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Using Concept Maps to Assess the Effect of Graphing Calculators Use on Students' Concept Images of the Derivative at a Point

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ABSTRACT

This study used concept maps to investigate the effect of using graphing calculators on students' understanding of the derivative at a point. The study looked for differences between the concept images that are held by students' who are using graphing calculators and the students who are not using them. Seventy one students enrolled in two different first semester calculus classes at two state universities participated in the study. Participants drew concept maps for the derivative. Analysis of the maps showed that students' in the experimental class showed more representations of the concept of the derivative at a point in their maps than the ones in the traditional class. This implies that the experimental group's concept image of the derivative at a point is richer than that of the traditional group.

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Graphing Calculators Use on Students' Concept Images of the Derivative at a Point

The calculus reform movement emerged in response to a major crisis in the teaching of calculus. Parts of the reasons for this crisis have been students' increasing drop-out and failure rates in calculus courses. Many students who finish calculus courses are unable to use the knowledge they acquired to their benefit in real life (Frid, 1994).

The National Council of Teachers of Mathematics [NCTM] *Standards* (1989) recommended the use of technology in the classroom to save students the trouble of lengthy computations and help them focus more on concepts and problem solving. These *Standards* stressed the use of computers and graphing calculators as tools in the 9-12 mathematics classrooms.

A similar call for the use of technology at college level came out in 1986. During a conference that was held at Tulane University, the participants called for the use of technology as a major part of the new calculus courses. Following this call, The National Science Foundation initiated its "Curriculum Development in Mathematics: A calculus program" in 1988. This program funded several calculus reform projects. To improve students' performance in calculus, the new courses focused on teaching the concepts, not only the techniques. Concepts were introduced algebraically, graphically, and numerically. Some of these courses incorporated writing, others relied on computers and graphing calculators (Culotta, 1992).

Advances in technology such as graphing calculators and computer software packages play a major part in calculus reform. Research on exams that consisted of conceptual and procedural questions showed that students who used technological devices in their calculus

course performed as well as or better than students without access to technology (Tucker & Leitzel, 1995).

This study investigated the impact of using graphing calculators on students' understanding of the derivative at a point. The study looked for differences between the concept images that are held by students' who are using graphing calculators and the students who are not using them. Also the study compared the students' concept images through their concept maps with an expert's concept map. This study will focus on answering the following two questions:

Question 1: Is there a difference between the concept images that are held by the students who are using graphing calculators and the students who are not using the calculators?

Question 2: To what extent does a student’s concept image through his concept map of the derivative at a point match an expert's concept map of the derivative at a point?

Concept Image and Concept Definition

The *concept image* consists of all the mental pictures in the individual’s mind that are associated with a given concept (Tall & Vinner, 1981). The student’s image consists of everything a student associates with the concept: symbols, words, pictures, etc. It takes years of all kinds of experiences to build that image, which changes as the individual matures and encounters new stimuli. Whenever the student processes new information, change may occur on existing concept images. New difficulties, conceptions, and misconceptions may be encountered and may cause ambiguity or conflicts with some parts of the concept image.

The *concept definition* “is a form of words used to specify the concept” (Tall & Vinner, 1981, p. 152). This may be a definition the student has learned, or it may be a personal reconstruction of a definition by the student. The concept definition is then the form

of words the student uses for his own explanations of his (evoked) concept images (Tall & Vinner, 1981).

Several researchers used the theoretical framework of the concept image in their studies (Ellison, 1993; Hart, 1991; 1996; Orton, 1983; Tall, 1989; Vinner, 1983; Vinner, 1992). The results of the research conducted by Orton (1983) showed that although students were able to compute derivatives, their concept image of the derivative was largely limited to a computational representation. Students' concept image of the derivative included only the ability to work with symbolic representation. This showed that students' knowledge of derivative was more procedural than conceptual. Orton suggested that the basic concepts of calculus needed to be revisited and reinforced at various times during the students' mathematics education. In addition, he suggested that students should understand the tangent as the limit of the secants. He recommended the use of graphing calculators or computers to provide graphical representations.

In an experiment with high school students, Zandieh (1998) studied the understanding of the derivative concept of nine students (five males and four females). The results of Zandieh's study showed that none of the students used the same order to learn the different aspects of the derivative. Also, students relied on their concept images, not on the formal definition of the derivative in their understanding of the derivative concept.

