Pattern Finding Skills of Pre-school Children

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This study investigates the pattern finding skills of pre-school children and the in-class pattern activities conducted by teachers. The research was designed as a descriptive survey study carried out with a total of 162 children aged 60-77 months from families with middle socio-economic status. The findings of the study revealed that the children's pattern-finding achievement was above average and that there was not a significant difference between the pattern finding skills of the children in terms of gender and age. Lastly, in relation to pattern categories, the results showed that the children were more successful in the repetitive pattern category and pattern sub-categories, and structures were effective in the children's pattern-finding performance. In addition, the teachers were found to use repetitive patterns more frequently and triple-structured patterns most frequently in in-class pattern activities.

Introduction

In the Pre-school Education Curriculum (MEB), (2013, p. 42) it was stated that mathematics education intends to contribute to the cognitive development of children, to foster children's positive attitudes towards mathematics to help children to understand why and how mathematical concepts are used. Through mathematics activities, children should notice the patterns around them and develop assumptions and experiences; they should be able to solve problems, reason and communicate by using mathematical concepts. In this context, when the objectives of pre-school mathematics education are considered, it can be said that studies on pattern finding occupy an important place in realising these objectives. Mulligan and Mitchelmore (2009) stated that in some of studies conducted with four to eight years old students, it has been found that an awareness of mathematical pattern and structure is both critical and salient to mathematical development among young students.

Generalisation of patterns is used as a route to algebra and a possible way to approach algebra and to develop algebraic thinking (Borralho and Barbosa, 2011; Radford, 2010; Warren and Cooper, 2008). Similarly, McGarvey (2013) has stated that exploring patterns in the early years is observed as an important introduction to algebraic thinking as children begin to notice similarities and differences between and among patterns, create rules to describe relationships, and eventually represent those relationships using symbols. NCTM (2010) emphasises that effective mathematical reasoning involves the ability to note patterns and structures in both real-world situations and symbolic objects; such reasoning enables the formation of generalisations in which the abstraction of ideas and relationships can take place. Therefore; the pattern finding skills gained during the pre-school period are important for children's development of mathematical reasoning skills, mathematical thinking skills and relationship-building skills (Papic and Mulligan, 2005).

The children in this developmental period have some informal mathematical knowledge. Children can usually count to a specific number in a rhythmic way, write down some numbers and recognise some geometrical shapes and patterns related to some shapes and numbers (Papic, 2007).

Almost all mathematics is based on pattern and structure (Mulligan and Mitchelmore, 2013). If defined in a simple way, a pattern is a form, a template, a model (or, more abstractly, a set of rules) (Swoboda, 2009). Patterns are constructions that are created by aligning objects, shapes or numbers in a specific order. Presenting the rules of these series can be thought of as mathematical modelling. Determining the rules that exist in the patterns is one of the important characteristics of mathematical thinking (Sovchik 1989; Worth 1990). Finding the rules in a pattern or creating a pattern requires knowing the basic characteristics and noticing and analysing the differences and similarities. Worth (1990, p. 53) states that all of these skills are related to problem solving. Searching for a pattern is not only a problem-solving strategy but is also related to the development of counting skills, arithmetic, proportional reasoning and the ability to search for structures in data (Papic 2007).

Smith (2001, p. 80) categorised the patterns in three groups: repetitive patterns, extending patterns and patterns based on relationships. A basic unit repeats in repetitive patterns (Olkun and Uçar, 2007), for example, 1, 2, 1, 2, 1, 2… etc. Extending patterns are marked by a regular increasing and decreasing in extending patterns, for example, 11, 111, 1111, … etc. The patterns based on relationships are obtained as a result of applying a process or processes to the elements of a set: 1, 3, 7, 15 … etc. This pattern, which consists of numbers to which one is added to two times the number preceding them, can serve as an example.

