Conceptual and Procedural Angle Knowledge: Do Gender and Grade Level Make a Difference?

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The study examined differences in students' conceptual and procedural knowledge of angles among two grades and gender. Participants were 382 sixth and 376 seventh graders from a metropolitan city in Turkey. Turkish students' conceptual and procedural knowledge of angles declined from sixth to seventh grade. Gender differences were found for procedural knowledge, but not for conceptual knowledge. Since conceptual and procedural knowledge of angles may have significant influences on the essential subsequent topics in geometry, we need to seriously consider the implications of these gender- and grade-related differences and pay attention particularly to males in Grade 7. The patterns of Turkish students' conceptual and procedural angle knowledge were discussed, and educational implications were offered.

Keywords: Conceptual knowledge; procedural knowledge; angles; geometry; gender; grade

Introduction

Theories of learning and instruction underlined the pivotal role that knowledge plays in students' cognitive development and learning process (De Jong & Ferguson-Hessler, 1996; Rittle-Johnson & Koedinger, 2005). According to Hiebert and Lefevre (1986), organized knowledge requires individuals to internalize a concept that is being taught, the meaning related to a procedure they are implementing, and interconnections built between them. Mathematical competence therefore relies mostly on how one develops knowledge of concepts and of procedures (Rittle-Johnson & Schneider, in press). Rittle-Johnson, Siegler, and Alibali (2001) have posited that conceptual and procedural knowledge do not develop independently but rather iteratively, with gains in conceptual knowledge leading to gains in procedural knowledge, which in turn trigger new gains in conceptual knowledge.

Numerous studies have demonstrated that structured and/or organized knowledge enables students to perform tasks successfully and remember more relevant information than if we have only memorized isolated facts and/or automatized procedures (e.g., Bransford, Brown, & Cocking, 2001; Rittle-Johnson & Schneider, in press). Unfortunately research has also suggested that students encounter difficulties with developing quality geometrical knowledge at all levels of schooling from kindergarten (e.g., Tsamir, Tirosh, Levenson, Barkai, & Tabach, 2016), elementary (e.g., Bartolini Bussi & Baccaglini-Franck, 2015, Clements, Battista, & Sarama, 1998; Smith, King, & Hoyte, 2014), and middle years (e.g., Clements & Battista, 1989; Owens & Outhred, 2006) to secondary (e.g., Battista, 1990; Chazan, 1993; Herbst & Brach, 2006; Jones, 2000; Hollebrands, 2007; Swafford, Jones, & Thornton, 1997) and even through university years (e.g., Couto & Vale, 2014; Mayberry, 1983). Researchers have shown that as the mathematics competencies shift in grade levels, males do gain a slight overall advantage over females, with large differences found in geometry (Battista, 1990; Casey, Nuttall, & Pezaris, 2001; Hyde, Fennema, & Lamon, 1990). The present study examines the important issue of potential differences between females and males in middle school using a Turkish sample

of 6th and 7th graders. The findings may lead to a better understanding of gender and grade level issues related to geometrical knowledge that is considered to be important for the development of spatial knowledge (Soury-Lavergne & Maschietto, 2015) and visual thinking (Battista, 2007; Clements & Battista, 1992).

The issue of individual differences on student knowledge of angles is particularly interesting, as it has not been fully elucidated in the literature. Furthermore, the existent literature on students' development of knowledge has been often investigated in the domains of counting (Gelman, Meck, & Merkin, 1986), single-digit addition (Baroody & Gannon, 1984), multi-digit addition (Hiebert & Wearne, 1996), fractions (Hallett, Nunes, & Bryant, 2010), decimal fractions (Moss & Case, 1999), percent (Lembke & Reys, 1994), and mathematical equivalence (Knuth, Stephens, McNeil, & Alibali, 2006). The topics studied in this bulk of studies have been mainly limited to mathematics, particularly arithmetics. By contrast, there has been very little research on geometry topics. There has, of course, been a vast amount of research on students' spatial and geometrical knowledge (Clements & Battista, 1992), but relatively few studies investigate individual differences in geometrical knowledge as opposed to conceptual and procedural angle knowledge (e.g., Mitchelmore, 1997, 1998). Whereas mathematical knowledge development is worth exploring, the individual differences in geometrical knowledge within a specific domain, especially the angles concept, would also be informative to curriculum designers and mathematics teachers.

The relation between conceptual and procedural knowledge is clearly more complex than has often been assumed and deserves closer investigation. As mentioned before, in some cases, algorithms support conceptual understanding and at a more complex level form the building blocks to understand the concepts whereas in some, with sound conceptual understanding students would develop their own algorithms. It is my pragmatic view in the design of this study that different aspects of the debate apply differently to particular mathematical concepts (e.g., angles), to different genders (i.e., females and males), and to different stages of development (i.e. grade levels). I am also convinced that mathematics teachers' access to these constructs and the related research allows them to make informed choices as to when to focus on different aspects of geometrical knowledge, and thus to have a better insight into the differentiation of the knowledge related to the angle concept between the two genders and the various grade levels. From a pedagogical point of view using conceptual and procedural knowledge can effectively inform teaching and learning of angles. A major challenge for mathematics teachers/educators is thus to foster students' knowledge of concepts and procedures, assess more purposefully, and to better ends have both females and males in different grade levels engaged in knowledge acquisition processes (e.g., extracting, structuring, and organizing), which would energize the links within and among the principles that govern angles (e.g., the angle that is 90 degrees is a right angle) and the action sequences to solve angle problems (e.g., finding angle measures between intersecting lines). Students being aware of the conceptual/procedural knowledge distinction can reflect on their learning and observe the nuances of teaching practices related to angles. If so, such a distinction may offer teachers much to alleviate the understanding of the diffrences underpinning students' conceptual/procedural knowledge. In this sense, the gender and grade level differences investigated in the present study offers discrepancies specific for conceptual and procedural knowledge of angles as well as hold parallels and provide directions that can be specified to account for other geometry topics (e.g., triangles) in measures of other cognitive constructs (e.g., instrumental understanding and relational understanding).

The Importance of Angles

Concept of angle comprises a fundamental portion of the content of the geometrical knowledge middle school students need to possess. Both national (Ministry of National Education [MoNE], 2013) and international (e.g., National Council of Teachers of Mathematics [NCTM], 2000) curriculum documents recommend that students develop some understanding of angles before entering high school. The MoNE, for example, recommended that students should develop the abilities to "use knowledge of basic geometric concepts and drawings including angles" (p. 7) to "select appropriate methods to solve problems related to angles" (p. 29) throughout Grades 5-8. Similarly, the NCTM recommended that students should come to the study of geometry in the middle grades with informal knowledge about angles; and with experience in visualizing and drawing angles (p. 233).

Obviously, the angle concept is central to the development of geometrical knowledge (Clements & Burns, 2000; Krainer, 1991) and important for students to understand at an early age (Mitchelmore, 1998). Further, the lack of a solid understanding of angle concepts can make understanding and working with functions more difficult later on (Moore, 2013). However, developing a thorough understanding of angles is challenging for many students (Clements & Battista, 1992). Students often experience difficulties with angle measurement (Clements & Burns, 2000; Freudenthal, 1983), operations with angles (Smith et al., 2014; Usiskin, 1987), abstracting and generalizing angles (White & Mitchelmore, 2010), recognizing angles in different contexts (Mitchelmore, 1997, 1998), solving real-life problems related to angles (Munier, Devichi, & Merle, 2008).