Concept Maps

A concept map is a graphic representation of a focal concept and its relation to other key concepts related to it in a subject matter. These concepts are connected together by lines that are labeled to convey meaningful relationships between them (Naidu, 1990).

To develop concept maps, key concepts and the relationships among them are identified and arranged around a focal concept and connected to it with lines that show

meaningful relationships between them. Concept maps are useful for a number of reasons: They give a holistic view of the concept in a way that can not be described in words; the visual symbols are easy to recognize; and they make it easier to scan for words, phrases, and ideas (Plotnick, 1997). Concept maps were chosen for this study to assess conceptual knowledge of the derivative at a point because they allow students to organize their own thoughts in a graphical way that is easy to follow.

In a research study that used concept maps, Williams (1994) compared the conceptual knowledge of function held by reform calculus students and traditional calculus students. Twenty eight students participated in this study, and were equally divided into two classes. The students were asked to construct a concept map of the function concept and complete an example-nonexample questionnaire during a taped interview. In addition to that, eight Ph.D.'s in mathematics were asked to draw concept maps of the function; four of those taught traditional calculus courses, while the others taught reform courses. For the analysis, the researcher compared the experts' maps with the students' maps. The results of the study showed that the concept maps of the two classes matched poorly with the experts' concept maps. It was also noted that the terminology used by reform course students was common in the reform text.

In developing his concept map, the researcher used the theoretical framework that was developed by Zandieh (1998). She described the concept of derivative as three layers of process-object. These were the ratio, the limit, and the function layers. These layers could be described graphically, numerically, and symbolically. She represented these layers as a set of three concentric circles: The smallest circle represented the first layer, the middle circle represented the second layer, and the largest represented the third layer.

In developing his map, the researcher focused on the derivative of a function at a point as two layers: the ratio and the limit. Those layers were described graphically, numerically,

and symbolically. The researcher's concept map was given to four experts to revise and to make any appropriate changes. The revised concept map (see Figure 1) which will be called the expert's concept map was used as the model to compare students' concept maps.

Method

Participants

Seventy one undergraduate students enrolled in two different first semester calculus classes participated in this study. Twenty-four students were enrolled in the experimental class, while 47 were enrolled in the other class, only 17 from the experimental class turned in their concept maps. The two classes were from two different universities in the U.S.A. The students in the experimental class attended classes at a large metropolitan university in the southwest. The students in the other class attended classes at a mid-western university.

The two classes were chosen with the approval of their instructors. Students were enrolled in each class by regular registration procedures. Students in the second university knew from the start that their class did not require graphing calculator. The students in both classes were asked to volunteer to be part of the study and were told about the study and its purposes; confidentiality was granted for all participants; and the researcher obtained releases from the students to use the data collected for academic purposes. For the experimental class, the use of TI- 83 graphing calculator was recommended, but other graphing calculators were permitted.

Treatment

Both classes used the same calculus book written by Stewart (1999). This calculus book begins with a detailed study of functions, limits, derivatives, the derivative formulas, applications, and a study of integration. The textbook was written in a way that it could be

used with or without graphing calculators. The “rule of three” has been emphasized through the textbook. The “rule of three” specifies that each concept should be introduced graphically, numerically, and symbolically, and that each component should be covered equally. Concepts in this book were introduced graphically, numerically, and symbolically. The instruction in the experimental class with the help of the graphing calculator emphasized the connections between symbolic, visual, and numeric representations. For example, the concept of function was introduced numerically by a table of values, visually by a graph, and algebraically by an explicit formula. The focus in the traditional class was on developing students’ skills in dealing with calculus concepts. This class lacked the visual representation of calculus concepts that can be achieved using graphing calculators. They had the chance to see the graphs in the book and the ones drawn by the instructor on the blackboard.

Procedure

Students in both classes attended a brief instruction session on concept maps on which the researcher explained to the students how to form a concept map. During the session, the researcher explained concept maps, gave an example, and emphasized the importance of the words written on the connecting lines. Then he provided each student with a copy of the concept map of function. This concept map was adapted from Williams (1994). Finally he asked each student to work individually to come up with a number of terms related to the derivative at a point, and to draw a concept map for the derivative at a point using the terms that they came up with as well as any others they could think of.

