ball & Schilling, 2008).

Mathematical Pedagogical Content Knowledge (MPCK)

Pedagogical content knowledge (PCK) combines an understanding of both content and pedagogy (Ball, Lubienski, & Mewborn, 2001). According to Shulman (1986), PCK comprises an awareness of students' difficulties and misconceptions about the concepts being taught, comprehension of the various ways in which the content is taught and represented, and an understanding of the teaching methods that make learning easy or difficult. PCK can be defined as the interaction between knowledge and the instructional strategies that help students to grasp the subject matter within the context of their cultural background and previous knowledge (National Council for Accreditation of Teacher Education, 2002). Carter (1990) defines PCK as the transformation of subject matter into a classroom curricular event.

Ball et al. (2008) divide mathematical pedagogical content knowledge into two sub categories: knowledge of content and students and knowledge of content and teaching. The former is a type of pedagogical content knowledge that combines an understanding of students with expertise in mathematics; in other words, content knowledge is intertwined with an awareness of how students think, know, and learn this specific mathematical content (Hill et al., 2008). The latter combines knowledge about teaching with knowledge about mathematics. Teachers need to be aware of which examples are appropriate as a starting point and understand the instruction design, the diverse representations of the explanatory concept, and how to evaluate the advantages and disadvantages of these representations (Ball et al., 2008).

In-service and Pre-service Teachers' MCK and MPCK

Findings from the extensive research into MCK and MPCK indicate that many in-service and pre-service mathematics teachers exhibit very low levels of both, as evident in their poor grasp of mathematical concepts, subjects, and components (Livy & Vale, 2011; Tutak, 2009). Despite the great interest in the field, relatively little attention has been paid to the knowledge appropriate for first- and second–grade mathematics (Mewborn, 2001).

While many first- and second-grade teachers are able to categorise addition and subtraction word problems and identify the distinction between problems and the strategies students employ to solve different kinds of problems, they find it difficult to organise this knowledge in a coherent network that integrates problem-distinction, problem difficulties, and student solutions. The teachers' success rate in organising word problems, characteristically correlates with student performance in solving these problems and not with the features of these problems (Carpenter, Fennema, Peterson, & Cary, 1988). Similarly Ma (1999), in a study comparing the mathematical knowledge of 72 Chinese and 23 American primary schoolteachers, finds that only one of the latter succeed in devising a word problem which suited the specific computation provided. In addition, while they were able to explain their solution processes for calculating addition and subtraction procedures, fewer than 20% of the participants have a conceptual understanding of the regrouping process and converting 10 into 10 units of one.

Similar findings have been obtained in relation to the numbers domain by Ball (1988) who report that, although the teachers in her study could perform algorithms correctly, they struggle to explain the algorithmic principle and its relationship to the place-value. According to Jones ' (2000) review, teachers also demonstrate limited knowledge of geometry; a domain that to date has attracted relatively little interest with far more studies addressing the numbers and operations domains (Tutak, 2009).

Research goal and questions

The primary goal of the current study is to discover whether the training of first- and second-grade mathematics teachers in the Early Childhood Education track provides them with adequate MCK and MPCK for the mathematical content of the first and second grades of primary school in four mathematical domains (numbers, arithmetic operations, geometry/measurements, and word problems). It also seeks to compare the MCK and MPCK levels of pre-service teachers in the initial and final stages of their training. In doing so, the study addresses the following questions:

1. To what extent do the first- and second-grade in-service mathematics teachers and pre-service teachers in the Early Childhood Education track, have limitations in their MCK/MCPK in four mathematical domains (numbers, arithmetic operations, geometry/measurements, and word problems)?
2. To what extent are the MCK/MCPK levels in four mathematical domains (numbers, arithmetic operations, geometry/measurements, and word problems) of pre-service teachers in their initial training different from pre-service teachers in their initial final stages of their training?

Method

Setting and Participants

The sample comprised of 300 subjects, all female, divided into 150 first- and second-grade in-service teachers and 150 pre-service teachers studying in the Early Childhood Education track, of which 75 were first-year students and 75 were third- and fourth-year students.

All of the in-service teachers had their qualification through the Early Childhood Education track. They were selected by a three-stage cluster sampling method: in the first stage, cities and towns were randomly select across the north of Israel. In the second stage, schools within those cities and towns were randomly selected with 37 schools participating in the study; and in the third stage, teachers with over five years' experience in the selected schools were randomly selected.

Most of the in-service teachers learned through their pre-service training through the Early Childhood Education track two or three courses in mathematics and mathematics education. In general the courses related to geometry and mathematics and indicated that most did not correlate to first- and second-grade curricula; more details about the course in the next section. The in-service teachers' background variables indicated a similarity between qualification processes of both in-service and pre-service teachers.

The Training Process for First- and Second-Grade Teachers

As noted above, although for many years first- and second-grade teachers could only train in the Early Childhood Education track in Israeli teacher training colleges, they can now also receive in their final studying year, some continuing education courses at the end of their training in the pre-school track. Pre-service teachers generally take three courses in mathematics and mathematics education.

Most colleges offer courses in geometry and mathematics. According to the syllabi, these courses cover the different topics in geometry and mathematics that are included in the curricula of grades K-9 such as: the concept of a point, a line, a ray, an angle, and a plane; the definition and classification of figures and polygons, perimeters and areas of geometric figures, systems of measurement, overlapping triangles and triangles similarity; and the delineation of the rules for adding, subtracting, multiplying, and dividing whole numbers, for primes and composite numbers, for word problems, and for fractions and decimals.