Part of the reason angles are difficult to understand may stem from the fact that an angle can be defined using a variety of representations. Clements and Burns (2000) described two common ways of representing angles: (1) the static representation depicting an angle as a pair of rays with a common endpoint, and (2) the dynamic representation displaying an angle as the amount of rotation necessary to bring one of its rays to the other ray without moving out of the plane. In practical terms, Henderson and Taimina (2005) described an angle as a geometric figure (i.e., a part of the plane included between two rays meeting at their endpoints), an angle as a dynamic figure (i.e., a turn or rotation), and an angle as a measure. Viewed together, an understanding of an angle as a multifaceted concept incorporating both two representations requires students to expose adequate knowledge of concepts and of procedures related to angles and that it is slow to develop over time as students move through grade levels (Casas-García & Luengo-González, 2013; Lehrer, Jenkins, & Osana, 1998). Therefore, it is important to understand individual differences in using knowledge of concepts and of procedures related to angles.

Knowledge of Angles

The term *angle knowledge* refers to knowledge about angle as a mathematical entity, which is embedded in a formal geometrical structure (Mitchelmore, 1997). It involves knowledge of definitions, facts, and theorems, which involves angles as well as knowledge of procedures, algorithms, and methods to solve relevant angle problems. From this perspective, angle knowledge can be thought of as consisting of two complementary types: conceptual and procedural. Within this categorization, *knowledge type* merely refers to what is known (Star, 2005) and that "the adjectives *conceptual* and *procedural* demarcate what type of knowledge is being characterized" (Star & Stylianides, 2013, p. 171).

Several researchers have offered definitions for both knowledge types (De Jong & Ferguson-Hessler, 1996; Hiebert & Lefevre, 1986; Rittle-Johnson & Koedinger, 2005). The conceptual and procedural knowledge framework derives essentially from the common view among researchers (Rittle-Johnson et al., 2001; Star, 2005, 2007) that conceptual knowledge is knowledge that is known *deeply*, while procedural knowledge is knowledge that is known *superficially*. In this regard, conceptual knowledge is characterized most clearly as a "connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (Hiebert & Lefevre, 1986, p. 3). Procedural knowledge, on the other hand, is composed of knowledge that is memorized by rote with some computational skills (Baroody, 2003). In this vein, researchers measured students' conceptual knowledge through tasks that mobilize declarative knowledge (i.e., primary level of relationships) in the context of "what" and "which" type of questions or that energize conditional knowledge (i.e., abstract level of relationships) in the context of "why" type of questions. Conceptual knowledge was therefore considered a combination of building relationships between existing bits of knowledge that mirrors primary level of relationships (i.e., declarative knowledge) and abstract level of relationships (i.e., conditional knowledge). Henceforth, these particular knowledge types were not classified as declarative and conditional knowledge rather introduced as conceptual knowledge. Students' procedural knowledge, on the other hand, was measured by tasks including the manipulation of algorithms that, to some extent, reflect "how" type of questions.

The following lines discuss their definitions and how they apply to knowledge of angles in detail. The definitions are elaborated upon here because the study was concerned with investigating gender- and grade-related differences in conceptual and procedural knowledge of angles.

Conceptual Angle Knowledge

In discussing conceptual knowledge, Star (2005) indicated that conceptual knowledge includes knowledge of concepts in a cognitive domain, including definitions and facts, and their interrelations. That is, the relationships built by conceptual knowledge pervade the individual facts and propositions so that all pieces of information are linked within and among one another in an incorporated network. As conceptual knowledge involves abstract and general principles (Byrnes & Wasik, 1991), it can be implicit or explicit, and thus does not have to be verbalizable (Rittle-Johnson, Schneider, & Star, 2015).
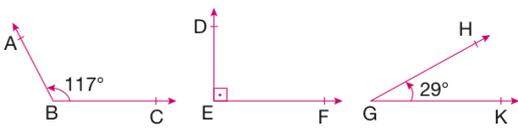
Conceptual knowledge relating to angles can be formed in many ways. For instance, one might recognize that an angle can be represented in three ways – as a turn, as a wedge, and as the intersection of lines. One might also recognize that all three representations have the common characteristic that they are sometimes used to define angle concept or to state angle measure. One might further recognize that in some instances one representation is more suitable than another for determining the type of angles. These examples do not constitute an exhaustive list of conceptual angle knowledge. Rather, they illustrate that knowledge of concepts grows as cognitive networks consisting of interrelations between discrete pieces of information about the representations that are used. As conceptual knowledge develops, such networks are likely to be realigned and reconfigured in a more effective manner (Hiebert & Carpenter, 1992).

Procedural Angle Knowledge

Procedural knowledge can be defined as knowledge of procedures, including subcomponents of a correct algorithm (Rittle-Johnson & Koedinger, 2005). From this description, it involves applying sequential action steps and automatized techniques for solving problems (Bisanz & LeFevre, 1990). Hiebert and Lefevre (1986) distinguish between two parts of procedural knowledge: 1) knowledge of formal language of mathematics, or symbolic representation of mathematical ideas, and 2) knowledge of rules or algorithms used to complete mathematical tasks.

Both types of procedural knowledge can be mapped to the case of angles. A student who recognizes that it is conventional to use the fact "the sum of complementary angles is $90°$" to represent a pair of complementary angles is demonstrating the first type of procedural knowledge. A student who can execute the "add-and-subtract" algorithm to calculate the unknown complementary angle demonstrates the second type of procedural knowledge, which encompasses knowledge of computational rules, techniques, and algorithms.

It should also be noted that conceptual and procedural knowledge are not always displayed in isolation. In particular, tasks influence students by directing their attention to a specific content and by indicating ways of processing information (Doyle, 1983). Accordingly, a task that reflects conceptual knowledge also incorporates procedural knowledge or *vice versa* depending on the task content. For instance, in relation to the items used in the present study, stating a correct definition for the right angle (see Item 6 in Figure 1) can be considered a matter of conceptual knowledge. It is not uncommon, however, for students to include the "add-and-subtract" algorithm for the complementary angles in a stated definition. Simply providing an accepted definition for right angle reflects knowledge of concepts. Including the "add-and-subtract" method reflects knowledge of procedure. Hence, when one states a definition for the right angle, there is the potential to display both conceptual and procedural knowledge. One's definition for the right angle might include a descriptive statement that it is a type of angle, which equals $90°$ along with the algorithm used for determining it.

Type of Knowledge	Item
Conceptual knowledge	**Item 6:** Define the right angle. **Item 12:** Given the figure below, label the angles as right, acute and obtuse. (I)　　　(II)　　　(III)
Procedural knowledge	**Item 11:** Given below the straight angle figure, find measure of the unknown angle.
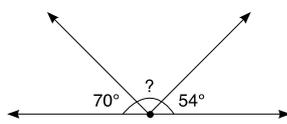
Item 15: In the figure below $s(\widehat{KOL}) = 55°$. Compute $s(\widehat{RON})$.
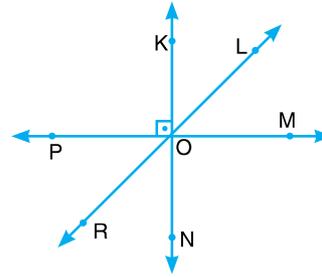
Figure 1. Specimen Items of the AT

Gender and Grade Level Differences

The effects of gender and grade level on conceptual and procedural knowledge in geometry are poorly understood. However, previous empirical findings related to mathematics provide some insight to guide further examination of patterns in which gender and grade level are related to conceptual and procedural knowledge of angles.

