Pedagogical Entrepreneurship in School Mathematics: An Approach for Students' Development of Mathematical Literacy

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Emphasis on the development of the inner qualities thought to make up pedagogical entrepreneurship and use of such entrepreneurship as a strategy in mathematics teaching and learning could be expected to strengthen students' development of mathematical literacy. This is because problem solving, local cultures and resources, authenticity and action competence are key elements in the development of mathematical literacy and in pedagogical entrepreneurship in mathematics. A foundation for research on an alternative mathematics teaching approach for future decades is suggested based on the relationship between pedagogical entrepreneurship in mathematics and literature on mathematical literacy and on the description of the contexts and teacher experiences of two best practice examples from the mathematics classroom. Emphasis on pedagogical entrepreneurship in school mathematics through problem solving and authenticity in a learning environment that applauds such approaches to learning of mathematics should be recognized as giving priority to the development of disciplinary literacy in mathematics.

Keywords: Pedagogical entrepreneurship, mathematical literacy, problem solving, school mathematics

Introduction

The ability to develop mathematical knowledge that can be utilised when faced with challenges and problems is a huge priority for school mathematics. The Organisation for Economic Cooperation and Development (OECD) refers to such competence as mathematical literacy, and defines it "as an individual's capacity to identify and understand the role that mathematics play in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OECD, 2009: 14). Developing students' mathematical literacy influences their confidence and competence in mathematics and prepares them for the use and future learning of mathematics in higher education and outside of school (OECD, 2013). The importance of prioritizing research on how mathematics can be taught to enable such development has been argued for (De Lange, 2003; Sfard, 2014), and the research community of mathematics education has for some time underpinned the relation of mathematical literacy to students' everyday lives (e.g., Carraher, Carraher, & Schliemann, 1986; De Lange, 2003;.Freudenthal, 1973; Haara, 2015; Tai & Lin, 2015).

Another issue of priority within school is entrepreneurship education. Entrepreneurship is considered a key factor in improving the well-being of citizens. It is also considered to be a key skill that is acquired through lifelong learning. Entrepreneurship includes the following two elements:

- The specific concept of training people to create a business; and
- "a broader concept of education for entrepreneurial attitudes and skills, which involves developing certain personal qualities and [that] is not directly focused at the creation of new businesses" (European Commission, 2004, p. 11).

The interpretation of what entrepreneurship education means may vary, due to the opportunity to relate the entrepreneurship concept to either of the two above elements. The broader concept of education for entrepreneurial attitudes and skills is represented by the term pedagogical entrepreneurship, which distinguishes entrepreneurship in education from the traditional core economic- and business-related aspects of entrepreneurship concepts. Pedagogical entrepreneurship is action-oriented teaching and learning in a social context where the student is active in the learning process and where personal features, abilities, knowledge and skills provide the foundation and direction for the learning processes (Haara, Jenssen, Fossøy, & Røe Ødegård, 2016; Røe Ødegård, 2003). Such a definition points to the development of inner qualities such as flexibility, the ability to analyse, dynamism, creativity, co-operativeness and proactivity (Haara & Jenssen, 2016). Furthermore, research on pedagogical entrepreneurship considers learning environment, authenticity and student activity as fundamental for the development of such qualities (Backström-Widjeskog, 2008; Ruskovaara & Pihkala, 2013); therefore, pedagogical entrepreneurship is based on constructivist and sociocultural theories on learning (Røe Ødegård, 2015).

An unresolved issue within the mathematics education research community is how to teach for mathematical literacy (Sfard, 2014). In a review article on mathematics education research on mathematical literacy Haara, Bolstad and Jenssen (2017) found that both researchers and teachers are uncertain about what to do in order to teach for students' development of mathematical literacy. They also found that researchers have concluded that specific attempts to work directly with mathematical literacy through mathematics alone does not work, and that it seems that teaching for mathematical literacy calls for something else than traditional teaching of mathematics. However, influence on mathematical literacy has been identified on several occasions through the application of mathematics to solving real-world problems, with attention to the development of students' self-regulation skills and mathematical knowledge, which are transferable to new tasks and problems (e.g., Gellert, 2004; OECD, 2005; Tai & Lin, 2015). Hence, problem-solving features, problem relevance and student activity are recognized as valuable for the development of mathematical literacy. Furthermore, problem-solving features are recognized within the field of pedagogical entrepreneurship through the similarities between the classic problem-solving process from Polya (1990) and the sharing of key features or inner qualities such as initiative, critical thinking, flexibility, co-operativeness, creativity and reflection (Haara & Jenssen, 2016; Haara & Jenssen, 2013; Tidåsen et al., 2015). In addition, emphasis on problem solving and teaching based on problem-solving approaches has proved to be empowering for the orchestration of more creativity and tolerance for ambiguity in school mathematics (Palmér, 2016; Smith & Stein, 2011), which in turn strengthen students' self-regulation and action competence through their increased participation in the development of the mathematics classroom activity. Thus, there seems to be a connection between pedagogical entrepreneurship based on problem solving, authenticity and student activity in school mathematics, and students' mathematical literacy development. However, the didactical and practical operationalization of teaching for mathematical literacy remains an unresolved issue. Pedagogical entrepreneurship may prove to be an approach that provides such an opportunity for operationalization. Based on this background, this article focuses on the following question: *In what way may a pedagogical entrepreneurship approach in mathematics teaching strengthen students' development of mathematical literacy?*