First- and Second-Grade Mathematics Curriculum

The mathematics curriculum of first-and second-grade in Israel comprises four sub-domains (Israel Ministry of Education, 2006): 1. Numbers; 2. Arithmetic operations; 3. Geometry/measurements; and 4. Word problems. The following details the contents for each domain:

1. Numbers - reading and writing of numbers up to 1000, odd and even numbers, the zero features, number line, digits value equations, inequalities, numbers divisibility by 2, 5, and 10;
2. Arithmetic operations - horizontal and vertical addition and subtraction up to 100, multiplication and division up to 6 * 6;
3. Geometry and measurements - length measurements, time measurements, area measurements, polygons, polyhedrons, translating and reflection;
4. Word problems - word problems in addition, subtraction, multiplication and division.

Instruments

MCK and MPCK assessment methods.

A proper assessment of teacher knowledge is based on an examination of content knowledge in specific domains rather than on the number of courses taken (Ball et al., 2001). Numerous studies of the MCK and MPCK of in-service and pre-service teachers have been conducted using various methods. Some employ items from already existing student tests such as the National Assessment for Educational Progress (NAEP) test, designed specifically for Grades 11 and 12 (Stevens & Wenner, 1996) and the Trends in International Mathematics and Science Study (TIMSS) test for Grades 4 through 8 (Wilkins, 2008; Tatto et al., 2008). Other tests address specific topics or domains taught in a particular grade or topics appropriate for two or more grades above. Southwell, White, Way, and Perry (2006) tested the mathematics knowledge of primary school teachers via tasks that involve four arithmetic operations: fraction operations, decimals, percentages, measurements, and word problems. The Learning Mathematics for Teaching (LMT) test employs tasks relating to Grade 8 numbers, arithmetic operations, functions, and algebra and relating to Grades 3–8 geometry as recommended by Hill, Rowan and Ball (2005). Tasks may also relate to the curricula of higher grades (Goulding, 2003).

The components used to assess MPCK also differ. Several studies have utilised items relating to different domains. Cheang et al. (2007), for example, employed 16 items relating to numbers, geometry, algebra, and data processing. Others have focused on the MPCK of specific mathematical topics - such as the mathematical knowledge of first- and third-grade teachers, by analysing the solution strategies adopted by their students to solve arithmetic operations (Hill et al.,2005) or the MPCK of first-grade teachers, by testing their ability to distinguish difficult problems, their general knowledge of strategies, and their specific knowledge of the strategies their students adopt to solve subtraction and addition word problems (Peterson et al., 1989).

Data in the current study was collected using two tools - a MCK test and a MPCK test, with the items on each representing the four sub-domains studied in first and second grade: numbers, arithmetic operations, geometry/measurements, and word problems. Some of the items were written by the researcher, while the remainder were collected from other sources including teacher evaluation projects, Ministry of Education tests of teacher professionalism, and various studies (for example, Ball & Bass, 2000; Hill et al. 2004).

MCK Test.

The MCK test was structured along the lines recommended by Ball et al. (2001) who stressed that assessment should relate to subjects that the participant is currently teaching or

> If k is the unit's digit of a three-digit number ($k \neq 0$) and the tens digit is twice as much as the unit's digit and the hundreds digit is three times as much greater than the unit's digit, what is the correct algebraic expression of this number?
>
> a) 3k + 2k + k
> b) 300k + 20k + k
> c) 100k + 20k + 3k
> d) Another answer

hopes to teach in the future. It consists of 30 items representing four sub-domains, numbers, arithmetic operations, geometry/measurements, and word problems. The results of internal consistency Cronbach's α for the overall test was high (0.89), the items were chosen to reflect first- and second-grade curricula. The following item was selected from the Ministry of Education professionalism test, to examine the mathematical content knowledge of the numbers-decimal representation and place-value.

This item is similar to items in the second-grade curricula, such as writing the extended name of the number 235 which will be equal to 200+30+5. Teachers with inefficient content knowledge are likely to choose option (a) as an answer. Another example, a question related to the reflection topic in first- and second-grade curricula is the following item:

The participants need to answer the reflection along the long side of the rectangle. Participants with inefficient content knowledge about reflection may be not successful in answering the question. Another example related to even and odd numbers in first- and second-grade curricula is the following item:

Participants with inefficient content knowledge about even and odd numbers may be not successful in presenting their answer based on the two meanings of even and odd numbers - that even numbers can separated into two equal groups whilst odd numbers cannot be separated into two equal groups; in addition, even numbers can be separated into pairs while odd numbers will have one element left over (For other examples see Appendix A).

MPCK Test.

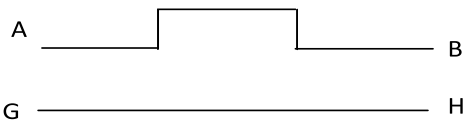
The items used to assess MPCK as proposed by Cheang et al. (2007) include comprehension of mathematical structures and relationships, understanding of difficulties,

> Explain why, when adding an even number to an odd number, the result is an odd number. Explain in two different ways, if possible

> Dina drew a rectangle and reflected it along one of its sides. The combination of the rectangle and the reflection produced a square. Was the reflection along the long or the short side of the rectangle? Explain your answer.

misconceptions, student solution strategies and suitable responses, knowledge of alternative teaching methods and the representation of various explanatory concepts. The 25 items (for more examples see Appendix B) represent four sub-domains: numbers, arithmetic operations,

Students answer that the two paths AB and GH are equal



Explain their comparative strategy

geometry/measurements and word problems. The results of internal consistency Cronbach's α for the overall test was high (0.90). An example of an item constructed by the researcher from the geometry and measurements domain examines the understanding of the development of students' thinking about length.