There is no specific study, so far as we know, on the existence of gender and grade level differences in conceptual and procedural knowledge of angles, although reports of individual differences in conceptual and procedural knowledge of fractions (e.g., Hallett, Nunes, & Bryant, 2010; Hallett, Nunes, Bryant, & Thorpe, 2012), addition/subtraction (e.g., Gilmore & Bryant, 2006), single-digit addition (e.g., Canobi, 2005; Canobi, Reeve, & Pattison, 2003), and single-digit multiplication (e.g., Mabbottt & Bisanz, 2003) do exist. In common, researchers concluded that students may take different pathways when acquiring conceptual and procedural knowledge and that there are individual differences in the way that they combine conceptual and procedural knowledge. Students' conceptual and procedural knowledge have been found to become more strongly correlated or more effectively possessed as they gain experience in the domain. That is, for instance, as they move from kindergarten to Grade 2 (Canobi et al., 2003), from Grade 2 to Grade 4 (Gilmore & Bryant, 2006), from Grade 4 to Grade 5 (Hallett et al., 2010) and to Grade 6 (Mabbott & Bisanz, 2006), from Grade 7 to Grade 8 (Schneider, Rittle-Johnson, & Star, 2011), and from Grade 6 to Grade 8 (Hallett et al., 2012). Some findings, however, did not support this developmental trend indicating that no significant differences in the iterative relations between conceptual and procedural knowledge appear among females and males attending to middle school (e.g., Rittle-Johnson & Star, 2007; Rittle-Johnson et al., 2001).

Collectively, previous research on students' geometrical task performance provides evidence that gender differences emerge as early as preschool years (Levine, Huttenlocher, Taylor, & Lanrock, 1999) and persist throughout years (Voyer, Voyer, & Bryden, 1995). Hyde et al. (1990) in a comprehensive meta-analysis indicated that participant age, which particularly reflects grade level, is the strongest predictor of task performance differences between females and males. They reported that males were more likely to outperform females on mathematics tasks that require procedural knowledge in high school and beyond. Indeed, there is evidence from other research (Bielinski & Davison, 1998) indicating that the gender gap tends to favour females in elementary school, with no gender differences in middle school, and with females outperforming males in high school.

More specifically, turning first to the gender differences, a broad body of literature has examined the performance differences between females and males on spatial/verbal (e.g., Battista, 1990) and graphics/diagrammatic tasks (e.g., Lowrie & Diezmann, 2011). In relation to the types of geometrical knowledge outlined in the present study, males' advantage might be particularly pronounced in the case of spatial task that require the possession of procedural knowledge (Halpern, 2000; Halpern, Benbow, Geary, Gur, Hyde, & Gernsbacher, 2007). Although performance differences in such tasks are widely acknowledged (Else-Quest, Hyde, & Linn, 2010; Hyde et al., 1990; Linn & Petersen, 1985), Spelke (2005) indicated that these differences between females and males have disappeared in the recent years. Nevertheless, a substantial body of literature (Vasilyeva, Casey, Dearing, & Ganley, 2009; Vasilyeva & Bowers, 2006; Voyer et al., 1995) has documented moderate differences favoring males in performing spatial tasks that require the generation and manipulation of geometric figures (procedural knowledge). In contrast, the verbal tasks that require using conceptual knowledge were not likely to show a male advantage: a small disparity was found in favor of females. A bulk of research also showed that females performed relatively well compared to males when using known algorithms (Kimball, 1989; Sabers, Cushing, & Sabers, 1987) and performing numeric computations (Kim, Plake, Wise, & Novak, 1990; McGraw, Lubienski, & Struchens, 2006) that require displaying procedural knowledge. Some studies have found no evidence of gender differences in logical reasoning (e.g., Battista, 1990) or geometric proof writing (e.g., Senk & Usiskin, 1983) that call for conceptual knowledge activation.

To account for these differences with reference to the knowledge of angles, Smith et al. (2014) designed a task-based interview to understand the way 3rd and 4th grade students build relationships among angles, angle types, and angle measurement (conceptual knowledge) as they completed the tasks. Although, the learning outcomes were reported for all the students who participated in the interview, it was interesting to detect that researchers chose two students (one female and one male) from Grade 3 to describe in detail regarding the criteria that the male student showed considerable gains in conceptual knowledge (from 20% to 100%) whereas the female student did not demonstrate any learning gains at all (answering zero questions correctly).

Moving to the grade level differences, qualitative studies on the development of angle concept using classroom observations (e.g., Clements & Burns, 2000; Devichi & Munier, 2013; Keiser, 2004) and student interviews (e.g., Mitchelmore, 1997, 1998; Mitchelmore & White, 1998, 2000; Simmons & Cope, 1990; Smith et al., 2014) throw some light on the developmental changes in conceptual and procedural angle knowledge. More specifically, these studies highlighted students' developing notions of angle in terms of displaying conceptual knowledge (Keiser, 2004; Mitchelmore & White, 1998, 2000; Simmons & Cope, 1990) and procedural knowledge (Clements & Burns, 2000; Mitchelmore, 1997) or exhibiting both conceptual and procedural knowledge (Devichi & Munier, 2013; Mitchelmore, 1998; Smith et al., 2014). Mitchelmore and White (2000) indicated that between Grade 2 and Grade 6 students evidently learned to represent physical situations (e.g., scissors, wall, etc.) using the standard angle model (conceptual knowledge) but many students, still in Grade 8, found it difficult to interpret the angles in terms of the similarities between different situations (procedural knowledge). Indeed, Mitchelmore (1997) found that students in Grade 2 had excellent knowledge of representing physical angle situations or grouping together angle situations into angle contexts (conceptual knowledge) and recognizing similarities between different angle contexts or building angle models (procedural knowledge). In contrast, Keiser (2004) indicated that students in Grade 6

struggled with using such multiple representations to define angle types by using the size of angles (conceptual knowledge) and thus had difficulties with conceiving of $0°, 180°$, and $360°$ angles (procedural knowledge). Mitchelmore (1998) supported this declining trend by showing that students in Grade 2, compared to students in Grade 4 and Grade 6, were more able to recognize dynamic (e.g., the action of opening a pair of scissors) angle-related similarities (procedural knowledge). On the other hand, students in Grade 4 and Grade 6 tended to easily recognize static (e.g., the result of the opening) angle-related similarities (conceptual knowledge) than students in Grade 2 did.

Further evidence for the existence of differences in conceptual and procedural angle knowledge between students in lower and higher grade levels was obtained by Simmons and Cope (1990) in an analysis of Grade 2-Grade 7 students' basic knowledge of angles. They found that students in higher grades (Grades 5, 6, and 7) were not able to estimate the size of $60°, 90°$, and $120°$ angles (procedural knowledge). Besides, neither students in higher grades (Grades 5, 6, and 7) nor in lower grades (Grades 2, 3, and 4) were able to label internal/external angles (conceptual knowledge). In a collective manner, although the Grade 4 classroom enrolled in Devichi and Munier's (2013) study was able to understand right angle (conceptual knowledge) and make estimations of angle size based on side length (procedural knowledge), the Grade 3 classroom was not. This result was not surprising given the findings from Clements and Burns (2000) emphasizing that even above-average ability students' in Grade 4 encountered difficulties with the definition of angle measure (conceptual knowledge) and the development of strategies for angle measure by establishing benchmarks such as $90°$ (procedural knowledge).