Theoretical framework

Pedagogical Entrepreneurship

Pedagogical entrepreneurship is a relatively new area of interest within schools and research on teaching and learning. While entrepreneurship education has been well supported (European Commission, 2004; OECD, 2009), it has been only recently that researchers have started to look into pedagogical entrepreneurship; for instance, through the investigation of the implementation of entrepreneurial methods in primary and lower secondary schools (Elo, 2015; Haara, Jenssen, Fossøy, & Røe Ødegård, 2016; Moberg, 2014; Palmér, 2016; Tidåsen et al., 2015). Implementation of pedagogical entrepreneurship in schools is of significant current concern according to the OECD (2010), EU (European Commission, 2010, 2011, 2013) and policy documents in several countries (Lund et al., 2011). Based on the key competence definitions from the OECD (2005), entrepreneurship may be referred to as a key competence that should be emphasised within education and lifelong learning (European Commission, 2007). Emphasis is placed on the importance of discovering resources and opportunities, and that this ought to be stimulated and developed in such a way that one might act innovatively in both future work and social situations. Based on this, the school system is encouraged to participate in the development of student qualifications that provide a foundation for the use of such resources in a productive and meaningful manner (Haara & Jenssen, 2016).

The implementation of pedagogical entrepreneurship processes in schools, however, has proved to be challenging (Haara, Jenssen, Fossøy, & Røe Ødegård, 2016). First and foremost, the concept has proved challenging to define, and both school implementation and research on school implementation of pedagogical entrepreneurship have struggled in the attempt to overcome the discussion of principles about the concept (Haara & Jenssen, 2016; Haara, Jenssen, Fossøy, & Røe Ødegård, 2016). Second, Haara, Jenssen, Fossøy, and Røe Ødegård (2016) identified several tensions between policy and practice in attempts to implement pedagogical entrepreneurship in schools. Among these are teachers' bewilderment about the concept and a lack of knowledge about how to make pedagogical entrepreneurship part of the subject they teach.

As clarified and defined in the introduction of this article, pedagogical entrepreneurship is not about supporting or opposing economy and business, but rather about paving the way for the development and growth of human beings. Pedagogical entrepreneurship is defined as action-oriented teaching and learning in a social context where the learner is active in his/her own learning, and where personal features, abilities, knowledge and skills provide the foundation and direction for the learning processes (Røe Ødegård, 2003: in Haara & Jenssen, 2016). Such features point to the development of inner qualities such as ability to analyse, flexibility, dynamism, creativity, cooperativeness and proactivity. Haara and Jenssen (2016) relate these features to self-regulated learning, and self-regulation stages as flexible goal-setting, planning, monitoring of progress and the ability to adapt learning strategies. The use of pedagogical entrepreneurship as an approach for learning becomes an opportunity for students to experience authenticity, for teachers to pay attention to students' ability to self-regulate, and for schools to emphasise students' action competence. This means that a pedagogical entrepreneurship approach concerns students' active learning and self-regulation, and action competence developed in authentic situations. The distinctiveness of pedagogical entrepreneurship mirrors the actual creation of something new, and learning and social interaction are central in this process (Sagar, 2014).

Pedagogical Entrepreneurship and Mathematics

Some important attempts to relate pedagogical entrepreneurship to mathematics have been reported. Tidåsen et al. (2015) reported on an initiative to relate entrepreneurial learning and problem solving in mathematics based on the basic features shared by pedagogical entrepreneurship and problem solving, and through a participative action research approach they involve both teachers and primary school students in the project. Through a small scale study with teacher education students, Haara and Jenssen (2013) identified and systematized characteristics related to both problem solving and pedagogical entrepreneurship. Furthermore, Haara and Jenssen (2016) elaborated on some possible consequences for mathematics in teacher education related to the recognition of a shared pattern between the phases of problem solving from Pólya (1990) and the characteristics of the stages in an entrepreneurial learning process. Actually, Haara and Jenssen (2016) pointed to the fact that such a process proves to be quite similar to the stages of traditional self-regulation development (Zimmermann, 2000). Self-regulation implies flexible goal setting, planning, monitoring of progress and the ability to adapt learning strategies to task demands (Hopfenbeck, 2011; Schraw, 2001). However, Haara and Jenssen (2016) pointed to the necessity of adding emphasis on action competence to the list of features emphasized by entrepreneurial learning. Pedagogical entrepreneurship also emphasizes the subsequent application of knowledge and reinforces its relevance for the students' future work. They therefore recommended future emphasis on action competence, problem solving and self-regulation in mathematics teaching and learning.