Mrs. Jackson wants to work with groups of students who are making the same kind of mistake. When she looks at a recent quiz to see how they approach the problem, she sees these three miscalculations:

III	II	I
1	1	1
32	45	38
14 +	37 +	49 +
19	29	65
64	101	142

Which contain the same kind of error? (Mark ONE answer only):
a) I and II
b) I and III
c) II and III
d) I, II, and III

This item is taken from Battista (2006) and it presents a basic-level student solution, which uses a holistic visual comparison, in which the students look only at the start and end points of the two paths and compare them. Another example is related to arithmetic operations (multidigit addition) in the following item taken from Hill et al., (2004).

The participants need to identify the strategy of the three students, and determine who has made the same error. The students I and II made the same error, carrying 10 instead of 20 (For other examples see Appendix B).

A correct answer gained one point, incorrect answers and non-responses gained zero points. Five scores were calculated for each participant for each test: overall score, numbers score, arithmetic operations score, geometry/measurements score, and word problems score. The total was derived from 10 scores.

Procedure

The in-service teachers performed the tests during the academic year, the first-year pre-service teachers at the start of the year, and the third- and fourth-year pre-service teachers at the end of the academic year. The two tests were administered individually at one-week intervals, the tests were distributed by the researcher. While no time limitation was imposed for completion, each test required an hour on average. The in-service teachers' test was administered at the beginning of the school day, and participants were asked to complete it by the end of the day, they did not do the tests in test conditions, but they were asked to do the test individually, the researcher collected the tests. The pre-service teachers' test was administered as part of a regular class.

Data analysis

SPSS version 21 was used to analyse the quantitative data. The means and SD for each group in the MCK and MPCK in four domains were calculated. For the MCK and MPCK one-way Anova tests were demonstrated to find if the difference between the different domains are significant. The post hoc Scheffé tests were demonstrated to find the source of the differences between the groups. Paired-samples t-test were demonstrated to find if the differences between the domains are significant.

Then, in order to specify the most difficult items in the MCK and MPCK questionnaires, the solution for each question were examined and the frequencies of incorrect answers to each question were calculated, in order to follow the common wrong strategies used.

Findings

MCK

The findings indicated that in-service teachers displayed a mean of 64 scores (on a scale of 0-100), third-and fourth-year pre-service teachers a mean of 46 scores, and first-year pre-service teachers a mean of 36 scores. Regarding the differences obtained across all the groups in all the domains, the highest mean was found in the numbers domain and the lowest mean in the geometry/measurements domain. (See Table 1).

Table 1: MCK Means/SDs of the three groups of the study

		M	SD	N
MCK	In-service teachers	64.35	20.29	150
	Third/fourth-year pre-service teachers	46.05	19.86	75
	First-year pre-service teachers	35.96	14.84	75
	Total	52.67	22.51	300
Numbers	In-service teachers	70.54	19.81	150
	Third/fourth-year pre-service teachers	60.19	16.48	75
	First-year pre-service teachers	48.38	21.83	75
	Total	62.41	21.44	300

Arithmetic operations	In-service teachers	67.77	23.26	150
	Third/fourth-year pre-service teachers	55.02	31.43	75
	First-year pre-service teachers	46.67	22.81	75
	Total	59.31	26.90	300
Geometry	In-service teachers	57.02	27.84	150
	Third/fourth-year pre-service teachers	31.66	24.34	75
	First-year pre-service teachers	21.33	16.25	75
	Total	41.76	29.07	300
Word problems	In-service teachers	70.22	29.19	150
	Third/fourth-year pre-service teachers	52.00	26.63	75
	First year pre-service teachers	43.56	27.92	75
	Total	59.00	31.15	300

A one-way ANOVA demonstrated that the differences between the groups in MCK in general and in each domain individually, were statistically significant. There was a statistically significant difference between the groups at MCK $F(2,97) = 62.07$ $p< 0.001$; There was a statistically significant difference between the groups at the MCK Number domain $F(2,97) = 33.07$ $p< 0.001$; There was a statistically significant difference between the groups at the MCK Arithmetic operations $F(2,97) = 18.14$ $p< 0.001$; There was a statistically significant difference between the groups at the MCK Geometry $F(2,97) = 61.31$ $p< 0.001$; There was a statistically significant difference between the groups at the MCK Word problems $F(2,97) = 24.06$ $p< 0.001$.

The source of the disparity between the groups in the MCK in general and in all domains was the in-service group. The post hoc Scheffé test showed that in-service teachers had significantly higher mean scores in the MCK in general and in all domains than the other two groups at the .05 level of significance. According to the differences between the two groups of pre-service teachers, the post hoc Scheffé test showed that Third/fourth-year pre-service teachers had significantly higher mean scores in the MCK and in the numbers and geometry/measurements domains (separately) than the First-year pre-service teachers at the .05 level of significance.