The Current Study

In studying conceptual and procedural knowledge, researchers have been overly focused on mathematics domains particularly arithmetics, while neglecting the potential importance that geometry domains such as angles might have. In addition, gender and grade level differences in conceptual and procedural knowledge of angles are under-researched. The present study aims to address these gaps. Specifically, I aimed to contribute to the literature by bringing the two types of angle knowledge (conceptual and procedural) on the scene and to show possible gender and developmental trends in the use of conceptual and procedural angle knowledge. I attempted to address the following research questions which had not been thoroughly elucidated in previous research: (1) Are there gender and grade level differences in terms of conceptual and procedural angle knowledge?; and (2) Are gender and grade level differences, if any, consistent across conceptual and procedural angle knowledge? In line with the exploratory nature of this study, no specific hypotheses were given a priori.

Method

Participants

Participants were from three public middle schools in İstanbul, a major metropolitan city in Turkey. The three schools were similar in student achievement, socioeconomic status, and school administration. The 382 sixth and 376 seventh grade students in three schools who were present on the days of data collection were invited to participate in the study. With 36 participating classes from Grade 6 ($N = 19$) and Grade 7 ($N = 17$), 758 students were the initial sample. Prior to data analysis with this initial sample, missing data

analysis was conducted by using Little's MCAR test (Little, 1988). In this regard, the item non-response in the data set (i.e., demographic and outcome variables) was examined. Results revealed that the data were missing completely at random (MCAR) ($p= .635$, $p >$ .05). Given the low percentage of overall nonresponse rate (3%) of the data and that, the data are MCAR; the listwise deletion procedure that would give unbiased estimates (Cohen, 1998) was used. Following that, 55 cases (22 sixth and 33 seventh graders) were eliminated due to the following: showing insincerity in their responses (e.g., scored 0 in the total test), not completing the test, multivariate outliers (6 cases), or missing demographic information (e.g., gender not specified), leaving 360 sixth and 343 seventh graders, with a total of 703 students (380 females and 323 males) (see Table 1).

Table 1
Distribution of Grade Levels by Gender

	Grade 6	Grade 7	Total
Females	177	203	380
Males	183	140	323
Total	360	343	703

Instrument

Angles Test (AT). All outcome variables were drawn from the students' scores on the AT developed by the researcher. The 18-item open-ended AT was designed to demonstrate student knowledge on angles (see Figure 1 for specimen items) within a broad base of fundamentals covered at both elementary (Grades 1-4) and middle (Grades 5-8) school levels.

In Turkey, *4 + 4 + 4 Education System* has been implemented since 2012-2013 academic year. Along the 12 years of schooling, students attend to primary (Grades 1-4), middle (Grades 5-8), and high (Grades 9-12) school. There are no requirements (i.e., national exams) for the transition from primary to middle school. Within the transition from primary to middle school, students make a school choice between public and private schools. Public schools accept students with regard to their place of residence. Because of such educational policies, parents of students in public schools cannot choose or exert influence over which schools their children attend. Private schools provide an alternative for parents who 1) are dissatisfied with the conditions of public schools, 2) can afford the tuition charged, and 3) receive financial aid. Some private schools are selective in their admissions, while others are not. Various foundation schools accept students according to the results of standardized exams conducted by their measurement and evaluation departments. To be accepted to public schools students are required neither to take level determination exams nor to pay tuition. It is also noteworthy that the same educational curriculum (MoNE, 2013) is implemented for each subject (e.g., Mathematics) in both public and private schools. At the time the study took place, in Turkey as in many countries, the angles topic is introduced to the students at the early years of schooling. Throughout the elementary years (Grades 1 to 4), the emphasis is placed on the basics of angles such as labeling the angles, identifying the corners and sides of an angle or classifying the angles with respect to their size (Conceptual Knowledge) and further finding angle measures (Procedural Knowledge). Drawing on a broad base of fundamentals built in these years, students move toward middle school (Grades 5 to 8) where they initially practice the applications of angles such as computing the measures of

complementary and supplementary angles given inside or outside the two dimensional figures (Procedural Knowledge). In these years, they also become more competent in building relationships between angles and angle measures (Conceptual Knowledge). When students move toward higher grades, they become relatively more comfortable with defining, classifying, and symbolizing angles (Conceptual Knowledge) and applying, implementing, and exploring angle-related algorithms (Procedural Knowledge).

In line with the recommendations of the Standards for Educational and Psychological Testing (AERA, APA, & NCME, 1999), the development and validation of the AT were conducted within five phases: question generation and refinement, pilot study, exploratory factor analysis, confirmatory factor analysis, and reliability analysis. The whole test is available upon request from the researcher.

In *Phase 1: Question generation and refinement*, an initial 30-item question pool on knowledge of angles (e.g., definitions, symbols, and facts related to the angle concept, properties related to the angle types, visual representations of different angle types based on their measurement, and angles word problems) was constructed in the light of learning objectives for angles (MoNE, 2013) utilizing national exam archives (e.g., MoNE Level Determination Exam Questions, 2010–2014) and journal articles (e.g., Mitchelmore, 1997; Smith et al., 2014). The test items were then refined through providing evidence based on expert evaluations (two mathematics teachers and a mathematics educator). The three experts confirmed that the content of the AT was appropriate to measure middle school students' basic knowledge of angles. In *Phase 2: Pilot study,* the AT was administered to 494 students attending to a public school other than the ones involved in the main study. Participants included 157 sixth, 181 seventh, and 156 eighth graders. A scoring scheme, which reflected the potential fully correct, partially correct, and incorrect responses was developed by the researcher. The correct, partially correct, and incorrect answers were scored 2, 1, and 0, respectively. The possible scores on AT ranged from 0 to 36. The researcher scored students' responses and then randomly selected 100 cases to be checked by the mathematics educator, who was involved in the present study for expert evaluation. In order to establish the extent of consensus on use of the scoring rubric for the AT, inter-rater reliability coefficient was computed. The intraclass correlation (ICC) value of .83 indicated a quite high reliability (Cohen, 1998) and the internal consistency of the scoring rubric.

In *Phase 3: Exploratory factor analysis (EFA),* the factor structure of the AT was investigated. A principle component analysis (PCA) with varimax rotation was run for all items, using the pilot data. The Kaiser-Meyer-Olkin measure of sampling adequacy was .85, exceeding the recommended value of .60 (Kaiser, 1974) and Bartlett's test of sphericity (Bartlett, 1954) provided a statistically significant result ($p < .05$) supporting the factorability of the correlation matrix. Subsequent investigations demonstrated the presence of four factors with eigenvalues greater than one; explaining 27.37%, 9.08%, 6.5%, and 6.05% of the variance, respectively. The total variance explained by these four factors was 49.02%. The scree plot was then investigated to select the correct number of factors to be extracted (Cattell, 1966). This inspection revealed a clear break between the second and third factors, and that first two factors explain the much more of the variance than the remaining factors. It was therefore decided to retain two factors for subsequent analyses. The second EFA was conducted using 18 items with an extraction to two factors. The two-factor structure explained a total of 36.46% of the variance, with Factor 1 contributing 27.37% and Factor 2 contributing 9.08%. Additionally, the factor loadings were strong (ranging from .43 to .79) indicating that each item on the related factor was at

a reasonable size (> .30) to define its relevant factor (Hair, Black, Babin, Anderson, & Tatham, 2006). The communalities were above .30 (from .32 to .85), demonstrating that all the items fit well with the other items in its factor (Hair et al., 2006). This analysis guided to form the names of the content/context that are represented in each of the two dimensions: *conceptual knowledge* and *procedural knowledge*. The scores on the first factor, conceptual knowledge (10 items), reflected students' knowledge of concepts, definitions, symbols, and facts related to the angles. The scores of the second factor, procedural knowledge (8 items), reflected students' knowledge of procedures, methods, and techniques related to the applications of angles.

