Teaching and Learning through Problem Solving: A New Zealand Perspective

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Introduction

The Core Mathematics qualification—a new post-16 pathway for further mathematics study—aims to:

> consolidate and build on students' mathematical understanding and develop further mathematical understanding and skills in the application of maths to authentic problems. …the Core Maths qualification should foster the ability to think mathematically and to apply mathematical techniques to a variety of unfamiliar situations, questions and issues with confidence. (Technical Guidance, DfE, 2014)

In this paper, I consider what could be involved in developing and teaching such a curriculum based on my own experience working with curriculum development, pedagogical best evidence research, and teacher learning projects in New Zealand. In particular, I focus on the scope of potential/desired learning outcomes and the associated opportunities to learn.

Desired Learning Outcomes

In looking to make a difference to those students who would have previously reached an endpoint in their mathematics learning, we need to provide opportunities that are different, engaging, and relevant to their future roles as numerate adults in society and the workplace. In recent interviews with students I explored why they thought they needed to learn mathematics. A 10-year girl responded:

> Girl: To get better, but I don't get better. I've done maths for years and I don't get better?
>
> Int: So how will maths help you in the future?
>
> Girl: To count money, to go to the shop. Not for a job cause I can't do maths.

If Core Maths is going to change the life pathway of students like this girl then there needs to be major changes to the 'what' and 'how' students in the Core Maths programme experience mathematics and the learning of mathematics. Setting out to provide a curriculum where students learn about mathematics and about themselves as a doer and user of mathematics through a problem solving approach will take considerable resolve. For a start, this approach challenges a strongly held belief of many teachers that students who have difficulties in traditional mathematics classrooms will have even greater difficulties with problem solving activities in inquiry-based classrooms. It also challenges the long-held belief of many within our society that only some people can do mathematics; that is, you either have a maths brain or you don't. We now know from many research studies that mathematics teaching that promotes growth mindset—based on an incremental orientation that views ability as malleable and able to increase with effort—is more effective than mathematics teaching aligned to an entity orientation that perceived ability and a fixed quality of self that does not increase with effort (Dweck, 2006).

So what learning outcomes would we want to see for Core Maths? It is clear from many reports on adult numeracy requirements (See ACME reports) that society needs people who are confident with mathematics, who can develop mathematical models and predictions, and who can justify, reason, communicate, and problem solve. However, being a powerful mathematical thinker outside of the classroom, even when using relatively basic secondary mathematics, is not something that we should take for granted. I know from personal observation in the context of building that those students with high-level school mathematics qualifications often struggle to apply that mathematics in a problem-solving situation in the field. Their confidence to transfer school-based, traditional textbook learning is sorely lacking. And to that end, the suggestion that Core Maths focus on more basic maths in complex situations (Hodgen & Marks, 2013) is an admirable goal.

To ensure transfer of mathematical skills and practices, the focus of Core Maths programmes must be mindful of the need to develop students' mathematical disposition, alongside the important conceptual understanding, procedural fluency, problem solving and reasoning components of mathematical proficiency (National Research Council, 2000). Mathematical disposition—the ability and inclination to use mathematics in one's real life, in the workplace, in the home, and in society—needs to be firmly embedded into the learning outcomes/goals of a problem-solving curriculum if there is to be short and long-term engagement by pupils and teachers. Particularly so in mathematics classrooms, where "students learn more than the mathematics—they learn what it is like to be a member of that community of practice and whether or not they want to participate" (Boaler, Wiliam, & Zevenbergen, 2000, p. 195).

Rethinking Learning and Teaching

Teaching and learning through problem solving (Schroder & Lester, 1989) requires that students rethink what it means to be a successful learner—what are the skills and ways of doing mathematics that are seen as important? Moreover, it requires that teachers rethink, what it means to be a successful teacher—especially expectations of how we as teachers define success and participation in the classroom (Boaler, 2008). In creating students' normative identities as 'doers' of mathematics rather than 'receivers' of mathematics we need to shift perceptions of teachers 'ethic of care" (Anthony & Walshaw, 2009) from one where the teacher makes sure that students are provided with more than tasks and practice opportunities needed for tests, to one where students are supported to take risks both in group and whole class settings, where authority is distributed and where students are obliged to communicate not just about what they know, but also about what they don't yet understand (Anthony, 2013). Working within a mathematical inquiry community will also support students to develop pro-social skills associated with collaboration, flexibility, and relational respect for diversity—all highly valued workplace skills (Kathy Sullivan, the first women astronaut to walk in space, NZ media interview June, 2015).

Learning through problem solving

For all students, the 'what' that they do in the classroom is integral to their learning. It is by engaging with tasks that students develop ideas about the nature of mathematics and learn that they have the capacity to make sense of mathematics. Mathematical tasks draw students' attention towards particular mathematical concepts and provide information surrounding those concepts. More than any other actions that teachers might take, posing tasks that engage students in thinking for themselves about mathematics through the act of problem solving is the main stimulus for student learning (Anthony & Walshaw, 2007; 2009). Problem solving tasks, in particular should:

- engage students in doing important mathematics,
- foster meaning making, understanding, and connections to other aspects of mathematics,
- be challenging, with the pathway to the solution not being immediately obvious to the students,
- require students to think, make decisions, and communicate with each other,
- foster the ability to ask questions, and
- use contexts or situations with which the students are familiar and which they see as potentially useful for them or connected to their lives.