The lowest MCK domain between the three groups was the geometry/measurements mean. As Table 2 indicates, the paired-samples t-test demonstrated a significant difference between the geometry/measurements mean and each of the other three MCK domains.

Table 2: Paired-Samples T-Test between the Geometry/Measurements Domain and Other MCK Domains

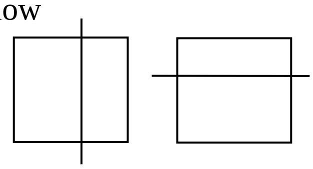
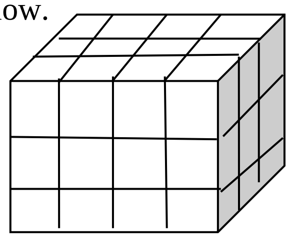
		M	SD	T
In-service teachers	Geometry–numbers	-13.51	27.92	-5.92*
	Geometry–operations	-10.74	23.74	-5.54*
	Geometry–problems	-13.20	22.68	-7.12*
Third/fourth-	Geometry–numbers	-28.53	23.90	-10.33*

year pre-service teachers	Geometry–operations	-23.36	28.86	-7.00*
	Geometry–problems	-20.34	24.18	-7.28*
First-year pre-service teachers	Geometry–numbers	-27.04	22.52	-10.40*
	Geometry–operations	-25.33	23.71	-9.25*
	Geometry–problems	-22.22	27.34	-7.03*

*p<.001

In order to specify the most difficult items in the MCK questionnaire, the frequencies of incorrect answers to each question were calculated and the solution examined. Two items belonging to the geometry and measurement domain were found to have the highest percentage of mistakes (see Table 3).

Table 3: Items in the MCK Questionnaire with the Most Wrong Answers

Item	Percentage of wrong answers	Common wrong answers
If a square is separated into two parts by a straight line, which polygon with the largest number of sides may you get?	85% of the answers	A rectangle. Participants drew a square and divided it into two rectangles as described below  It is important to note that among all the answers suggesting a rectangle, the most common answer was to cross the square into two overlapping parts via either a vertical or a horizontal line.
Write the dimensions of a box whose volume is three times the box drawn below.  1 cm 1 cm	73% of the answers	The dimensions are 9,9,12.

MPCK

The findings demonstrated that the general MPCK mean was lower than the MCK mean in all the domains in all the groups. The highest mean of 54 scores (on a scale of 0-100) was observed among in-service teachers, third-and fourth-year pre-service teachers showed a mean of 21 scores, and first-year pre-service teachers a mean of 16 scores. (See Table 4).

Table 4: MPCK Means/SDs of the three groups of the study

		M	SD	N
MPCK	In-service teachers	54.50	26.93	150
	Third/fourth-year pre-service teachers	21.57	21.72	72
	First-year pre-service teachers	16.81	16.34	75
	Total	37.00	29.35	297
MPCK Numbers	In-service teachers	34.42	34.42	150
	Third/fourth-year pre-service teachers	40.28	49.21	72
	First-year pre-service teachers	23.33	27.97	75
	Total	47.59	40.73	297
MPCK Arithmetic Operations	In-service teachers	52.54	30.81	149
	Third/fourth-year pre-service teachers	23.02	22.81	72
	First-year pre-service teachers	20.38	19.39	75
	Total	37.21	30.59	296
MPCK Geometry	In-service teachers	52.34	30.31	149
	Third/fourth-year pre-service teachers	21.64	27.96	72
	First-year pre-service teachers	15.33	20.91	75
	Total	34.99	32.18	296
MPCK Word problems	In-service teachers	52.49	33.59	149
	Third/fourth-year pre-service teachers	9.13	16.49	72
	First-year pre-service teachers	10.38	17.06	75
	Total	31.27	34.10	296

One-way ANOVA proved the differences in MPCK in general and in each domain individually to be statistically significant between the groups. There was a statistically significant difference between the groups at the MPCK $F(2,294) = 85.30$, $p<.000$; There was a statistically significant difference between the groups at the MPCK Numbers F(2,294) = 30.61 ,p<.000 ; There was a statistically significant difference between the groups at the MPCK Arithmetic Operations F(2,294) = 50.49 ,p<.000 ; There was a statistically significant difference between the groups at the MPCK Geometry F(2,294) = 53.43 ,p<.000; There was a statistically significant difference between the groups at the MPCK Word problems F(2,294) = 95.22 ,p<.000.

The source of the disparity between the groups in the MPCK in general and in all domains individually was the in-service teacher group. The post hoc Scheffé test showed that in-service teachers had significantly higher mean scores in the MPCK in general and in all domains than the other two groups at the .05 level of significance. The sole significant difference between the two groups of pre-service teachers lay in the numbers domain as demonstrated by the post hoc Scheffé; this showed that the Third/fourth-year pre-service teachers had significantly higher mean scores in the MPCK numbers and domain than the First-year pre-service teachers at the .05 level of significance

The lowest MPCK mean was found in the word-problems domain. The paired-samples t-test indicated that the differences between the word-problem mean and the other MPCK domains were significant among the pre-service teacher groups but not among the in-service teachers (see Table 5).