The identification of some shared characteristics between entrepreneurial learning and problem solving provides a starting point for a common understanding of the potential for pedagogical entrepreneurship in school mathematics. Problem solving emphasizes analysis, heuristic strategies, processes and checking the viability of a suggested solution, rather than repeating explicit algorithms or formulas. The nature of a problem-solving environment is that it emphasizes problem authenticity and an active learning environment - characteristic concepts of pedagogical entrepreneurship (Backström-Widjeskog, 2008).

Mathematical Literacy

The broad definition of mathematical literacy given by the OECD in 2009 was referred to in the introduction to this article. Some years earlier, De Lange (2003) defined mathematical literacy as the comprehension and application of mathematics through reasoning, thinking and interpreting via problem solving in order to develop an applicative knowledge of mathematics. Tai and Lin (2015), Gatabi, Stacey and Gooya (2012), and Edo, Hartono and Putri (2013) picked up on this relationship and connected problem solving and mathematical modelling to mathematical literacy. When posed with a real-world problem requiring mathematics, the problem must be formulated in mathematical terms and solved by applying mathematical concepts and procedures. Then, the solution has to be interpreted to provide an answer, and that answer must be checked for viability in relation to the original problem. Hence, problem solving and mathematical modelling are key processes in mathematical literacy. Other definitions of mathematical literacy have been suggested (Colwell & Enderson, 2016), but there seems to be common agreement within mathematics education research that mathematical literacy is a person's ability to engage in higher-order thinking skills specific to mathematics that allows the understanding and application of mathematics in real-life, everyday situations. Mathematical literacy therefore concerns the ability to make use of mathematical knowledge, the ability to pose and solve mathematical problems in a variety of situations and the motivation to do so, as well as the ability to develop one's self-regulation through such processes (OECD, 2013).

The commonalities of attempts to define mathematical literacy promote a constructivist vision of learning (Colwell & Enderson, 2016), or even a socio-cultural vision of learning, because of the emphasis on students' activity, application of mathematics and problem solving. Students need to be engaged in learning mathematics in ways they find relevant and applicable. Teachers therefore need to find opportunities to provide students with environments where they can construct mathematical ideas, and then guide and support the students' development as self-regulated mathematics thinkers and users (Haara, 2015). Practical activities may provide such opportunities (Haara, 2015; Moyer, 2001), especially if they are related to subjects that the students find motivating. Orchestrating for learning in accordance with such requirements calls for the acknowledgement of a constructivist, or, more precisely, a socio-cultural approach, to learning mathematics. A socio-cultural approach to learning is based on the principle that knowledge is constructed through collaborative social and cultural activities. The motivation to learn stems from participation in culturally valued collaborative practices in which something useful is produced. Therefore, conversation and common activities are crucial features for learning, and the participant's development is recognized by changed participation in the practice situation (e.g., Carraher, Carraher, & Schliemann, 1986).