A framework that can guide thinking about tasks, proposed by Stein, Grover, and Henningsen (1996), highlights that the consideration of classroom tasks by teachers involves multiple phases. Collectively, these phases serve to mediate the student learning:

- Firstly, teachers attend to the task as presented in curricular material/instructional material. Considerations are influenced by their goals, their mathematics knowledge, and their knowledge of their students.
- Secondly, the task as set up by the teacher in the classroom is influenced by the classroom norms, the instructional expectations and obligations, and the students' learning habits and dispositions.
- Thirdly, the task is interpreted and implemented by the student.

Rich problem solving tasks

In the first step noted above, identifying the task, there are many possible sorts of tasks that a teacher might select, ranging from practice/worksheet tasks, 'open-middled' task, open-ended task (Sullivan et al. 2015), and modelling type tasks (Meyer, 2015). However, when assessed in terms of opportunities to engage in cognitively challenging mathematical thinking, not all tasks are created equal. Smith and Stein (1998) provide a useful task categorisation that distinguishes those tasks which require low levels of cognitive demand (e.g., memorisation tasks and tasks that use procedures without connections) versus those that require high levels of cognitive demand (e.g., using procedures with connections and 'doing' mathematics tasks). Problem solving and modelling type tasks typically require high-levels of cognitive demand and are more likely, therefore to lead to the greater learning gains for all students (Anthony & Walshaw, 2009).

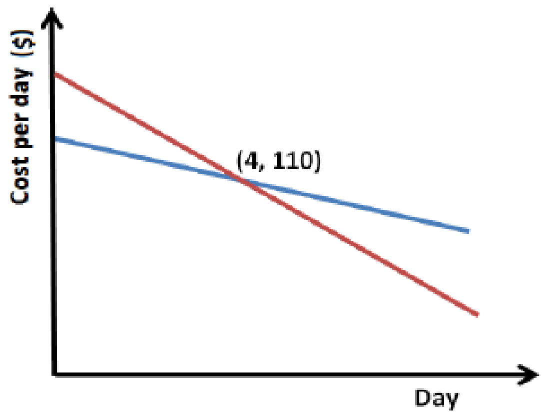
Rich tasks include those that afford opportunities for students to interpret and develop multiple representations, to evaluate mathematical statements, to make conjectures, justification and explanations, and encourage analysis and reasoning of solutions and the making of mathematical generalisations. For example in the following activity (Figure 1), students can solve this task by using either an arithmetic, geometric, or conceptual approach (see http://nzmaths.co.nz/level-5-rich-learning-activities).

Activity:

Task: A car rental business has two rental schemes, red and blue for rentals up to ten days. These schemes are advertised in their brochures with this graph.

Use the following information to work out how many days of rental would carry the same total cost on either the blue or the red scheme.

The area under the graph gives the total cost of renting.



The schemes each follow a linear pattern, cutting the vertical axis at 120 and 170.

Both blue and red schemes cost $110 on the 4th day of rental.

Figure 1: Renting a Car

Another rich task format is the open-ended or middle task (see Figure 2) that encourages the use of multiple solution strategies and or multiple solution responses. Less well defined than closed tasks, the openness of the tasks means that:

> students are less prompted to recall a rule or procedures as a way of solving the tasks, and so need to consider the meaning of the concepts involved, make decision about processes for undertaking the tasks, and consider the possibility of multiple responses, and thinking about appropriate ways of communicating results. (Sullivan et al., 2013, p. 58)

- $2\_ \times 3\ \_ = \_\ \_0$ What might the digits be?

- You describe a situation with the expression $5x$. What might the situation be?

- A rectangle has a length of x cm and a width of 4 cm. What algebraic expression can describe features of the rectangle?

Figure 2: Open tasks examples

Another form of rich tasks is those that involve authentic contexts. Here we see that the mathematics used is not necessarily of an advanced level, but the process of mathematising from the real situation, selecting the appropriate mathematics, and completing the calculation with confidence, including the checking of the solution process and reasonableness, is of central importance. In the workplace being able to solve these types of problems is often essential to financial or reputational viability of a business (see Figure 3 example), or with the example of drug calculations, to the health and well-being of a patient (Wright, 2012).

The foundation on a building project has twenty-four wooden piles (circular posts) each of diameter 230mm.

The holes for the piles, each of a diameter of 450mm, have been excavated to a depth of 1.5m.

You need to order the concrete mix – to be delivered on site How many cubic metres of concrete should you order?

Figure 3: Ordering the Concrete

Fostering Learning Communities

Opportunities to learn mathematics depends significantly on both what is made available to learner—the task—and on the learning community that is developed. Because learning mathematics in a classroom is an inherently social process, our pedagogical approach should serve capitalize on the affordances of social interactions. Rich tasks, highlighted above, provide suitable opportunities for group task discussions—all of which provide occasions for students to learn to engage with other's ideas, and to experience a variety of "alternative conceptual systems that are potentially relevant to the interpretation of a given situation" (Lesh & Zawojewski, 2007, p. 789).