There are some researches on the pattern finding skills of children in the pre-school period. Papic (2007) explained how experiences in the early years of schooling should focus on identifying, justifying and transferring various patterns using a variety of materials if a sound understanding of patterning was to develop in students. Papic and Mulligan (2005), in their semi-experimental study, implemented a program that allowed children in the pre-school period to find patterns. The results of this study revealed that the program improved the children's pattern finding skills. In another experimental study by Papic and Mulligan (2007), a program that consisted of activities based on problems was implemented. The results showed that the children could see simple repetitive structures by using one repetitive unit and could represent the patterns in different spatial forms (restricted zones, cages, squared paper), sequences and sequences of numbers). In a similar experimental study by Mulligan, Mitchelmore, English & Crevensten (2013) significant differences were found in the children's pattern finding skills, structural development levels and making connections between mathematical ideas and process when compared to the control group.

Two studies on the pattern finding skills of children in pre-school were found in Turkey. A pilot study was carried out by Tarım (2012) to determine the pattern finding skills of children in pre-school. This study found that the pre-school children were successful in finding more repetitive patterns. In another study, Tarım (2015) tried to determine the effect of working in cooperative groups on children's pattern finding skills. The results of the study showed that working in cooperative groups positively affected the development of children's pattern finding skills.

There have been a limited number of studies carried out in Turkey and abroad on pre-school children's pattern finding skills that are important for the development of mathematical reasoning skills, mathematical thinking skills and relationship-building. That is why, more studies were thought to be needed. This study aimed to determine the

pattern finding skills of pre-school-aged children. In line with this purpose, the study aimed to answer the following questions:

- What are the children's pattern-recognition achievements in general?
- Do the children's pattern-recognition achievements differ by gender or age?
- What are the children's pattern-recognition achievements according to the structures of patterns in the repetitive pattern category?
- What are the children's pattern-recognition achievements according to the structures and the sub-categories (in the presentation that used the picture of the object and in the presentation that used the number) of patterns in the repetitive pattern category?
- What are the children's pattern-recognition achievements according to the structures and the sub-categories (in the presentation in which object pictures were used and in the presentation in which the location changed) of patterns in the extending patterns category?
- What are the children's pattern-recognition achievements according to the structures and the sub-categories (sub-categories requiring counting forward and counting backward) of patterns in the category of patterns based on relationships between numbers?
- What are the structures of the patterns that the pre-school teachers use in the scope of mathematics activities?
- What are the materials that the pre-school teachers benefit from when creating the patterns?

Method

Participants

The pre-school education in Turkey is based on the voluntary basis. It is given in state or private pre-schools or in the pre-schools which are in the elementary schools. The study was conducted in 4 independent pre-schools. The study sample consisted of children aged 60-77 months old. The children came from families of middle socio-economic status in the province of Adana. The average age of the children was 68 months; there were 88 females and 74 males (162 total). Most of the children's mothers were housewives (69.5 %), and the second most prevalent occupation was civil servant (24.7 %), then in the health sector (12.3 %), engineer-academicians (8.6 %), finance sector (3.1 %), workers (2.5 %) and other occupations (9.3 %).

Regarding fathers' occupations, the order was as follows: civil servants (23.5 %), self-employed (21 %), workers (17.9 %) and engineers, doctors, lawyers, academicians (14.8 %). Though very low, the percentages of the other sectors were finance, management, health and education.

All the teachers whose students were interviewed, were interviewed too. Semi-structured interviews were conducted with all teachers; 4 female and 2 males (6 total) to obtain the pre-school teachers' thoughts about the pattern finding activities that they use in the scope of mathematics activities. Instead of their real identities, the teachers were coded as T1, T2, T3, T4, T5 and T6 in line with their order of being interviewed. T1 has an occupational experience of 15 years and is a graduate of the School of Education, Department of Child Development. T2 has an occupational experience of 20 years and

is a graduate of the Child Development and Pre-School Teaching Department. T3 has an occupational experience of 24 years and is a graduate of the School of Education, Pre-School Child Development Department. T4 has an occupational experience of 19 years and is a graduate of the Pre-School Teaching Department. T5 has an occupational experience of 3 years and is a graduate of the Child Development Department. T6 has an occupational experience of 13 years and is a graduate of the School of Education, Pre-School Teaching Department.