Table 5: Paired-Samples T-Test between the Word-Problems Domain and Other MPCK Domains

		M	SD	T
In-service teachers	MPCK problems-numbers	-10.48	32.97	-3.88*
	MPCK problems–operations	-0.04	31.77	-0.01
	MPCK problems–geometry	-1.51	28.94	-0.48
Third/fourth-year pre-service teachers	MPCK problems–numbers	-31.15	45.50	-5.80*
	MPCK problems–operations	-13.88	23.97	-4.91*
	MPCK problems–geometry	-12.51	24.06	-4.41*
First-year pre-service teachers	MPCK problems–numbers	-12.95	27.72	4.04*
	MPCK problems–operations	-10.00	19.33	4.48*
	MPCK problems–geometry	-4.95	20.11	2.13**

*p<.001, **p<.05

In order to specify the most difficult items in the MPCK questionnaire, the frequencies of incorrect answers to each question were calculated and the solution examined. Two items belonging to the word-problems domain were found to have the highest percentage of mistakes (see Table 6).

Table 6: Items in the MPCK Questionnaire with the Most Wrong Answers

Item	Percentage of wrong answers	Common wrong answers
Write a word problem that appropriates to the following exercise. ? + 4 = 1 2 Pay attention to the variable position in the exercise.	86% of the answers	Different word problems with the initial mount 4 such as: Sami has 4 marbles. His friend gave him some marbles, so now he has 12. How many marbles did his friend give him?
Sort the division word problems for the second grade into two categories. What characterises each category?	84% of the answers did not include a partitive division or measurement division	The two categories most frequently offered were long division and regular division.

Comparing MCK and MPCK

As Tables 1 and 4 demonstrate, each of the groups studied revealed a higher MCK than MPCK mean. A paired-samples t-test between the means for each of the groups, showed that each group also displays a higher mean and statistically-significant difference between MCK and MPCK. In relation to the in-service teachers A paired-samples t-test indicated that scores were significantly higher for the MCK (M = 64.35, SD = 20.29) than for the MPCK (M = 54.50, SD = 26.93), t (149) = 5.58, $p < .001$**.** In relation to third- and fourth-year pre-service teachers, a paired-samples t-test indicated that scores were significantly higher for the MCK (M = 54.09, SD = 19.63) than for the MPCK (M = 21.57, SD = 21.56), t (71) = 9.411, $p <$ .001**.** According to first-year pre-service teachers, a paired-samples t-test indicated that scores were significantly higher for the MCK (M = 35.96, SD = 14.84) than for the MPCK (M = 16.81, SD = 16.34), t (74) = 11.37, $p < .001$**.**

Discussion

The findings of the current research indicate that both first- and second-grade in-service teachers experience difficulties with both MCK and MPCK with respect to the contents taught in these grades. This result is consistent with Malzahn’s 2002 study according to which almost half of the second-grade teacher participants reported their need to improve and deepen their mathematical and pedagogical content knowledge.

The participants studying in the Early Childhood Education track - both those at the beginning of their studies and those in their final year, also demonstrated poor MCK and MPCK levels. This may be attributed to their failure to remember school-level mathematics (Hamlet, 2007) or to a general weakness in mathematics.

The MCK and MPCK differences between third-and fourth-students and first-year students were only significant in some domains. While this might suggest a deficiency in their training, the disparities between the in-service teachers and MCK and MPCK in general and in each domain individually, indicate that the MPCK of in-service teachers differed and was higher than at the end of their teacher training, as has been proposed in other studies. Experienced in-service teachers, however, showed almost the same MCK scores as student teachers at the end of their training (Kleickmann, Richter, Kunter, Elsner, Besser, Krauss, & Baumert, 2013) - a disparity which may derive from experiential learning on the job (Cheang et al., 2007).

The lowest score among the three research groups was found in the geometry/measurements MCK - a finding consistent with and highlighted in other studies conducted among teachers (Jones, 2000; Baturo & Nason, 1996). This result may also reflect a lack of knowledge and poor academic training (Jones, 2000) with novice teachers being expected to teach the subject based on the geometry they learned in high school.

The predicted research finding regarding geometry/measurements was confirmed, showing that pre-service teachers are reluctant to teach geometry during their practicum. Inadequate training in geometry/measurements may influence the attitude of teachers towards this subject, and their reluctance to teach it may adversely affect their students' geometric abilities (Tutak, 2009) and explain why geometry is often ignored in early education (Sarama & Clements, 2009). The cycle can be broken by special training and intervention programs in geometry/measurement which are designed to expand MCK and deepen MPCK (Nason, Chalmers & Yeh, 2012).

The weakest domain in MPCK was word problems. Participants exhibited a greater capacity to solve word problems and less ability to deal with tasks relating to writing word problems in specific exercises, particularly in cases in which the initial group is unknown. They also demonstrated difficulty in sorting division problems, despite the fact that both types form part of the second-grade curriculum. The findings of the present study in this regard are consistent with those of Carpenter et al. (1988).

The two weakest MCK and MCPK domains among the teachers were reported as also the weakest among the students. The latter regarded word problems as a difficult subject (Rogers, 2004) with geometry/measurements ranking as the weakest domain among students from different countries who participated in the TIMSS (Tatsuoka, Corter, & Tatsuoka, 2004). The correlation between teachers' knowledge and students' achievements is verified in the literature (Hill et al., 2005; Peterson et al., 1989).While numbers was viewed as the strongest MCK and MPCK domain among the three study groups, the level of knowledge in this domain is also relatively poor. The findings demonstrate that the numbers domain forms the basis of all the other domains.