Analysis

A general analysis of 64 students’ concept maps (47 traditional, 17 experimental) will be given. The purpose of this analysis is to describe students’ concept images and to look for

differences between the two classes that would correlate with the use of graphing calculators and to get insights about students’ understanding of the derivative at a point.

The researcher investigated students' ability to connect the different representations of the concepts. In order to make a decision and to assess students' abilities, the researcher compared between the students' concept lists and the connections they used in their maps. The researcher also investigated the connections between the different concepts that the students made in their concept maps and evaluated the validity of these connections. This was determined based on the accuracy of the linking words that the students used in their maps. The researcher studied the student maps to determine, if possible, a student’s most dominant concept image of the derivative at a point: symbolical, graphical, numerical, or perhaps something else.

To investigate the effect of using graphing calculators on students’ concepts, the researcher looked for concepts that appeared in only one of the two groups’ lists. Then students' maps were studied to determine the images they held about the derivative. The researcher checked the accuracy of these images using the linking words that students used in their maps.

To conduct the analysis, the researcher formed three lists; the expert’s list (see Table 1) was formed first. In the same manner, the researcher formed two lists, one for the experimental group, and one for the traditional group (see Table 2). The numbers in the columns in the students’ lists represent the percentage of the students who mentioned the specific concept in their maps.

Results

The students' concept lists indicated that 4% of the students in the traditional group mentioned the symbolic definition of the derivative compared to 0% from the experimental group. This actually is not a big difference between the two groups. It tells us that both groups

had a weak symbolic representation of the concept of the derivative of a function at a point.

The graphical representation as slope of a tangent line was mentioned by 59% of the students in the experimental compared to 43% of the students in the traditional group. The physical representation of derivative as velocity was mentioned by 35% of the experimental group compared to 40% of the traditional group, that is, the two groups' physical representation is almost the same. Also, the students in the experimental group attributed more importance to the derivative as a rate of change compared to the traditional group, with 47% of the experimental group mentioning that compared to 28% from the traditional group. Moreover, the results showed that differentiation rules were mentioned by almost the same number of students in the two groups, which means that the perceived importance of differentiation rules was the same for both groups.

The previous results tells us that the students' concept image of the derivative at a point is richer for the experimental group compared to the traditional group, because students in the experimental group mention more representations than the ones in the traditional group.

Discussion

To answer the first question about the difference between the concept images held by students in the two groups. The results from analyzing students' concept maps showed that students in both groups had a weak symbolic representation of the concept of the derivative of a function at a point. Only 4 % of the students in the traditional group mentioned the symbolic definition of the derivative compared to 0% from the experimental group. This actually is not a big difference between the two groups. The most dominant representation for both groups was the graphical representation as slope of the tangent line, with more students in the experimental group mentioning this representation (59% compared to 43%). This

indicates that because of the emphasis on the visual representation in the experimental group, more students in this group mentioned the graphical representation than the traditional group.

Almost the same percentage of students in both groups mentioned the physical representation of the derivative as velocity (35% experimental compared to 40% traditional). The rate of change was the second dominant image of the derivative for the experimental group (47% compared to 28% from the traditional).

In regards to the second question, the students' concept maps in both groups did not match the expert's map. When compared with the expert’s list none of the students in the two groups mentioned the average rate of change (first layer) in their maps. For the second layer, none of the students in the two groups dealt with the derivative concept numerically. For a sample of students' maps see Figure 2, Figure 3, Figure 4 and Figure 5.

In general, students in both groups have drawn weak concept maps in comparison with the expert’s concept map. None of the students in the two groups mentioned the numerical representation in their maps. Most of the students in the traditional group did not put any links in their maps. Among the ones who put the links, some of them had very weak connections, for example “second derivative used to calculate things that are cool”, and “velocity type of derivative at a point”. There was only one student who used links in his entire map. Almost the same phenomenon occurred among the experimental group students: Some of the students in the group had formed weak links, for example “Derivative has a function”, and “Derivative has a graph”.

Conclusion

The analysis of the students’ concept maps showed that the students in the experimental group mention more representations than the ones in the traditional group. This implies that the experimental group’s concept image of the derivative at a point is richer than

that of the traditional group. This indicates that the students in the experimental group had a richer concept image of the derivative at a point compared to the traditional group students. Students in the experimental group had the opportunity to view the derivative concept using different representations, and the visual image was emphasized through the use of the graphing calculator technology. This learning environment might affect the students' understanding of the derivative concept.