Data Collection Tools

As a data collection tool, a 16-item pattern test developed by Tarım (2012) was used after being rearranged. This rearranged test was shared with three experts on mathematics education and three pre-school teachers to get their ideas. In line with their thoughts, the questions were revised again, and the test was finalised. The KR-20 reliability coefficient was calculated as 0.88 in the reliability test that was conducted for the data obtained with the data collection tool.

The items in the final form of the test were distributed according to the pattern categories as repetitive patterns (11 items), extending patterns (2 items) and patterns based on relationships between numbers (3 items). The patterns in the repetitive category were divided in two sub-categories, i.e., the presentation that used object pictures (9 items) and the presentations that used numbers (2 items). The patterns in the extending category were divided in two sub-categories, i.e., the presentation in which object pictures were used (1 item) and the presentation in which location change was given a place (1 item).

Though the children's sense of numbers was out of the scope of this study, it was thought that the children at this age group already acquired some skills related to numbers. Therefore; the test included items based on the relationship among numbers.

In the patterns based on the relationships-between-numbers category, two sub-categories were formed; in one, two items were based on counting forward, and in the other, one item was based on counting backward. In addition, the structures of each item in the test were expressed symbolically by using letters or numbers. For example, a repetitive triple structure was expressed symbolically as a,b,c-a,b, and a quartet structure was expressed symbolically as a,b,c,d-a,b,c,d. The items in the pattern test and the structure of those items are presented in Table 1.

Semi-structured interviews were used to obtain the thoughts of pre-school teachers about the pattern finding activities that they used in mathematics activities. Some questions were asked about the teacher's personal information, i.e., their teaching background, the school where they worked and their educational background. In addition, some questions asked what types of patterns the teachers used during their in-class activities, what materials they used in mathematics activities while creating patterns, and what the characteristics were of the objects or shapes they used. The children's teachers were informed about the content and they were asked for permission. All teachers voluntarily participated in the study. Appointments were taken from these teachers in line with their free time. The teachers were interviewed in the most convenient room of the school. Just before the interview, the teachers were asked if they agreed to the audio recorder being used. One teacher did not permit the audio

recorder to be used, so her interview was recorded in a written way. The interviews lasted 25-35 min. 6 teachers were interviewed once.

Table 1.
The Categories in the Pattern Test and the Structures of the items

Pattern Categories	Sub-categories	Number of item	Structure of items
Repetitive	Object pictures were used	1, concrete object	a,b,c-a,?,c
		2, concrete object	a,b,c-a,?,c
		3, abstract object	a,b,c,-a,b,c,-?,?,?
		4, concrete object	a,b,c,-a,b,c,-?,?,?
		5, abstract object	a,b,c,d-a,b,c,d-?,?,?,?
		6, concrete object	a,b,c,d-a,b,c,d-?,?,?,?
		7, abstract object	0,0,a,a-0,0,a,a-??
		8, abstract object	0,0,0,a-0,0,0,a-?,0,?,?
		9, abstract object	aa,aa,0- aa,aa,0-aa,?,?
	Numbers were used	15	2,2,1-2,2,1, ?, ?
		16	3, 2, 5-3, 2, 5-?, ?
Extending	Object pictures were used	10, abstract object	a,aa,aaa,?,?
	Object-location change	11, abstract object	I, −,II,=,III, ≡, ?, ?
Patterns based on the relationships between numbers	Based on counting forward by one	12	5, 6, ?, 8, ?
		13	12, ?, 14, 15, ?
	Based on counting backwards by one	14	12, 11, 10, ?, 8, ?