Pedagogical Entrepreneurship and Mathematical Literacy

Emphasis on mathematical literacy in school mathematics has been stressed for some time (e.g., Colwell & Enderson, 2016; De Lange, 2003; OECD, 2009, 2013; Sfard, 2014). However, how to teach for mathematical literacy seems to be an unresolved issue within the mathematics education research community (Haara, Bolstad & Jenssen, 2017). Colwell and Enderson (2016) report that it has proved difficult to implement mathematical literacy in mathematics teaching largely because of resistance from teachers who find it challenging to consider reading and writing as part of school mathematics (Hall, 2005; Siebert & Draper, 2008). Making the effort to define literacy within the practices that mathematics teachers use in their teaching has proved to be the way past this scepticism (Moje, 2008; Shanahan & Shanahan, 2012). The introduction of disciplinary literacy within mathematics, or in other words, mathematical literacy, has therefore moved along a path similar to the approach Haara and Jenssen (2016) suggest for implementing pedagogical entrepreneurship in school mathematics. Colwell and Enderson (2016) suggest that emphasis on mathematical literacy is given priority in teacher education, and, as such, is incorporated in several mathematics courses rather than narrowed down to one mathematics methods course. These recommendations are another two commonalities shared with the perspective on pedagogical entrepreneurship in teacher education addressed by Haara and Jenssen (2016). A fourth commonality between emphasis on mathematical literacy and pedagogical entrepreneurship in school mathematics is the necessity of reliance on a learning environment that emphasizes student activity and authenticity. To develop self-regulation and action competence in mathematics, students need to be given opportunities and challenged to use and develop personal features relevant for such competences. This calls for emphasis on inner qualities that are central to both mathematical literacy and pedagogical entrepreneurship. A difference is nevertheless present between pedagogical entrepreneurship and mathematical literacy because of the possible application of pedagogical entrepreneurship to any subject in school and the students' general development of inner qualities comprised by pedagogical entrepreneurship, but not in the same sense as the distinction between disciplinary and content area literacy (Colwell & Enderson, 2016). Through its general perspective, pedagogical entrepreneurship comprises both disciplinary and content area literacy. According to Colwell and Enderson (2016), content area literacy is an approach to integrate literacy in a content area (for instance, mathematics), which incorporates general literacy strategies that may work in any content area to help students read and write information. This is something else and less tangible for the mathematics teacher than disciplinary literacy (here: mathematical literacy). Disciplinary literacy is based on the distinctive character of the subject (here: mathematics), and therefore the teacher may strive for application and authenticity in the teaching of the subject within the subject, and thereby development of action competence and self-regulation skills for the learning student in the subject at hand. In other words, within mathematics, emphasis on learning through pedagogical entrepreneurship can be identified to a considerable extent as emphasising students' development of disciplinary literacy in mathematics. Pedagogical entrepreneurship is about students' active learning methods and learning strategies, self-regulation and action competence developed in authentic situations (Haara & Jenssen, 2016). This calls for disciplinary knowledge and skills in mathematics, problem solving approaches and authenticity through emphasising content relevance, application of nearby natural and societal resources and co-operation with the local community. De Lange's (2003) initial initiative for a definition of mathematical literacy is based on the same demands. The combination of problem solving, authenticity and student activity paves the way for students to develop action competence, both in general and in mathematics specifically, thereby strengthening their ability to understand and apply mathematics in everyday life, make use of mathematical knowledge, pose and solve mathematical problems in a variety of situations, and develop their self-regulation—or, in other words, strengthen their mathematical literacy.

Methods

Research Perspective and Validity

An action research perspective within mathematics teaching and learning implies active action where the mathematics teacher and the students are involved in actual teaching and learning situations. Such a real-world perspective on classroom research, and the fact that the author of this article was the teacher involved, locate this article within an action research perspective (e.g., McNiff, 2002; Ulvik, Riese and Roness, 2016), and then more precisely as influenced by small-scale intervention (Stylianides & Stylianides, 2013) and self-study methodology (Cochran-Smith & Lytle, 2009; Zeichner, 2007). Within action research, the self-study methodology is a form of practitioner research. This method asks me as a teacher to reflect on my practice for the purpose of improving it and the practice of others (Hamilton et al., 1998). The methodology aims "to understand teaching from the inside out rather than the outside in and to simultaneously put what is learned into practice" (Loughran, 2004, p. 154). In this article, the choice of methodology served two main purposes: the professional development of the participants (teacher and students involved), and enhanced understanding of how pedagogical entrepreneurship may serve as an approach for emphasising students' development of mathematical literacy. First, this perspective was chosen because of the necessity of analysis related to possible normative effects provided by two small, best practice example interventions. The methodology was also chosen because of the proximity I could accomplish as both a teacher and a researcher in relation to the examples. Hence, in order to answer the research question in a trustworthy manner and offer mathematics teaching arguments for a "…reframed thinking and transformed practice of the teacher…" (LaBoskey, 2004, p. 844), I made it a priority to be as close as possible to the actual classroom activity, and thereby sacrificed some observational distance on the altar of relevance. This means that I chose the problems to use in the teaching, organised, taught and tutored, and was responsible for the analysis of data. This provided an intended and appreciated interaction between practice and research focus with awareness of the possible influence such a perspective may have on both the actual practice and the theoretical understanding of the practice (Kemmis, 2009).

The choice of methodology challenged my role as researcher because of the proximity to the production of data. This made it necessary for me to remain aware of my roles as researcher and teacher, and of the validity of the two best practice examples presented in the article. Therefore, I considered it important to the project's overarching self-study-perspective to follow Feldman's (2003) criteria for securing validity in a self-study-based project:

- To provide a detailed description of how data were collected and to make explicit what counts as data.
- To provide detailed descriptions of how the presentation of data was constructed.
- To include exploration and discussion of other ways to represent the same self-study; one data set can lead to a variety of representations.
- To show why one approach was chosen instead of others.
- To provide evidence of the value of the changes in mathematics teaching and learning. Self-study is a moral and political activity. If self-study were to result in a change of mathematics teaching, then there should be some evidence of its value.