In *Phase 4: Confirmatory factor analysis (CFA)*, the AT was administered to 205 students attending to a public school other than the one involved in the pilot study. Participants included 88 sixth, 62 seventh, and 55 eighth graders. Procedures for conducting CFA have been described elsewhere (e.g., Jöreskog & Sörbom, 1993) and are not further detailed here. The analysis was conducted by using LISREL 9.2 (Jöreskog & Sörbom, 2012). Fit indices recommended by Jöreskog and Sörbom (1993) were the ratio of chi-square to the degrees of freedom ($\chi 2/df$), root mean square error of approximation (RMSEA), root-mean-square residual (RMR), goodness-of-fit index (GFI), adjusted-goodness-of-fit index (AGFI), and comparative fit index (CFI). Schreiber, Stage, King, Nora, and Barlow (2006) suggested substantively interpretive models with chi-square ratios of three or less, a RMSEA from .06 to .08, a RMR below .08, a GFI above .90, an AGFI above .90, and a CFI above .90 as good fitting. In brief, results of the CFA revealed that the obtained fit indices for the two-factor model were $\chi 2$ (451.42, N = 205) = 1.21, RMSEA = .05, RMR = .06, GFI = .97, AGFI = .91, and CFI = .96. These indices suggested that the model fits the data, thus confirming that the underlying structure of the AT is formed by two factors that assess conceptual and procedural knowledge.

Finally, in *Phase 5: Reliability analysis* results showed that the Cronbach's alpha coefficients were satisfactory (Cohen, 1998) for both conceptual (α = .77) and procedural (α = .71) knowledge, as well as for the total knowledge of angles on the AT (α = .83).

Data Collection

The study was conducted during the fall semester of the 2015/2016 academic year. At each school, the study was first described to the headmasters as well as the mathematics teachers, and their consent was obtained accordingly. Students who agreed to participate in the study completed the 40-min AT test during their regular instruction time in school. The researcher administered the test to participants with the assistance of mathematics teachers at each school. In the presence of teachers, teachers did not communicate with their students and assist in the data collection. Prior to the administration, students were provided with oral instruction including the following: (1) directions on the AT to be read to the class, (2) a reminder that students not spend too much time on each item, (3) a remark that students answer all items, and (4) the approximate time required to complete the AT. Along with the AT, students' self-reports on their school, classroom, gender, and grade were collected. Teachers and students were assured of confidentiality and that identification codes were used rather than student names. The data for this study is drawn from a larger data-set on the investigation of prerequisite angles skills and associated misconceptions of middle grade students.

Data Analysis

To examine research questions, a multivariate approach to analysis of variance was performed with two independent (gender and grade level) and two dependent (conceptual knowledge and procedural knowledge) variables. A 2 x 2 multivariate analysis of variance (MANOVA) was conducted to examine the mean differences in terms of knowledge of angles for females and males and for those of different grade levels. MANOVA is an extension of analysis of variance for use when you have more than one dependent variable (Pallant, 2013). It is a robust statistical method that controls or adjusts for the increased risk of Type 1 error (i.e., the incorrect rejection of a true null hypothesis) (Tabachnick & Fidell, 2007). For multivariate results Wilks' Lambda was applied. For multivariate results, the Wilks' Lambda criterion was applied using the significance level .05. To present the effect size partial eta squared (partial η^2) was calculated separately. Preliminary assumption testing on sample size, normality, linearity, multicollinearity, and homogeneity of variance matrices (i.e., Levene's test, $p > .05$) was conducted and no violations were detected. All the analyses were conducted by using IBM SPSS 21.0 (SPSS Inc., 2012).

Results

Descriptive Statistics

The means and standard deviations of conceptual and procedural knowledge scores were presented by gender and grade level in Table 2. From an overall perspective, middle grade students' procedural knowledge scores were lower than their conceptual knowledge scores (see Table 4). Besides, there seemed to be a general pattern of reduced scores in students' conceptual and procedural knowledge for both females and males in Grade 7. In general, both the conceptual and procedural knowledge scores were lower in Grade 7, gender differences seemed apparent in procedural knowledge but not in conceptual knowledge (see below for significance tests). Correlations among two knowledge measures were .67 in Grade 6 and .55 in Grade 7 (see Table 3).

Table 2

Means and Standard Deviations of Conceptual and Procedural Knowledge Scores by Gender and Grade Level

		Grade 6		Grade 7		Total	
		M (SD)	*n*	*M (SD)*	*n*	*M (SD)*	*n*
Conceptual Knowledge	F	11.89 (4.31)	177	11.09 (4.41)	203	11.46 (4.38)	380
	M	11.83 (4.43)	183	10.59 (4.10)	140	11.29 (4.33)	323
	T	11.86 (4.37)	360	10.88 (4.29)	343	11.38 (4.35)	703
Procedural Knowledge	F	7.63 (3.40)	177	6.83 (3.01)	243	7.21 (3.21)	380
	M	6.84 (3.27)	183	5.18 (2.95)	140	6.12 (3.24)	323
	T	7.23 (3.35)	360	6.16 (3.09)	343	6.71 (3.27)	703

Note. N = 703 with the total AT scores. F, M, and T stands for Females, Males, and Total, respectively.

Table 3

Variable Correlations for Grade 6 (upper triangle) and Grade 7 (lower triangle) and Internal Consistency Estimates (alpha)

	1	2	α (6th)
1. Conceptual knowledge	-	.67**	.79
2. Procedural knowledge	.55**	-	.68
α (7th)	.77	.65	-

Note. Grade 6: $n = 360$; Grade 7: $n = 343$. **All correlation coefficients are statistically significant, $ps < .01$.

In addition, mean scores and percentages were calculated for each item (see Table 4). The mean scores along with the percentages represented, to some extent, the acquisition level of knowledge, and thus used as indicators of the most difficulties that students encountered in conceptual and procedural knowledge. Not surprisingly, the results in Table 4 demonstrated that overall, the mean scores of conceptual knowledge items were higher than procedural knowledge items within sixth and seventh graders as well as among the total sample of middle graders. As can be noticed, students were the most competent in identifying whether the given angle measure represents the right, acute, and obtuse angle (Item 12C) and defining the right triangle (Item 6C) in Grade 6 ($M = 1.90$ and $M = 1.56$, respectively) and Grade 7 ($M = 1.87$ and $M = 1.47$, respectively). In contrast, they had the most difficulties with evaluating the angle measure given the exterior angles of a triangle (Item 17P) and finding the angle measure given the two intersecting lines (Item 14P) in Grade 6 ($M = .28$ and $M = .56$, respectively) and Grade 7 ($M = .19$ and $M = .41$, respectively). In other words, there seemed to be a general reduction in procedural knowledge for both sixth and seventh graders, but the reduction tended to be greater for students in Grade 7.