Procedure

The data was obtained through the interviews with the children and the teachers. The interviews with children were performed individually in a quiet room in the school where they studied. Before starting the interview, a short preparation speech was given for children to understand the procedure. Next, two sample questions were discussed with children as a preparation. After being sure that the children understood the situation, the items in the pattern test were shown to the children one-by-one, and they were asked with which object or number the gaps in the pattern would be filled. In some items, the children wanted to continue the pattern. This process was explained on a sample question. Example:

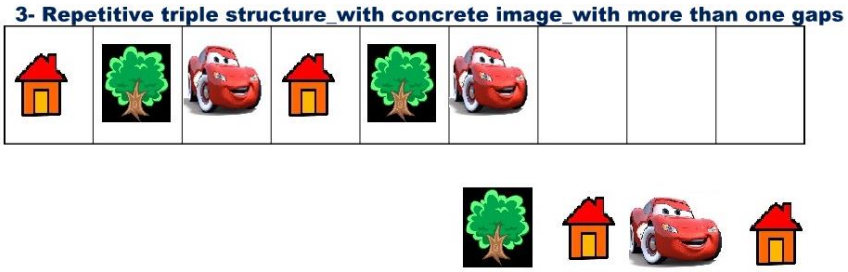

Figure 1. Pattern Test

With which of the following should the empty boxes be filled according to sequence above? If the answer was correct, it was recorded. If it was false, the interviewer made denominations such as, "Let's look at it together! How are these pictures sequenced?" and "house-tree-car, house-tree-car" to encourage the child to say the pattern aloud. Next, the interviewer asked the question, "With which of these pictures should the empty boxes be filled, respectively?" to the child again. Then, the answer was recorded. The interviews took 20-25 minutes. The results of the interviews were recorded in the interview form. Similarly, the interviews with the teachers were conducted individually in a quiet room in the school where they worked. These interviews took approximately 20-25 minutes and were recorded with a voice recorder device.

Data Analysis

At first, it was investigated whether the data obtained in the study met the assumption of normality. It was determined by the results of the analysis that the data did not meet the assumption of normality. Thus, it benefited from non-parametric tests (Mann Whitney U and Kruskall Wallis) during the analysis process. The frequencies and percentage values according to some variables were calculated in the description of the data.

The data obtained from the teachers through semi-structured interviews were analysed by descriptive analysis, a qualitative data analysis method. The records of the interview were computerised and transcribed. Then, the transcribed files were read question-by-question and evaluated according to the codes and themes that were formed before.

Findings

In this study, which was conducted to determine the pattern finding skills of children in the pre-school period and investigate the pattern activities that the teachers used in the scope of mathematics activities, the findings are presented under two headings: the findings obtained from the interviews with the children and the findings obtained from the interviews with the teachers.

The findings obtained from the interviews with the children

When the data were analysed, it was observed that the children received an average score of 10.34 (SD=4.20) out of 16 items. Children received an average score of 6.65 (SD=2.60) on the 11 repetitive pattern questions, an average score of 0.90 (SD=.78) on

the 2 extending pattern questions, and an average score of 1.53 (SD=.86) on 3 questions related to the patterns based on the relationships among numbers.

With respect to gender, it was observed that the female and male children received similar scores in terms of the average scores (U=3252.50, p>.05). Similarly, no significant difference was observed between the age groups in terms of total scores ($\chi 2$ (2)= 3.48, p>.05). Because of this, the data was described as a whole according to the pattern categories without considering the variables of gender and age. In this context, the data on the repetitive pattern category is presented in Table 2.

Table 2.
The Percentages and Frequencies of the Correct Answers Given to the Patterns in the Repetitive Category

Category	Sub-category	Item number	Item structure	f	%
Repetitive (*M*=6.65; SS=2.60)	Object pictures were used in the presentation	1, Concrete object	a,b,c-a,?,c	148	91.4
		2, Concrete object	a,b,c-a,?,c	148	91.4
		3, Abstract object	a,b,c,-a,b,c,-?,?,?	114	70.4
		4, Concrete object	a,b,c,-a,b,c,-?,?,?	127	78.4
		5, Abstract object	a,b,c,d-a,b,c,d-?,?,?,?	98	60.5
		6, Concrete object	a,b,c,d-a,b,c,d-?,?,?,?	104	64.2
		7, Abstract object	0,0,a,a-0,0,a,a-??	128	79.0
		8, Abstract object	0,0,0,a-0,0,0,a-?,0,?,?	91	56.2
		9, Abstract object	aa,aa,0- aa,aa,0-aa,?,?	55	34.0
	Numbers were used in the presentation	15	2,2,1-2,2,1, ?, ?	107	66.0
		16	3, 2, 5-3, 2, 5-?, ?	96	59.3

The percentages of the correct answers given to the items that included the patterns in the presentation that used object pictures, were higher than the items that have similar structures in general. The achievement decreases as the complexity of the patterns increases.