I do believe that I have fulfilled these criteria, although it can be difficult to document impressions from observational data collected during hectic and activity-based mathematics lessons when one is the teacher orchestrating the activities. Important allies are the outcome of the lessons, and, of course, a thorough theoretical framework established through discussion of research done by peers.

Data Collection and Reflections on Impressions from Data

Because of the nature of the self-study method design, the available data stem from only two sources: literature studies and the teacher's classroom experiences; therefore, this is not a regular, qualitative study ready to provide arguments for the reliability of empirical data. However, this study does provide close contact with actual teaching and tutoring experiences, as well as research perspectives on these experiences through the discussion of possibilities and necessities regarding a possible relationship between pedagogical entrepreneurship in mathematics and mathematical literacy. The study therefore also contains facets of a case-study perspective (Yin, 2014) through its purpose of developing an understanding of the teaching experiences related to the described mathematics lessons and the reliance on data that is normally difficult for others to replicate. On this occasion, it would probably be nearly impossible to retrieve the same data, because of the teacher's close involvement in both producing and collecting data, and the choice not to film or audio-record anything. The settings in each lesson would call for others to be in charge of filming, and thereby to choose which data to collect. Such a data collection process is irrelevant for this article, but it would probably be valuable for an article with another perspective, for instance, on the teacher's role in mathematics teaching through pedagogical entrepreneurship. The theme and design for this article requires that the teacher/researcher is part of the collected data, and thereby, in an unconscious manner, chooses the experiences and impressions that will be subject to analysis. This may seem to be a rather careless and unpredictable way to work in classroom research, but this is not an article about mathematics teaching and learning seen from the outside; rather, it is an article about mathematics teaching possibilities experienced from the inside.

The two best practice examples presented below were lessons produced in different settings. Two different groups of teacher education students were involved, and I had not previously been the mathematics teacher for these students. One group was involved in example 1, and the other group was involved in example 2. Each of the examples represents the first lesson I had with each group, which lasted for about 90 minutes. In each lesson a group of about 25 students participated, in a classroom atmosphere where I emphasised student activity in groups and students' plenary sharing of ideas and input to the problem at hand. I also emphasised to be present and available, but not keen on reducing the problem to an exercise for the students. The students should be allowed to develop and try out their own strategies, and then discuss these processes with the other students and the teacher.

My reflections on impressions from the data are based on two rounds of personal notes from my recall of the lesson experiences: a first round immediately after each lesson was finished, with emphasis on organizational, progress and result-based impressions, and a second round, months later, with emphasis on process-related learning outcomes. In the second round of reflections, my notes from round one and the theoretical framework presented earlier in the article were included.

Pedagogical Entrepreneurship for Students' Mathematical Literacy Development – Two Examples

Operationalization of teaching for students' mathematical literacy development through pedagogical entrepreneurship may, of course, be done in many contexts. Nevertheless, some orchestrating ingredients should be present. First, a setting that implies some degree of authenticity must be chosen. Second, the element of relevance for the students needs to be given attention. Third, the students must be allowed to be active and to own the problem or situation at hand. Finally, it must be mathematically possible for the students to reach a viable solution to the problem (and to possibly learn new mathematical content necessary to deal with the problem in the process). In the following paragraphs, I present two such examples:

Example 1: Barbie's bungee jumping. Some years ago, Wæge (2007) introduced the bungee jump of a Barbie doll to mathematics teaching and learning. In short, the task (or, for the doll, definitely the problem!) is to figure out how many standard rubber bands are required to be tied together in order to make a bungee cord long enough for the doll to perform a qualified bungee jump from a defined height and get her hair wet from a water-filled wash-pot or bucket on the ground below the jumping place, without breaking her neck in the attempt. In other words, this is serious business.

First, the students were thoroughly introduced to the task at hand and divided into groups of three or four. Next, a Barbie doll was left in their protective but slightly unpredictable custody. For the mathematical part of the problem, the students were given a location for the jump, in their case, a ledge on the top of a staircase that would give the doll an approximately 5-metre-long dive (it was measured to exactly 4.80 metres). They were also provided with a pile of rubber bands to be tied together and a 2-metre-long measuring tape. After an initial round of questions about the necessity of accuracy, and some discussion of the value of modelling in mathematics, the groups entered the phase of problem analysis: how can we decide how many rubber bands we need? After some rounds of discussions, all groups entered a trial phase to collect data from the doll's "jump" from shorter ranges. Rubber bands were tied together as the doll jumped from higher and higher bases, and the length of each jump was registered. Then, the data were plotted into a coordinate system, with the number of rubber bands on the horizontal axis and the measured jump length (measured from the jumping point and downwards) on the vertical axis. Through studies of the plotted point (and not the use of the least square method for a straight line on this mathematical level), the students reached the conclusion that the relation between the number of rubber bands and jump length can be represented by a linear function. Hence, this is an obvious lower secondary school problem. Some discussion then needed to be devoted to the a and b in $y = ax + b$, a being the length of a rubber band stretched by the weight of the doll, and b being the height of the doll, and then of course the number of rubber bands needed to tie the bungee cord around the doll's ankles. After some more detailed preparations and discussion about tiredness in the bungee cord, all groups were ready to wish their stunt Barbie doll a safe but tense and hair-washing journey. An impressive crowd gathered for the bungee jump show, and seven Barbie girls made their "jump". Four of them had quite impressive jumps, and almost reached the wash-pot on the floor, while two of them had jumps that were much too short to say that it was money well spent. Sadly, the last one did not survive. She made the jump all right, but smashed into the wash-pot and floor before the bungee cord was fully stretched.