The productiveness of any group is determined to a large extent by the established participatory and communication practices (Hunter, 2008; Walshaw & Anthony, 2008a). Teachers need to take responsibility to establish the participation practices based on respect and intellectual risk taking. Establishing what it means to be accountable to the learning community will involve explicit discussion about participants' rights and obligations concerning contributing, active listening and valuing other's contributions (Hunter & Anthony, 2011).

Fostering Mathematical Discourse

A mathematical inquiry community should create a space that focuses on learning rather than performing. For teachers to value and build on students' thinking it is essential that opportunities for interaction, between peers, between student and teacher, and teacher and student are created. Mathematical talk is valuable on so many levels (Walshaw & Anthony, 2008b). For example, mathematical talk:

- helps students clarify and organise their thoughts,
- facilitates personal and collective sense making,
- supports building connections between representations and multiple strategies,.
- enables students to use others as a resource of ideas to challenge and broaden understanding,
- helps students learn mathematical language,
- enables the sense of authority to moves from teacher to discipline,
- provides a resource for teachers – build on their thinking,
- supports development of mathematical identity, and
- enables students to see mathematics as created by communities of learners.

With mathematical discourse now widely recognised as a valuable resource and activity for the learning community—both students and teachers—we turn our attention to ways in which teachers can facilitate or orchestrate mathematical talk in the classroom. Research studies (e.g., Chapin, & O'Connor, 2007; Hunter, 2008; Walshaw & Anthony, 2009) have identified a range of high-leverage practices that effective teachers use to elicit and respond to students' thinking in productive ways. Hunter (2008) provides a communication framework for supporting students to engage in group discussions that includes introducing students to friendly argumentation and explicit teaching of explanatory justifications. Kazemi and Hintz (2014) describe a range of teacher talk moves that support classroom discourse:

1. Revoicing: "So you're saying…"
2. Repeating: "Can you repeat what she said in your own words?"
3. Reasoning: "Do you agree or disagree and why?"
4. Adding on: "Would someone like to add on to this"?
5. Wait time: "Take your time…"
6. Turn-and talk: Turn and talk to your neighbour…"
7. Revise: "Has anyone's thinking changed?"

In terms of orchestrating whole-class discussion Smith and Stein (2011) describe five practices for effectively using students' responses in whole-class discussions:

1. *Anticipating* students responses prior to the less
2. *Monitoring* students work on and engagement with the tasks
3. *Selecting* particular students to present their mathematical work
4. *Sequencing* students' responses in a specific order for discussion
5. *Connecting* different students' response and connecting the response to key mathematical ideas.

Explicit Teaching

As noted above, supporting students to engage in rich mathematical tasks and learn through the process of problem solving requires that we provide appropriate tasks and opportunities for mathematical inquiry within group and whole-class scenarios. But for many students this type of engagement will be new, and our teaching needs to be explicit in terms of expectations and support. In helping students become active problem solvers, learners who engage in the process of thinking and reasoning, sense-making through problems, we need to ask and support students to:

- Elaborate their answers and explain why their solutions work.
- Think about what they know before solving a new problem.
- Make connections among problem-solving strategies.
- Make connections among representations: drawings, numbers, contexts, and concrete materials.
- Work through their errors and misunderstandings.
- Reflect on their learning.

Only with explicit teaching about the practices of learning and doing mathematics will teachers be able to maintain the high-level cognitive demands of rich tasks. In doing so, we need to reaffirm that 'doing' maths requires struggle, with an emphasis on the making meaning rather than speedy completion of tasks. In supporting students to engage and struggle with tasks we can look more closely at the planning of tasks. This involves considering the launch of a task where key contextual features and key mathematical ideas are discussed —while avoiding the suggestion of a particular solution method (Jackson et al., 2012). In considering the background knowledge which students are likely to bring to the task, planning should also consider *enabling* prompts for those students who might have difficulty starting on the tasks, and *extending* prompts to challenge students to move to generalisation (Sullivan et al., 2013).

Conclusion

Core Maths presents an exciting opportunity; in creating a new and or alternative pathway for students Core Maths offers an opening, as designers and teachers, to revisit and rethink about the priority learning goals and outcomes for mathematics education. In doing so, however, it is equally important that consideration is given to the pedagogical practices that will support such learning goals. To support more students to participate and achieve in mathematics— to challenge normative ideas about who can and who cannot participate in mathematics—this paper has argued that we need to provide opportunities to learn mathematics as a social, collaborative, participatory experience—one that puts students' mathematical thinking at the centre of instructional planning and action. Only then can we provide *all* students with opportunities to engage with authentic mathematical practices and reasoning experiences. In looking to enhance students' understanding of mathematical and statistical concepts in a way that will make a difference and transfer to their future lives it will be important that students have opportunities to make mathematical and statistics 'real', to experience mathematical and statistical thinking and reasoning in the context of problems solving and modelling activities that relate to future pathways.

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