When the repetitive triple structured items (structure: a,b,c-a,b,c; item numbers 1, 2, 3, 4, 16) were analysed, the percentages of correct answers in the first (91.4 %) and the second (91.4 %) items were found to be higher than the ones in the third (70.4 %) and the fourth (78.4 %) items. When the answers given to these items were analysed,

children were found to be more successful in the first and second items, which required filling only one gap, than in the third and fourth items, which required filling three gaps.

The lowest percentage of the correct answers among these triple structured items was observed in item 16 (3,2,5, 3,2,5,?,?, which required filling two gaps in the number presentation. Some of the children who could not answer this question correctly gave answers such as 3,2,5, 3,2,5, 2,5, and some of them gave answers such as 3,2,5, 3,2,5, 6,7.

When the repetitive quartet structured items (structure: a,b,c,d-a,b,c,d; item numbers 5, 6) were analysed, the percentages of correct answers in the item in the concrete object picture presentation (64.2 %) were found to be higher than the ones in the abstract object picture presentation (60.5 %).

The lowest achievement in the repetitive patterns was on the ninth item (structure: aa,aa,0- aa,aa,0-aa,?,?; 59.3%; see Table 2). The data about patterns in the extending category is shown in Table 3.

Table 3
The Percentages and Frequencies of the Correct Answers Given to the Patterns in the Extending Category

Category	Sub-category	Item number	Item structure	f	%
Extending (*M*=0. 90 SS=.78)	No change was made in object-location	10	a,aa,aaa,?,?	91	56.2
	Change was made in object-location	11	I, −,II,=,III, ≡, ?, ?	21	13.0

Table 3 shows that the achievement on the patterns in the extending category was lower than the achievement on the patterns in the repetitive category. However, more than half of the children (56.2 %) gave correct answers by noticing the extending structure. The percentage of the correct answers (13 %) decreased substantially after the change of location was added to this structure, and 87.3 % of the children who made mistakes in this type of pattern displayed the behaviour of repeating the initial unit. The data about the patterns based on the relationships between numbers are shown in Table 4.

Table 4
The Percentages and Frequencies of the Correct Answers Given to the Patterns Based on the Relationships between the Numbers

Category		Sub-category	Item number	Item structure	f	%
Patterns based on the relationships between numbers	(*M*=1.53 SS=.86)	Based on counting forward	12	5, 6, ?, 8, ?	137	84.6
			13	12, ?, 14, 15, ?	124	76.5
		Based on counting backward	14	12, 11, 10, ?, 8, ?	87	53.7

As shown in Table 4, the percentage of the correct answers in the counting-forward patterns was quite higher than the percentage of the correct answers in the counting-backward patterns. Moreover, when the answers given to the counting-forward patterns were evaluated, it was observed that the achievement percentage was higher in the patterns in which small numbers were used.

The Findings about the Interviews with the Teachers

Table 5 shows the themes and codes that were produced during the analysis of the data about the pattern categories that the pre-school teachers used in mathematics activities, the materials they used while creating the patterns and the characteristics of the materials used to create the patterns.

Table 5
The Themes and Codes which were obtained from the teachers' interview data

Theme	Code	Teachers
Pattern Categories	Repetitive patterns with maximum triple structure	6 (T1, T2, T3, T4, T5,T6)
	Based on the relationships between numbers	2 (T2, T4)
The materials used in the patterns	Shapes/Pictures (Worksheets)	5 (T2, T3, T4, T5, T6)
	Objects	5 (T1, T2, T4, T5,T6)
	Numbers	2 (T2, T4)
The characteristic which was used	Colours, size, shape	6 (T1, T2, T3, T4, T5, T6)

As Table 5 shows, three themes - pattern categories, the materials used in the patterns and their characteristics, and the codes about these themes, were obtained.