Example 2: Awning assembly. Shortly after the bungee jump session, I received a phone call from a peripheral acquaintance. He contacted me on behalf of his brother, who worked in a local company that sold and assembled awnings. To be able to help customers more precisely in the discussions of what a fully assembled awning would look like on their wall, and for practical help in the actual assembly process, he was looking for a straightforward and operational formula for the relation between the length of the awning, the end of the awning's height from the ground, the assembled height of the awning at the wall, and the angle between the awning and the wall it would be mounted on. The simple question to me was: "Can you help?"

The local company's problem was as authentic as one can possibly ask. This was actually a real problem! The opportunity to help with mathematical issues in the local environment was appreciated both by the teacher education students I chose to introduce the problem to and their teacher (me). Other content was put aside, and time and space were invested in the awning assembling problem.

First, the students had to analyse and understand the problem. What was this all about? Sketches were made to sort the information and understand the practical challenges that the awning assembler faced. In Figure 1, the information has been structured in a sketch illustrating the problem from the side. Point A represents the assembly height of the awning, point D is the ground, the length AD = h represents the house wall, point B is the end of the fully stretched awning (hence AB = k), and point C is the height above the ground of the fully stretched awning (hence CD = x). The angle CAB = r is the angle between the awning and the house wall. The question the students then faced was: How can we present a connection between r, h, k and x?

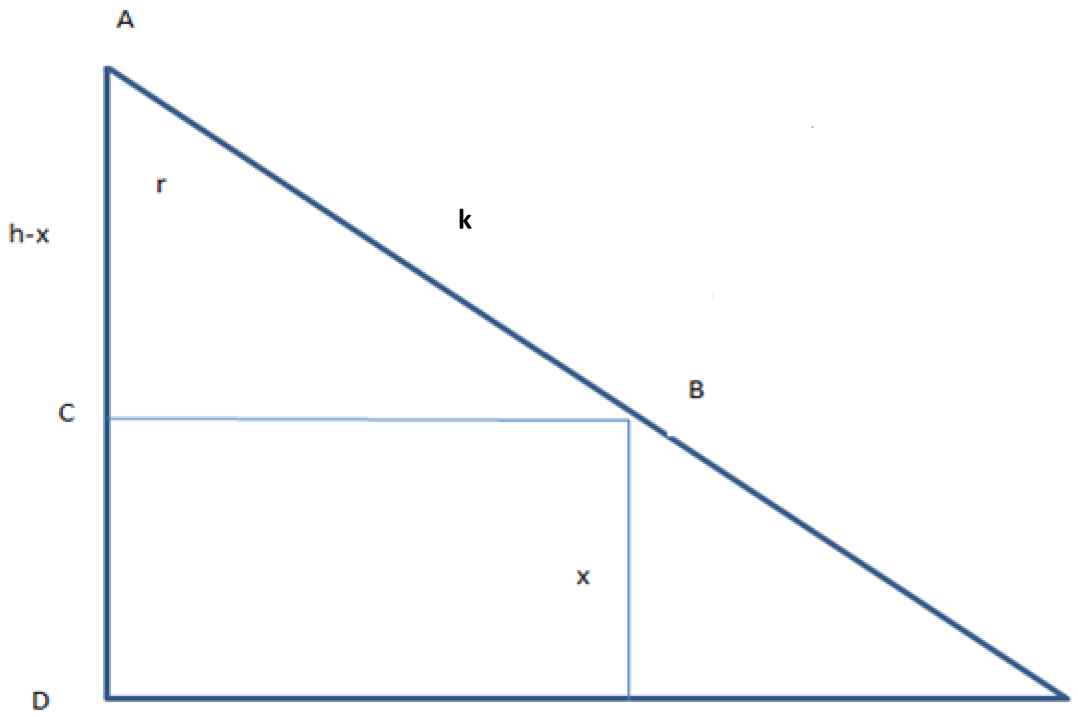

Figure 1. Sketch illustrating the awning assembly problem from the side

First, it was recalled that the trigonometrical functions sine, cosine and tangent represent ratios between two sides in a right-angled triangle. In this particular problem, the cosine of the angle r is the ratio of the length of the adjacent side (h–x) to the length of the hypotenuse (k). This means that:

$\cos r = \frac{h-x}{k}$ and that $(h-x) = k \cdot \cos r$

In a plenary session the students then made the setup shown in Figure 2 (in a spreadsheet developed live, with the teacher as the secretary and critical friend), in which the awning assembly company would be able to set k, h, and r in order to be able to see the impact from one variable on the resulting height above the ground for the end of the fully stretched awning (Point B in Figure 1). To be able to choose x would prove less interesting because of its lack of practical potential as a supplier of terms for the awning assembly decision. One rarely chooses the height of the end of the fully stretched awning (point B in Figure 1) as a term for the assembling of an awning, but it is rather an important issue when it comes to deciding where to place the awning (point A in Figure 1, and thereby the length h), the length of the awning (the length k), and the angle between the wall and the awning (r). In Figure 2, a screenshot of the setup is shown, with starting points for k, h and r, and linear increase (k and h) and linear decrease (r) for the three variables. These three variables could be changed separately, and thereby work practically in relation to each other to show the impact on the length AC (h–x) and finally on x, for the ultimate answer to the awning assembler's question of height above ground for the fully stretched awning.

4	**For varying h, k, r we have:**				
5					
6	AC=AB*cos r				
7					
8	**This gives:**				
9					
10	x= h - AC				
11					
12					
13	Then we have:				
14	Start value length of awning k:		1		
15	Start value assembling height h:		2		
16	Start angle r between k and h:		90		
17					
18	**This gives:**				
19	**k**	**h**	**r**	**AC**	**x**
20	1	2	90	0,00	2,00
21	1,5	2,1	85	0,13	1,97
22	2	2,2	80	0,35	1,85
23	2,5	2,3	75	0,65	1,65
24	3	2,4	70	1,03	1,37
25	Please note that k, h and r can be fixed, in order to follow the influence from one variable at the time				

Figure 2. Screenshot from spreadsheet setup of relation between the four magnitudes involved in the awning assembler's calculations

Identifying the necessity of using the trigonometric cosine function has implications, not only regarding the immediate solution of the problem at hand, but also for the level of mathematical knowledge necessary to be able to deal with the problem. Trigonometric functions are not normally part of the mathematics curriculum until students reach upper secondary school (e.g., Kunnskapsdepartementet, 2006). A lower secondary school teacher therefore has to decide whether to invite the students to learn mathematical content that is not part of their curriculum, and whether they have the prerequisite skills for such a pedagogical challenge, or to let the problem wait until the students meet this mathematical issue through the progression suggested in the curriculum and textbooks. "What can you learn and use together with others, and then later on manage alone?" paraphrases the scaffolding scenario built by Vygotsky. This real-life problem offers students the opportunity to co-operate, contribute to the development of the local environment and touch an authenticity level that they are rarely introduced to in mathematics classes. Problem solving is work on problems that one does not have an immediate solution to, and it may call for introduction to mathematical content that one is not already familiar with. This issue is the same for the bungee jumping problem, but in that example, the element of authenticity and development of the local environment is, of course, not that obvious.

Discussion

Learning through pedagogical entrepreneurship in mathematics challenges more traditional, acquisition-based teaching processes. Through inductive, participative processes, the goal is to reach beyond simply reproducing the curriculum. The challenge is to have the students become more involved in their own professional development. Pedagogical entrepreneurship may be a problem-based strategy that demands preparation for student activity and participation in which various interaction processes contribute to the transmission of tacit, private knowledge into explicit and shared knowledge. Pedagogical entrepreneurship can stimulate in-depth learning and become a learning strategy that addresses the prevailing understanding about what ought to be emphasized in school in the future (NOU, 2014). To learn through pedagogical entrepreneurship demands the type of teaching priorities that activate each student's interests and resources and underpin the ability to see possibilities, to take the initiative and to organize targeted work. Active students thus become driving forces for their own learning. Through an approach to the mathematical subject or problem based on co-operation and dialogue between students and teacher, they become part of a learning community (Wenger, 1998). The students must, within these frames, be active and give and receive responses, both to each other and to the teacher. The way the teacher uses these contributions in teaching and tutoring has consequences for the student's choice of a dual work process both to solve the problem at hand and to nurture the development of action competence and self-regulation skills (pedagogical entrepreneurship), or choice of a single approach to solve the problem at hand (problem solving). The students' development of mathematical literacy depends on the opportunity to experience environments where they can construct mathematical ideas. The teacher must orchestrate such opportunities, and then guide and support the students in their work, but not deprive them of the possibility to be creative, strive and make decisions. This is crucial phases in the students' self-regulation development. The combination of problem solving, authenticity and student activity paves the way for students to develop action competence in mathematics. They will be more experienced with mathematics related to everyday life, and thereby strengthen their ability to understand and apply mathematics in a variety of situations. They will strengthen their mathematical literacy. The two examples above, and the described approach to the problems they present, provide such opportunities.