Gender and Grade Level Effects on Conceptual and Procedural Knowledge of Angles

As can be observed in Table 5, results revealed that there was a statistically significant difference between females and males on the combined conceptual and procedural knowledge, $F(2, 698) = 17.03$, $p = .00$; Wilks' Lambda = .95; partial $\eta^2 = .047$. In addition, there were statistically significant differences among Grade 6 and Grade 7 students on a linear combination of conceptual and procedural knowledge, $F(2, 698) = 12.97$, $p = .00$; Wilks' Lambda = .96; partial $\eta^2 = .036$. It is noteworthy, the partial η^2 values were small based on Cohen's criteria (1998), demonstrating that both gender and grade level had a small effect size. More specifically, this implied that gender and grade level accounted for 4% and 3% of the total variation in the combined knowledge of angles, respectively.

When the results for conceptual and procedural knowledge (i.e., dependent variables) were considered separately, there was a statistically significant difference between sixth and seventh graders in conceptual knowledge, $F(1, 699) = 9.57$, $p = .00$; partial $\eta^2 = .014$, a small effect size. Further inspection of the mean scores indicated that sixth graders ($M = 11.86$, $SD = 4.37$) performed slightly better than seventh graders ($M = 10.88$, $SD = 4.29$) in tasks that required knowledge of definitions, facts, and symbols related to angles. Despite reaching statistical significance, the actual difference in conceptual knowledge between grade levels were quite small. Indeed, the small partial η^2 value indicated that 1.4% of total

variance in conceptual knowledge was attributable to differences in grade level. On the other hand, for conceptual knowledge the effect of gender did not reach statistical significance (see Table 5). Notice that gender effect ($\eta^2 = .001$) on the conceptual knowledge scores is trivial (negligible effect).

Table 4
Means and Percentages of Conceptual and Procedural Knowledge Items Across Grade Level

Item	Content	Mean scores			Percentages (%)		
		Grade 6	Grade 7	Total	Grade 6	Grade 7	Total
1C	State the unit of angle measurement.	.88	.66	.77	41.7	30.6	36.3
2C	State the number of sides and corners of an angle.	1.22	1.28	1.25	40	45.5	42.7
3C	Define angle concept.	.56	.48	.52	15.6	12.5	14.1
4C	Define acute angle.	1.03	1.03	1.03	18.3	20.1	19.2
5C	Define obtuse angle.	1.13	.94	1.03	28.1	15.5	21.9
6C	Define right angle.	1.56	1.47	1.51	67.8	65	66.4
7C	Define straight angle.	1.22	1.01	1.12	51.9	42.3	47.2
8C	Define full angle.	1.06	.80	.94	47.8	34.4	41.3
9P	Represent an angle measure in degrees symbolically.	.86	.83	.84	9.7	7.9	8.8
10C	State different types of angles.	1.32	1.35	1.33	41.9	44.3	43.1
11P	Find the unknown angle in a straight angle.	1.36	1.21	1.28	58.6	50.7	54.8
12C	Identify the right, acute and obtuse angles.	1.90	1.87	1.89	93.1	92.4	92.7
13P	Evaluate the truthness of a statement using the right angle definition.	1.51	1.34	1.42	64.7	53.6	59.3

14P	Find the unknown angle in a complete angle.	.56	.41	.49	18.3	11.7	15.1
15P	Find the unknown angle in a right angle.	.80	.66	.73	33.3	25.7	29.6
16P	Represent a right angle symbolically.	1.03	1.08	1.05	21.9	23.3	22.6
17P	Find the unknown angle using opposite angles.	.28	.19	.23	6.1	2.9	4.6
18P	Represent the truthness of a statement about opposite angles visually.	.84	.45	.65	35.8	17.8	27

Note. C: Conceptual knowledge, P: Procedural knowledge

Table 5
2 x 2 MANOVA Results for Gender and Grade Level Differences in Conceptual and Procedural Angle Knowledge

				Univariate					
Source	Multivariate			Conceptual knowledge			Procedural knowledge		
	F^a	p	η^2 partial	F^b	p	η^2 partial	F^b	p	η^2 partial
Grade	12.97	.00*	0.036	9.57	.00*	0.014	25.97	.00*	0.036
Gender	17.03	.00*	0.047	0.74	0.38	0.001	25.57	.00*	0.035
Grade x Gender	1.75	0.17	0.005	0.43	0.51	0.001	3.17	0.07	0.005

Note: Multivariate F ratios were generated from Wilks' Lambda statistic. aMultivariate df = 2, 698, bUnivariate df = 1, 699. * p < .05

An inspection of the mean scores, however, showed that females ($M = 11.46$, $SD = 4.38$) were slightly more able to describe angle types and/or represent angle measures with respect to these types than males ($M = 11.29$, $SD = 4.33$). Combined, although females remained higher than males in the conceptual knowledge scores, the difference in scores favouring females in sixth and seventh grades became minimal (see Figure 2).
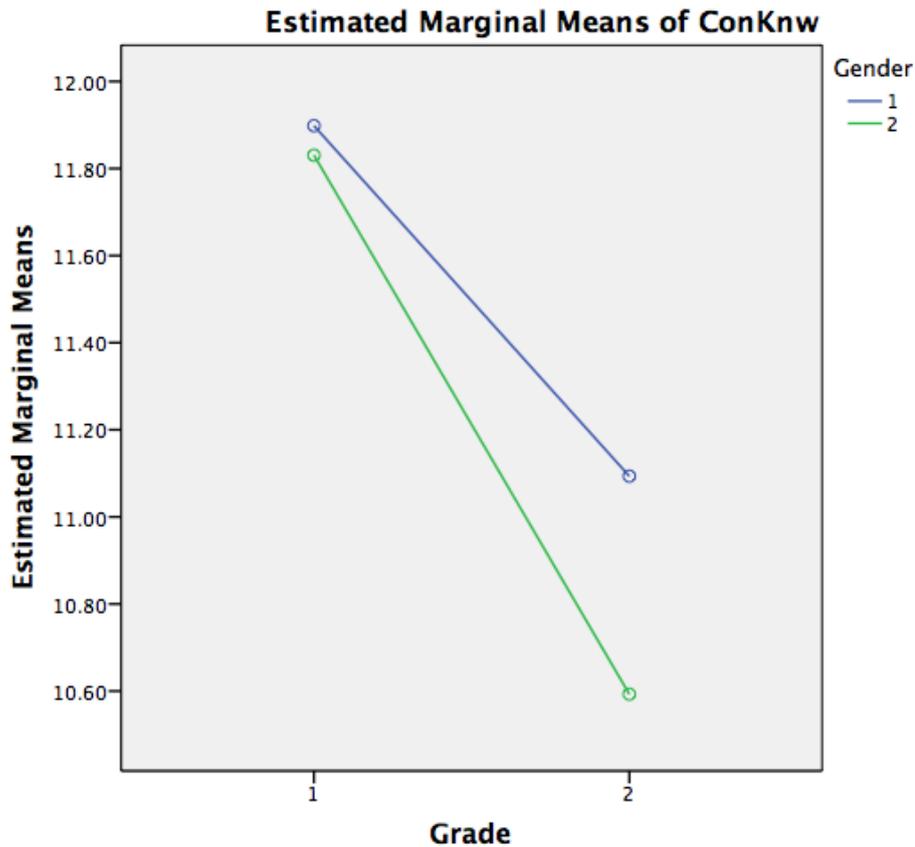
Figure 2. Conceptual Knowledge Scores for Females and Males