Most significantly, the results reveal a difference between MCK and MPCK levels; the latter being higher than the former in all the groups. This finding is consistent with the literature (Turnuklu & Yesildere, 2007). This disparity reflects the fact that MPCK includes MCK and also encompasses the analysis of solution strategies, misconceptions, teaching methods, and other components. All the groups likewise encounter content knowledge to a far greater extent, especially in high school and during academic training. Similarly, MCK components and the use of this knowledge are more prevalent than those of MPCK in everyday life.

The current study findings regarding the low levels of MCK and MPCK among first- and second-grade in-service and pre-service teachers in the content they will teach has been highlighted in various studies (Amarto & Watson, 2003; Morris, 2001; Livy & Vale, 2011), and they emphasise the need to reinforce MCK in teacher-training programs and continuing education courses for teachers (Tutak, 2009; Hill et al., 2005).

Summary and recommendations

The study's findings indicate that Grade 1 and 2 MCK and MPCK levels among first- and second-grade in-service and pre-service teachers are low across all the domains. The principal conclusion is, therefore, that the training process of first- and second-grade teachers in the Early Childhood Education track is inadequate.

These results highlight the importance of revising the professional development programs of first- and second-grade mathematics teachers. In addition, they emphasise the need to expand the program of the Early Childhood Education track by adding courses in mathematics and mathematics-education courses. Mathematics and mathematics education courses should deal with the mathematical and pedagogical content aspects of first- and second-grade curricula and should offer special intervention programs in various mathematics domains, in particular in geometry/measurements and word problems - the two domains considered the weakest areas by both in-service and pre-service teachers. The recommendation of enrichment courses is based on the findings of other studies which have indicated that first-graders demonstrate higher levels of achievement in reading and mathematics in schools where teachers report increased coursework in these subject areas during their training (Croninger, Rice, Rathbun, & Nishio, 2007).

In light of the current findings and in order to offer adequate stimulation for the cognitive development of every child, it is recognised that besides general educational knowledge, teachers also require knowledge of developmental psychology and competence, mathematics didactics and mathematics itself (Thiel, 2012). There is, therefore, a need to adapt mathematics specialisation and integrate it within the Early Childhood Education track, thus enabling teachers to qualify with two majors: mathematics and early childhood. This process will provide teachers with a wide background in mathematical approaches, thus equipping them to employ different methods of teaching mathematical concepts and providing them with both expertise in mathematics and an awareness of how to communicate with children. By making mathematics teaching a specialty, it can be expected that the mathematical aptitude of teachers will increase, giving them a greater opportunity for study and for collegial interaction (Ma, 1999).

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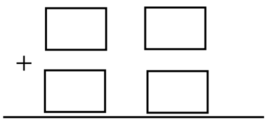
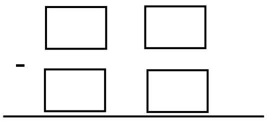
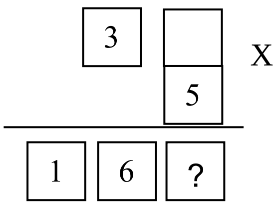
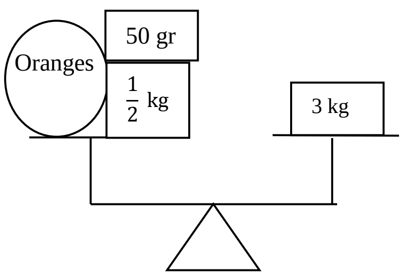
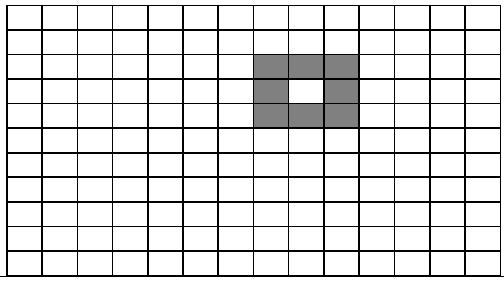
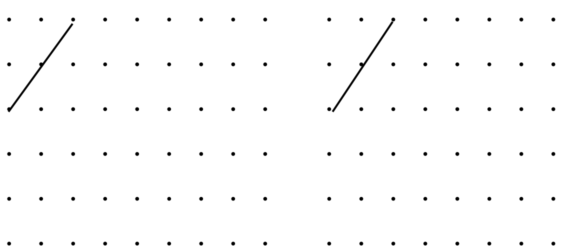
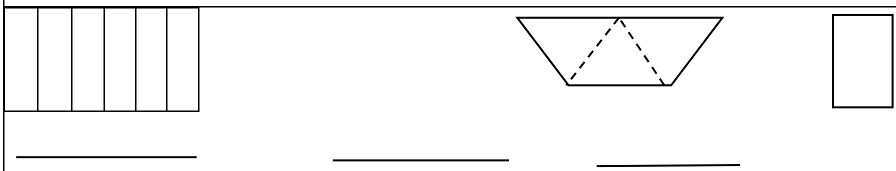
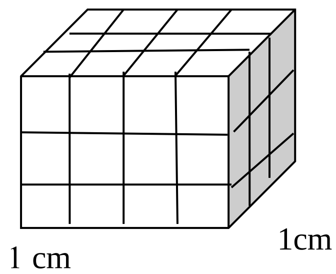
Appendix A