Simultaneously, there will be expectations about the students' ability to handle such challenges as the two examples provide, both regarding mathematical content and exploitation of the autonomous frames provided by a pedagogical entrepreneurship approach. Pedagogical entrepreneurship as a learning strategy may appear to be demanding for students who are familiar and content with traditional teaching, organization and methods, and mathematical literacy is not necessarily highlighted explicitly in school curriculums or textbooks (e.g., Gatabi, Stacey, & Gooya, 2012; Kunnskapsdepartementet, 2006). Such challenges may lead to a noticeable clash between cultures, either when the teacher uses priorities that are not necessarily coherent with school curriculum priorities or when the teacher does not necessarily offer a suggestion for the solution of a problem. Prior research has shown that students find the latter change of perspective unfamiliar and challenging, probably because their experiences in solving mathematical exercises usually include a comparison with a set answer, and mathematics teachers tend to reduce problems to exercises through their desire to help students (Schoenfeld, 1993; Smith & Stein, 2011). Instead, through use of a pedagogical entrepreneurship approach the teacher introduces an alternative way to learning and must therefore be aware that his/her role may change from traditional instructing to prioritizing student participation and acting more as a tutor and less as an expert or guarantor for a "correct" answer. This means that it is not possible to ignore teachers' views about learning and their opinions of what is the best approach when emphasizing a process-based learning strategy in teaching. This raises questions about whether mathematics is mainly about calculation rules and algorithms, or whether it is also about making assumptions, reasoning about consequences of these assumptions, and creative thinking. Such questions may challenge the beliefs teachers have about teaching and learning of mathematics (e.g., Haara, 2015; Moyer, 2001) and teacher identity (Sfard & Prusak, 2005).

Might emphasis on pedagogical entrepreneurship in school mathematics then prove to be an alternative for teaching mathematical literacy? Pedagogical entrepreneurship brings authenticity, a process-based culture and a problem-solving approach to the classroom. On the other hand, students' development of mathematical literacy is not highlighted just by giving priority to problem solving. The problems must be worked with in an environment where initiative, creativity, sharing and co-operation prevail, they should be attached to settings that the students see the relevance in (here: authenticity), and they ought to provide or introduce work strategies that the students can apply to new problems in the future.

Conclusion

In this article, I have attempted to examine in what way a pedagogical entrepreneurship approach in school mathematics may strengthen students' development of mathematical literacy. This has called for a review of the relationship between pedagogical entrepreneurship in mathematics and mathematical literacy. This involves identification of elements from pedagogical entrepreneurship and their relation to problem solving in mathematics, and presentation of two examples where a pedagogical entrepreneurship approach have been used. The conclusion is that emphasis on the development of the inner qualities of pedagogical entrepreneurship and use of a pedagogical entrepreneurship approach in school mathematics can be expected to strengthen the students' development of mathematical literacy. This is because problem solving, local cultures and resources, authenticity and action competence are key elements in the development of mathematical literacy, as well as in pedagogical entrepreneurship in mathematics. The two examples presented show that it is possible to plan for such operationalization in mathematics teaching already in lower secondary school. This operationalization would be based on acknowledgement of students' needs for development of knowledge and action competence for their future application of mathematics in situations that call for mathematical considerations. What do they need to know? When do they need mathematics? How do they know which mathematics and strategies to apply when they realize that a mathematical approach is required? The latter question is central. Students' learning through pedagogical entrepreneurship in school mathematics is a possible operationalization of mathematics teaching and learning for mathematical literacy development, which in turn helps students' develop self-regulation and competence in choosing and applying the right mathematics when relevant. The two examples therefore serve as confirmation of the possibilities that exist within school mathematics when it comes to emphasising the development of the inner qualities of pedagogical entrepreneurship, and how use of pedagogical entrepreneurship as a strategy in mathematics teaching and learning will strengthen the students' development of mathematical literacy.

A call for change in the teaching of mathematics to develop students' mathematical literacy requires empirical research both on how mathematical literacy is currently taught and how it may be taught in schools in the future, the results of such attempts and their implications. Approaching students' development of mathematical literacy through pedagogical entrepreneurship in mathematics is a promising attempt and should be investigated thoroughly through classroom research.

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