Based on the patterns that the teachers used most, it is observed that all of the teachers stated that they used the patterns in the repetitive category, but two of them (T2, T4) expressed that they also used the patterns based on the relationships between numbers. None of the teachers stated that they used patterns in the extending category.

In the interview, T1 expressed, "I'm trying to use double repetitive or triple repetitive patterns by using objects. For example; yellow Lego, blue Lego, yellow Lego, blue Lego... or we can work with this pattern by adding one more colour and making it triple repetitions" and stated that s/he generally used double or triple repetitive structures. Similarly, T4 said, "We generally use simple patterns, for example, one triangle, one square, one triangle, one square, etc. In addition to this, we do exercises of counting forward rhythmically through songs. In this way, we try to make the children notice the sequence relationship between the numbers" and added that they used both repetitive patterns and the patterns based on the relationship between numbers.

When the materials that the teachers used in their activities were analysed according to their answers to the interview questions, it was observed that they usually used objects (T1, T2, T4, T5, T6) and shapes/pictures/worksheets (T2, T3, T4, T5, T6) in their activities of creating and finding patterns. Two teachers (T2, T4) said that they also used numbers in the study of patterns. Moreover, all teachers mentioned that in their worksheets, they made use of figures and pictures and included questions that guided the children to continue the path in the pattern.

T2 emphasised that s/he more often did activities of patterns in class by using shapes/pictures in the worksheets: "We study the patterns by using worksheets. We generally have patterns consisting of shapes or pictures in the worksheets. The children complete the gaps." T4 stated that s/he used worksheets and also numbers in the pattern activities: "I use worksheets. I use patterns based on counting forward rhythmically. However, I do not use patterns based on counting backward rhythmically."

During the interviews, the teachers stated that they generally used shapes and pictures in the activities of creating and finding patterns in the classroom. When they were asked what characteristics of these shapes and pictures were used while creating patterns, all of the teachers remarked that they pay attention to the characteristics of colour, size and shape. When the teachers were asked if they considered more than one characteristic simultaneously, all of the teachers said they used the patterns that were created by taking only one characteristic into consideration.

T6 expressed that s/he used the patterns that regarded the characteristic of size: "I want the children to continue the patterns such as big box, small box, big box, small box." T4 expressed that s/he used the patterns that regarded the characteristic of shape: "We usually use simple patterns, for example, one triangle, one square, one triangle, and one square". T5 expressed that s/he used the patterns that regarded the characteristic of colour: "I also do pattern studies according to the colours of the objects, such as red pencil, blue pencil, red pencil....."

Discussion

The findings of this study revealed that the achievement of the pre-school children in pattern finding was above average. The children displayed the greatest achievement in repetitive patterns. It can be evaluated that this result was partially consistent with the following objectives of the Pre-School Curriculum (2013, p.22) about the concept of

patterns: i) They form a pattern with objects by looking at the model, ii) they say the rule in a pattern that consists of a maximum three repetitive objects, and iii) they say the object that is missing in a pattern and complete the pattern. The curriculum limited the concept of pattern with the only repetitive pattern category for this age group. The children displayed remarkable achievement in the extending pattern category and the patterns based on the relationships between numbers category, as their teachers stated, despite the lack of experience with these categories. Similarly, in their study, Rittle-Johnson, Fyfe, McLean and McEldoon (2013) stated that young children could duplicate and extend patterns, and some showed a deeper understanding of patterns.

When the variable of gender was considered, it was observed that the female and male children received similar scores. This finding is aligned with the literature results. Unutkan (2007) showed, in a study that investigated the mathematical skills of pre-school children that the mathematical skills of the children did not differ according to gender. On the contrary, Lauzon (2001) emphasised that the children's achievement in mathematics differs according to gender in favour of the male children as age increases. However, it did not differ in the younger ages according to gender.