The MCK items test and the source of each item

<table>
<tr><th>Item</th><th>Source</th></tr>
<tr><td>Ms. Dominguez was working with a new textbook and noticed that it paid more attention to the number 0 than the old book. She came across a page that asked students to determine whether statements about 0 were true or false. Intrigued, she showed them to her sister - who is also a teacher and asked her what she thought. Which statement(s) should the sisters select as true? Mark YES, NO, or NOT SURE for each item below:
<table>
<tr><th></th><th>Yes</th><th>No</th><th>Not sure</th></tr>
<tr><td>a) 0 is an even number</td><td></td><td></td><td></td></tr>
<tr><td>b) 0 is not really a number but a placeholder in writing large numbers</td><td></td><td></td><td></td></tr>
<tr><td>c) The number 8 can written as 008.</td><td></td><td></td><td></td></tr>
</table>
</td><td>(Hill et al. 2004)</td></tr>
<tr><td>258a2 is a 5-digit number; “a” is the tenth digit. What could be “a” if the given number is divided by 2? Write all the possibilities.</td><td>Ministry of Education professionalism test</td></tr>
<tr><td>Given that all the digits in a 3-digit number are even and equal, is the given number divisible by 10 without a remainder? Explain your answer.</td><td>Ministry of Education professionalism test</td></tr>
<tr><td>If k is the unit’s digit of a three-digit number ($k \neq 0$), the tenth digit is twice as much as the unit’s digit and the hundredth digit is three times as much as the unit’s digit. What is the correct algebraic expression of this number?
a) $3k + 2k + k$
b) $300k + 20k + k$
c) $100k + 20k + 3k$
d) Another answer</td><td>Ministry of Education professionalism test</td></tr>
<tr><td>Explain why, when adding an even number to an odd number, the result is an odd number. Explain in two different ways, if possible.</td><td>Researchers</td></tr>
<tr><td>If “a = 685–269,” complete the equation 785–169 = □ by using “a”</td><td>Ministry of Education professionalism test</td></tr>
</table>

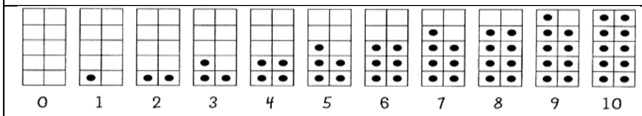
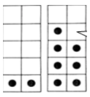
Let N be a positive integer. If you divide this number by 2, you get a remainder 1. If you divide this number by 3, you get a remainder 1. If you divide this number by 6, you get a remainder 1. What is the smallest number that has all these three properties?	Ministry of Education professionalism test
Use the numbers 3, 4, 6, and 9. Write them in the following squares in order to get the greatest result.  	Researchers
Use the numbers 3, 4, 6, and 9. Write them in the following squares in order to get the smallest positive result.  	Researchers
Given the following multiplication exercise, write the missing numbers in the two empty squares.  	Researchers
If $4 \times m = k$, (Circle the correct answer) m is a multiple of 4. k divided by m is 4. m divided by k is 4. all of the above answers are wrong.	Researchers
The following figure describes a balance, with a bag of oranges and standard masses in both pans. The two sides are balanced.  What is the weight of the bag of oranges?	Ministry of Education professionalism test

Dina drew a rectangle and reflected it along one of its sides. The combination of the rectangle and the reflection produced a square. Was the reflection along the long or the short side of the rectangle? Explain your answer.	Ministry of Education professionalism test
Given a shape with a greyed area. Sketch network lines with a different form (different pattern) that has the same area of the given shape. 	Ministry of Education professionalism test
In each net draw a rectangle with a vertex laying in a point of the net and one of its edges is on the drawn segment. 	Ministry of Education professionalism test
If a square is separated into two parts by a straight line, which polygon with the largest number of sides may you get?	Researchers
 The above figures are cover faces (without the bases) for three solids. Can you know what the solids are?	Ministry of Education professionalism test
Write the dimensions of a box whose volume is three times the box drawn below. 	Ministry of Education professionalism test

Given a triangular pyramid and triangular prism that have congruent bases, if we glue the base of the pyramid to the base of the triangular prism, vertex-to-vertex, we get a polyhedron. A. How many vertices does the polyhedron have? __________ vertices. B. How many sides does the polyhedron have? __________ sides. C. How many faces does the polyhedron have? _____________ faces.	Ministry of Education professionalism test
How many centimetres are there in 1.5 m?	Researchers
How many meters are there in 250 cm?	Researchers
A game began at 11:30 and ended at 14:15. How long did the game last?	Researchers
The difference between two numbers is 72. What is the larger number if the smallest is 123?	Researchers
The son's age is 29; the father is 32 years older than his son is. How much is the sum of their ages?	Researchers
Rami bought 3 kg apples and paid for them with N Shekels. Simon also bought apples at the same store, and he paid 40 Shekels. What was the quantity of apples that Simon bought? Express your answer using N.	Ministry of Education professionalism test