For procedural knowledge, there was a statistically significant difference between students attending to Grade 6 and Grade 7, $F(1, 699) = 25.97$, $p = .00$; partial $\eta^2 = .036$, a small effect size. Sixth graders performed relatively better ($M = 7.23$, $SD = 3.35$) than seventh graders ($M = 6.16$, $SD = 3.09$) in tasks that required knowledge of algorithms, methods, and techniques related to angles. Additionally, there was a statistically significant gender difference in procedural knowledge, $F(1, 699) = 25.57$, $p = .00$; partial $\eta^2 = .035$, a small effect size. Females ($M = 7.21$, $SD = 3.21$) scored higher than males ($M = 6.12$, $SD = 3.24$) in the applications of measuring angles. The small partial η^2 values indicated that 3.6% and 3.5% of total variance in procedural knowledge can be explained by grade level and gender, respectively. Viewed together, there seemed to be a general reduction in procedural knowledge for both females and males in Grade 7, but the reduction tended to be greater for males (see Figure 3).
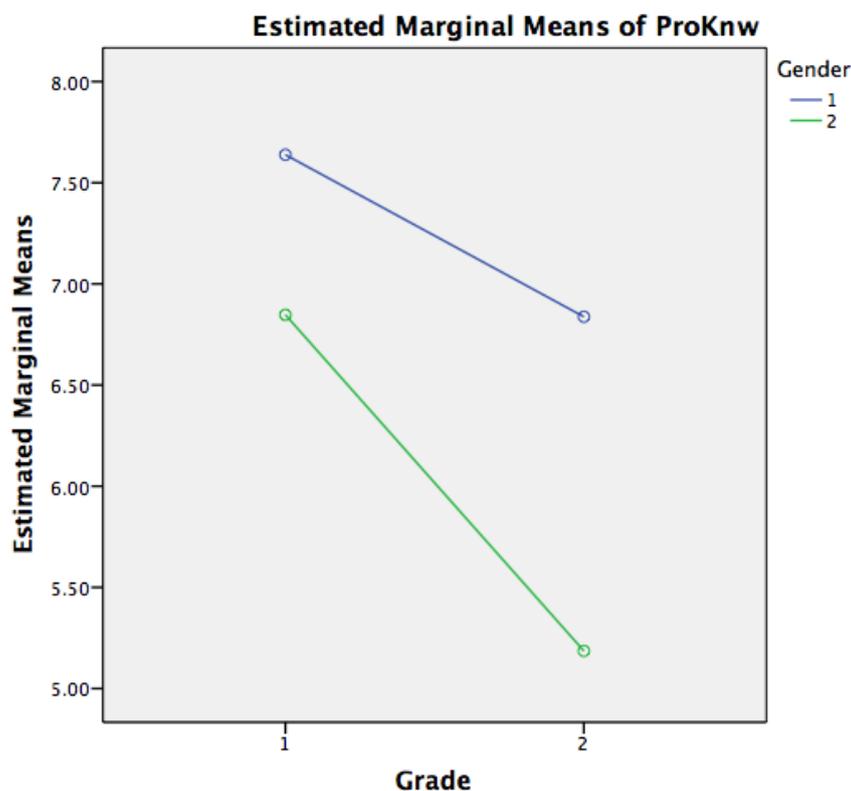
Figure 3. Procedural Knowledge Scores for Females and Males

Discussion

Conceptual and procedural knowledge in mathematics have been studied extensively in the recent past. However, how students acquire conceptual and procedural knowledge with reference to individual differences related to gender and grade level has rarely, if ever, been examined. In the current study, I extended the existing research by examining the individual differences in conceptual and procedural knowledge within the context of angles.

The results support the idea that gender and grade level differences in conceptual and procedural angle knowledge do exist. The present study started by validating the two constructs that are generally accepted as the most common types of mathematical knowledge (Hiebert & Lefevre, 1986). Two components of knowledge applied to a specific geometry domain – conceptual and procedural angle knowledge – were tested in a CFA model, which provided support for the integrity of the measurement. Then using these validated constructs, potential gender and grade level differences were examined. Particularly, for the factor grade, the results showed that students in higher grades tended to have lower scores for both knowledge constructs. However, gender differences were somewhat different for each construct. For the research questions, the analysis found that:
(1) There were statistically significant mean differences in conceptual and procedural angle knowledge favoring students in Grade 6.
(2) There was a statistically significant mean difference in students' procedural angle

knowledge favoring females.
(3) Although no statistically significant gender differences were found for conceptual angle knowledge, inspection of the mean scores revealed female superiority.
(4) Females and males had very similar patterns showing lower conceptual and procedural angle knowledge in higher grade levels of schooling.

Gender Differences

Results revealed that females had higher procedural angle knowledge scores than males. This corroborates the findings in mathematics achievement studies cited in the introduction. Importantly, when females and males were split according to grade level the same gender patterns were observed at sixth and seventh grade levels. The data agree with studies using Grade 6 and/or Grade 7 samples (Hallett et al., 2012; Mabbott & Bisanz, 2006; Schneider et al., 2011). They also suggest that mathematical knowledge gender patterns typically seen in higher grade levels can surface in early years of schooling (Levine et al., 1999; Voyer et al., 1995), in contrast to studies which found no gender differences in middle school samples (Rittle-Johnson & Star, 2007; Rittle-Johnson et al., 2001). More importantly, consistent with previous research on geometry in general and spatial thinking in particular (e.g., Kimball, 1989; McGraw et al., 2006), there was some evidence from this sample suggesting that reduced procedural angle knowledge may occur with respect to gender in favour of females.

Several reasons for apparent angle knowledge differences between females and males have emerged from the literature. On the one hand is the biological argument, frequently offered by researchers (e.g., Benbow & Stanley, 1982; Halpern, 2000), indicating that sex hormones modulate performance on specific cognitive tasks. The environmental argument, on the other hand, have often focused on the differential experience and socialization (for reviews see Friedman, 1989; Voyer et al., 1995), stating that, for instance, no gender differences in mathematics would be found if females were to take the same number of math-related courses as males. A range of alternative explanations may be possible for the female superiority in angle knowledge found in this study. First, it is possible that female students in the present study were likely to answer procedural angle knowledge questions given the evidence from previous research: When gender differences occur in samples of young children, females often score better on tests of computation that require procedural knowledge and males score better on tests of application that require conceptual knowledge (e.g., Kimball, 1989).

A second possibility is that the test (i.e., AT) used in the present study appears to be somewhere in between, with questions covering content found in elementary and middle school geometry texts, and no gender differences arose in conceptual angle knowledge. Indeed, when test items cover content that is taught and learnt almost exclusively in the classroom, no pattern of gender differences tends to be found (e.g., Senk & Usiskin, 1983). This also holds, in part, for the differences in procedural knowledge favouring females. Previous research (e.g., Kimball, 1989; Spelke, 2005) showed that when dealing with familiar material females are confident and motivated, and thus do better on classroom math exams that cover relatively more familiar material (i.e., routine procedural knowledge questions). In contrast, when dealing with unfamiliar material boys are more confident and motivated, and thus do well on standardized tests that are more likely to involve challenging material (i.e., non-routine conceptual knowledge questions). Accordingly, to the males in the present study, the AT, might have appeared to be less of a challenge and perhaps not worth the effort.