When the percentages of the correct answers given to the patterns in the repetitive category were analysed, it was observed that the percentages of the correct answers given to the object picture patterns were higher as a result of comparing the items that have similar structures. It was observed that the achievement decreased as the complexity of the patterns increased. This result could be expected when considering the development characteristics of the children in this period. Another reason for this result might be because the teachers use simple structured patterns that consist of double or triple repetitive structures more during in-class pattern activities and the object pictures are used in the presentation of these patterns.

In this study, the pattern questions were created to continue the existing structure in the pattern and to fill in the missing gaps in the pattern. The children displayed greater achievement in continuing the existing structure in the pattern than in completing the gap in the pattern appropriately. One reason for this achievement might be due to the children's experience with continuing the existing structure in the pattern. The interviews with the teachers support this finding. The teachers expressed that they studied the questions more that aimed to make the children continue the existing structure patterns that were in the worksheets or workbooks.

The children had the greatest difficulty with the question in the repetitive category that required completing two gaps and in the number presentation. Some of the children who could not answer this question correctly showed a tendency to repeat the last two units, such as 3,2,5, 3,2,5, 2,5, without noticing that the pattern in the question had a triple repetitive structure. Some of them displayed a tendency to give answers focused on the sequence relationship of the numbers, such as 3,2,5, 3,2,5, 6,7. In the interviews, the teachers stated that they did more activities of counting forward rhythmically. This makes us think that the children had more experience with the sequence relationship of the numbers, and therefore, they featured this relationship in these types of patterns.

When the mistakes that the children made in extending patterns were taken into account, it was observed that they judged the patterns as repetitive patterns without noticing the extending structure in the pattern. It was observed that 87.3 % of the children who made mistakes in these types of patterns displayed the behaviour of

repeating the initial unit. In the interviews, teachers stated that they did not use the patterns in the extending category during their in-class activities. It can be considered a remarkable result that more than half of the children noticed the extending structure in the patterns, even though the teachers reported that they did not use the patterns in the extending category during their in-class activities.

When the answers given to the patterns based on the relationships among numbers were analysed, it was observed that the percentage of the correct answers was quite higher in the counting-forward patterns than in the counting-backward patterns. Counting forward rhythmically can be considered the first informal mathematics activity that children try to put into practice, which may be why the children were more successful in the counting-forward patterns. Piaget (1973) stated that children in the pre-operational stage do not yet have. The principle of reversibility is related to the children's ability to come back to the starting point by following a specific reasoning sequence. A child's ability to count backward depends on the development of the principle of reversibility (Buldu 2010). The lower achievement in the counting-backward patterns can be explained by some children not having developed the principle of reversibility yet. In the interviews, the teachers stated that they did activities based on counting forward more, and they did only a few or no activities based on counting backward. This also might be another reason for the low achievement in the patterns based on counting backward.

Consequently, the results of this study showed that pre-school children aged 60-77 months i) had an average achievement in finding patterns, ii) did not display a significant difference in their pattern finding skills according to gender or age, iii) were more successful in the patterns in the repetitive category of patterns, iv) could display considerable achievement in the patterns in the extending category even though this category was not included in the in-class activities, furthermore, v) the sub-categories and the structures of the patterns were effective on the performances of the children in finding patterns, and vi) the teachers mostly preferred patterns in the repetitive category that were in the worksheets in their class mathematics activities.

In this context, it can be recommended to raise the awareness of the pre-school teachers about the categories of patterns (repetitive patterns, extending patterns and the patterns based on the relationships between numbers). In line with this, the children can be provided with more experiences about patterns and work with patterns that are in different categories and are presented in different ways. Papic, Mulligan, Highfield, McKay-Tempest and Garrett (2015) stated that children could explain patterns and pattern structures, view patterns from different orientations and use various materials to create complex patterns. So similar research about the prospective studies can be conducted with both pre-school aged children and children in the first and second grades to obtain a more comprehensive descriptive study about the early childhood period. The effect of using different ways of presentation (e.g., using the object pictures, using the objects themselves, etc.) on the pattern finding skills of children can be investigated. Experimental studies that investigate the effects of different programs that support the development of children's pattern finding skills can be conducted. The mistakes that the children made in pattern finding studies can be investigated more thoroughly through a qualitative study.

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