Appendix B

The MPCK items test and the source of each item

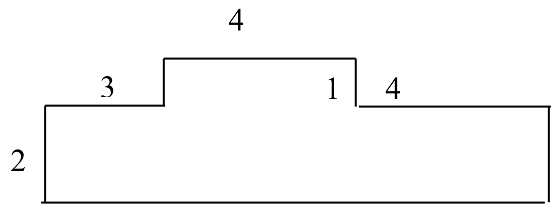
Item	Source
 The teacher showed her students many patterns of numbers from zero to 10 (shown above), and asked her students to represent the number 27 used the patterns. Rami chose two patterns and ordered them in the following way:  Explain Rami's solution. Which pattern would you choose?	(Losq 2005)
The teacher presented the series "90, 63, 38, 7" to the students and asked them to circle the even numbers. In checking the answers, she discovered that one of the students did not circle any of the numbers. A. What does this fact reveal regarding the student's understanding of the odd-even domain? B. What numbers would you present to test a student's understanding of the nature of even numbers?	(Ball & Bass 2000)
Mrs. Jackson wants to work with groups of students who are making the same kind of mistake. When she looks at a recent quiz to see how they approach the problem, she sees these three miscalculations: III: 1 / 32 / + 14 / 19 / 64 II: 1 / 45 / + 37 / 29 / 101 I: 1 / 38 / + 49 / 65 / 142 Which contain the same kind of error? (mark ONE answer only): a) I and II b) I and III c) II and III d) I, II, and III	(Hill et al. 2004)

<table>
<tr>
<td>Mrs. Jackson wants to work with groups of students who are making the same kind of mistake. When she looks at a recent quiz to see how they approach the problem, she sees these three miscalculations:

$$\begin{array}{ccc} \text{I} & \text{II} & \text{III} \\ \begin{array}{r} 4\ \ 12 \\ 5\ 0\ 2 \\ -\quad\ 6 \\ \hline 4\ 0\ 6 \end{array} & \begin{array}{r} 4\quad 15 \\ 3\ 5\ 0\ 0\ 5 \\ -\qquad\quad 6 \\ \hline 3\ 4\ 0\ 0\ 9 \end{array} & \begin{array}{r} 6\ 9\ 8\ 15 \\ 7\ 0\ 0\ 5 \\ -\qquad\ 7 \\ \hline 6\ 9\ 8\ 8 \end{array} \end{array}$$
Which contain the same kind of error? (mark ONE answer only):
a) I and II
b) I and III
c) II and III
d) I, II, and III</td>
<td>LTM learning mathematics for teaching</td>
</tr>
<tr>
<td>One student uses the following method to solve a subtraction exercise.

$$\begin{array}{r} -\ 3\ 7 \\ \underline{1\ 9} \\ -\ 2 \\ \underline{2\ 0} \\ 1\ 8 \end{array}$$
How do you think the teacher should respond?
Tell the student the method works only in this exercise but not in other exercises.
Tell the student it is not correct method according to the logic of mathematics.
Tell the student to test this method on all the numbers in subtraction exercises.
Tell the student that the method only works in certain exercises.</td>
<td>(Ball & Hill 2002).</td>
</tr>
<tr>
<td>Rami solved the multiplication problem in the following way:

$$\begin{array}{r} 2\ \ \\ 35 \\ *\quad 4 \\ \hline 2\ 0\ 0 \end{array}$$
Explain how Rami solved the exercise.</td>
<td>(Kamii & Dominick 1998)</td>
</tr>
<tr>
<td>Rank the following exercises according to their first-grade learning sequence:
12 + 4 = ?
4 + 3 = ?
8 + 4 = ?</td>
<td>Researchers</td>
</tr>
</table>

<table>
<tr><td>Tammy solved the addition exercises, some of them in the right way and some in the wrong way, as described in the exercises below:
$$\begin{array}{ccccc} 85 & 42 & 8 & 18 & 46\\ + & + & + & + & +\\ \underline{6} & \underline{56} & \underline{16} & \underline{30} & \underline{3}\\ 19 & 98 & 15 & 48 & 13\end{array}$$
What is the answer to the sixth exercise according to Tammy's strategy?
$$\begin{array}{c} 72\\ +\\ \underline{5} \end{array}$$
</td><td>Lim-Teo, Chua, Cheang & Yeo 2007)</td></tr>
<tr><td>A pupil solved the following exercises:
$$\begin{array}{ccc} 604 & 546 & 648\\ - & - & -\\ \underline{238} & \underline{308} & \underline{217}\\ 242 & 431 & 434\end{array}$$
Explain her strategies.</td><td>Ministry of Education professionalism test</td></tr>
<tr><td>Given the exercise: 3 + 12 : 3 – 2 x 2 =
Suggest three solutions students may mistakenly offer.</td><td>Researchers</td></tr>
<tr><td>Rami, Dani, and Sami calculate the perimeter of the following shape.


Rami's response was 28.
Dani's answer was 26.
Sami's response was 27.
Explain how each student has calculated the perimeter?</td><td>Researchers</td></tr>
<tr><td>A student claimed that the two paths AB and GH are equal.


Explain the student's comparative strategy.</td><td>Researchers</td></tr>
</table>

Rami is required to complete the following shape as a 6-sided polygon. Rami completed it as a 5-sided polygon with 6 vertices. However, he was sure that he had drawn a polygon with six sides and six vertices.  Complete the shape from Rami's perspective.	Researchers
When teaching children about measurement for the first time, Mrs. Ho prefers to begin by asking them to measure the width of their book first using paper clips and then using pencils. Why do you think she prefers to do this rather than simply teaching the children how to use a ruler?	(Lim-Teo et al. 2007)
Write six word problems represented by six given number sentences: 5 + 7 =? 6 +? = 11 ? + 4 = 12 13 – 4 =? 15 –? = 9 ? – 3 = 9	(Carpenter, et al., 1988)
Sort the division word problems for the second grade into two categories. What characterises each category?	Researchers