A cautionary note, however, should be made regarding the results. Recall that although no significant gender difference in conceptual knowledge was found, an overall inspection of the mean scores showed that females were more apt to perform well on conceptual knowledge questions than males did. Whatever the content of the AT questions might require conceptual or procedural knowledge, the whole test itself, might have reminded students of a classroom math test which was designed to cover what has been learned in class. A quantitative study such as that presented here cannot provide any clear answer as to which of these explanations best account for the existence of gender differences in procedural knowledge and non-existence of gender differences in conceptual knowledge, although it may serve to guide further research in more productive directions and provide considerable practical import. These gender-related issues need to be further investigated in future research. Conducting interviews with students and teachers could provide researchers a clear lens for understanding the big picture.

Grade Level Differences

Regarding developmental changes, results were consistent with previous research suggesting that students tend to have reduced conceptual and procedural angle knowledge as they mature (e.g., Mitchelmore, 1997; 1998). Supporting the downward developmental trend found in other research (e.g., Mitchelmore & White, 2000), the lower conceptual and procedural angle knowledge scores in higher grade levels call for serious attention. Because both conceptual and procedural angle knowledge tend to have significant influences on geometrical knowledge (see Clements & Burns, 2000; Krainer, 1991; Mitchelmore, 1998), the assisting potentials and facilitating roles of these cognitive constructs need to be capitalized. The tendency of weakening conceptual and procedural angle knowledge over middle school years implies that there may be something wrong in the classroom environment or in the curriculum content that need attention.

On the basis of the literature, there may be several possibilities for a consistent trend of decline in conceptual and procedural angle knowledge. A range of alternative explanations may be possible for the unfavourable trend found in the present study, and in some other studies. First, it is possible that as students grow up, they gradually learn to put value in the importance of education (Wigfield & Eccles, 2000), and hence schooling (Yeung, 2011). In a related vein, for some students who find that geometry instruction does not match their beliefs, conceptions, and attitudes they may become less motivated in paying attention to the acquisition of conceptual and procedural knowledge and thus fail to display knowledge of angle-related concepts and procedures. As females and males may have different beliefs, conceptions, and attitudes in, for instance, mathematics (e.g., Leder, Pehkonen, & Törner, 2006) and geometry (Duatepe-Paksu & Ubuz, 2009), it is likely that they display a somewhat different developmental pattern. The second possibility concerns students' educational experiences in geometry classes. If some teachers focus on conceptual knowledge and others focus more on procedural knowledge throughout different grade levels, this would lead to individual differences in acquiring conceptual and procedural knowledge of angles. It should be noted, however, that the participants in the present study were from three schools. Although these schools have some freedom in the choice of teaching approaches, they are required to follow a national curriculum. It is possible that different teaching approaches can, and probably would, affect the conceptual and procedural angle knowledge but that these approaches may need to be more thorough than the disparity that exists across schools in the same geographical region and working under a common mathematics curriculum. Although grade level differences may not be

alarmingly great, the relatively consistent findings of research showing a general decline of conceptual and procedural angle knowledge in Grade 7 call for a re-examination of geometry classes in higher grades. As "differences are not deficiencies" (Halpern, 2000, p. 8), in any case, there may be a range of alternative explanations for the developmental patterns found here. These patterns need to be replicated in different grade levels for generalization and the reasons for such patterns need further examination.

Implications

Whereas the developmental patterns of conceptual and procedural angle knowledge tended to be very similar for females and males, there were some subtle gender differences that may have important educational implications. First, the differences between females and males were larger in the later years than early years of schooling (e.g., $M = 6.83$ for females' procedural knowledge in Grade 7 vs. $M = 5.18$ for males). Although the differences were not large (e.g., $\eta^2 = .04$ for this comparison), mathematics educators may ask: Is it the mathematics curriculum content or the pedagogy that has led to the males' relatively lower conceptual and procedural knowledge? Can something be done to the middle school curriculum to make conceptual and procedural knowledge more appealing to males? Previous research has shown that although more males than females have positive attitudes towards mathematics (Hannula, 2002), their sense of competency tended to drop faster than females (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002). Teachers may provide versatile support for males in higher grade levels suffering from conceptual/procedural knowledge activation: (1) prompt males to grow positive attitudes towards conceptual and procedural knowledge by putting emphasis on the importance of knowledge of concepts/procedures, and the relationships among them, and (2) develop multidimensional tests including both conceptual and procedural knowledge tasks in order to increase males' confidence levels in the activation of these knowledge types. For instance, teachers may use multiple representations from the history of the angle concept such as introducing students first how the early Greeks attempted to classify everything in life into one of ten Aristotelian categories (Aristotle, 1980): substance, quantity, quality, relation, time, place, position, possession, passion, and action, and then asking them to categorize an angle in one of these categories (i.e., the angle is a *quantity*, namely, a distance between the lines containing it). Based on the categorizations, teachers may further request students to draw an angle and then to draw various other angles that could reflect the unfolding of the activation of conceptual and procedural knowledge.

Limitations

There are a number of limitations in the present study that should be mentioned. The major limitation is the inference of developmental changes on the basis of cross-sectional data. To fully answer the research questions of the study, future research should collect longitudinal data at different time points. The developmental patterns can be more fully investigated by comparing differences with repeated-measures designs or by testing growth curve models. Second, the data were collected from a single type of school (i.e., three public middle schools). Future studies may also consider including different types of schools (e.g., private) to examine the generalizability of the patterns of results, as different school types may have different structures, cultures, and expectations that may influence students' knowledge acquisition. In a related vein, because the sample came from mostly

families in a low-to-middle socioeconomic location, the developmental patterns may not generalize to other students with higher socioeconomic status.

Also the present study used items including only angle concepts, relations, and measures. Since student knowledge activation in specific cognitive tasks may differ from one another (Lowrie & Diezmann, 2011) and performance patterns may differ across different mathematics content (Spelke, 2005), further research should investigate the patterns of development in specific tasks (e.g., diagrammatic) and content (e.g., triangles). Because females and males may differ in their conceptual and procedural knowledge as they grow up (Bielinski & Davison, 1998), the patterns may not be the same in certain specific tasks and content. Finally, gender and grade level-related differences in conceptual and procedural knowledge may vary among different cultures (Kimball, 1989). Cultural characteristics may have specific influences on students' development in conceptual and procedural knowledge, which may influence performance in geometry in different ways. Further research may therefore consider using samples from different countries to facilitate cross-cultural comparisons.

Conclusions

To conclude, the present study found that both females and males tended to have lower conceptual and procedural angle knowledge in higher grade levels, which are known to affect mathematics learning outcomes. Whereas males tended to have lower conceptual and procedural angle knowledge scores than females, females' higher scores did not sustain through later years of schooling. Since conceptual and procedural angle knowledge have important influences in geometry achievement, mathematics teachers, mathematics educators, and curriculum designers need to consider ways to improve and sustain these knowledge types, particularly in higher grade levels for both females and males. Whilst the findings imply the need for urgent attention to creating a productive classroom environment in which middle school students would find challenge and engage themselves actively in displaying conceptual and procedural knowledge, further research is required to determine the causes/correlates of individual differences in different types of knowledge. In order to improve students' learning of geometry, further theoretical analysis of geometry topics that includes not only knowledge of the concepts and the procedures required by students, but includes the interrelationship between these two types of knowledge is the focus of future research.

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