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Table 1

Experts' Concept List from the Experts' Concept Map

Average rate of change
Difference quotient
Average velocity
Table of values
Slope of secant line
Algebraic definition
Instantaneous rate of change
Instantaneous velocity
Slope of tangent line
Table of values near a point
Mathematical definition
Derivative at a point
Local straightness
Linear approximation

TABLE 2

Students' Concept Lists in Percentages

Concept	Experimental	Traditional
Critical Points	12	23
Max./Min.	18	38
Velocity	35	40
Slope	59	43
Rate of Change	47	28
Concavity	6	13
Acceleration	35	2
Limit	29	28
Secant	12	0
Tangent	29	32
Differentiation Rules	47	43
Inflection	18	6
Symbolic Definition	0	4

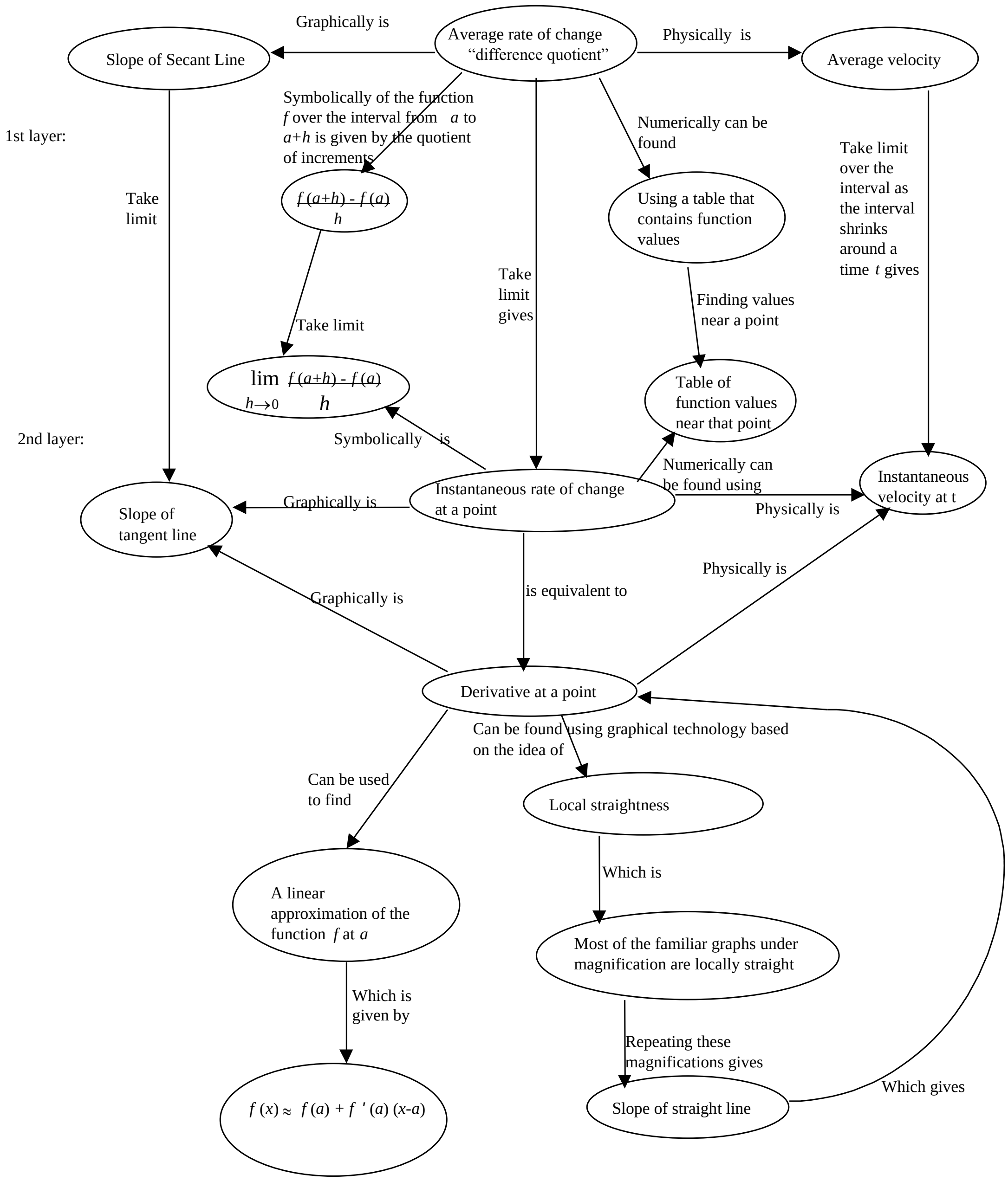

Figure 1. Experts' concept map

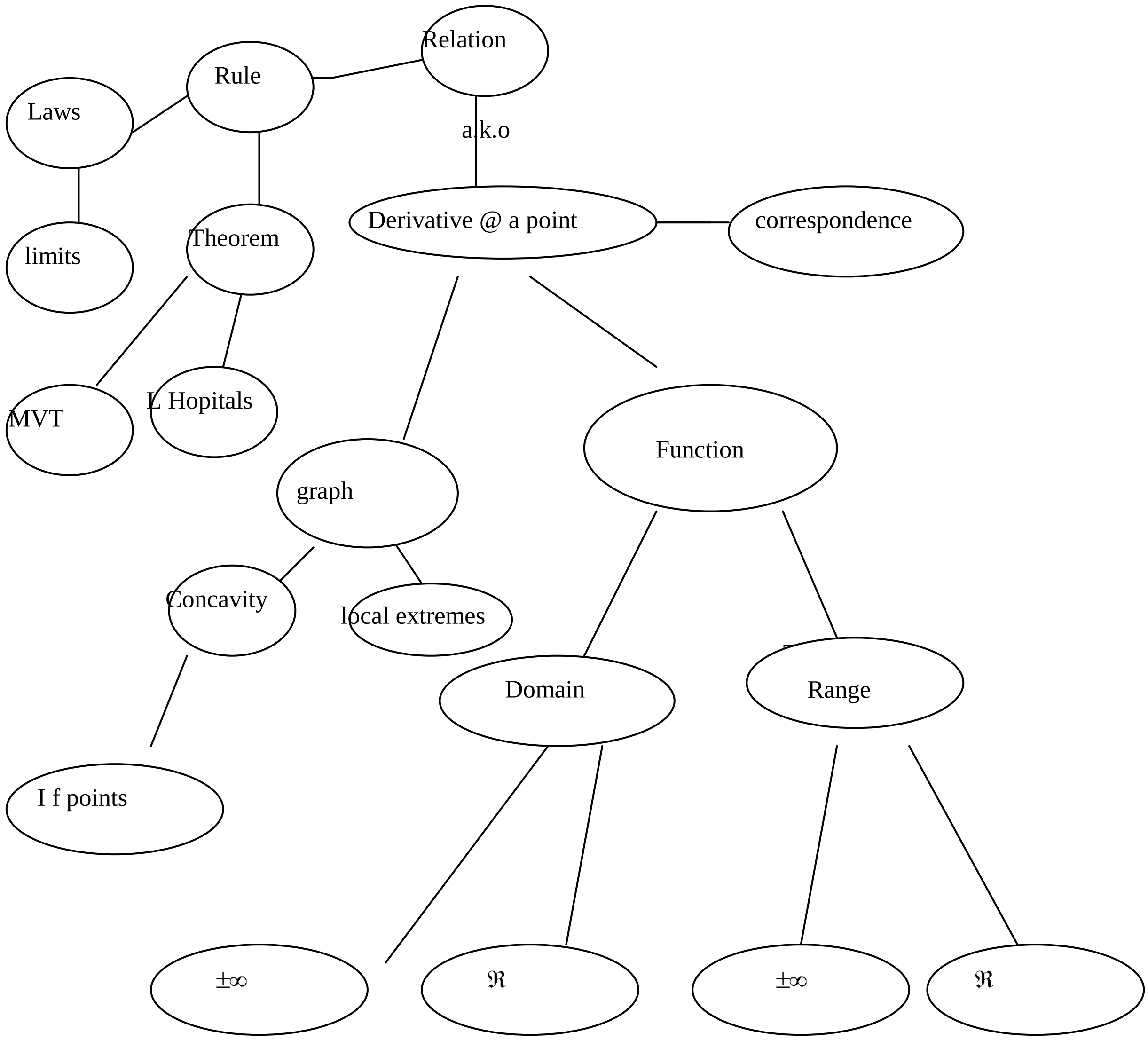

Figure 2. A traditional student concept map first sample

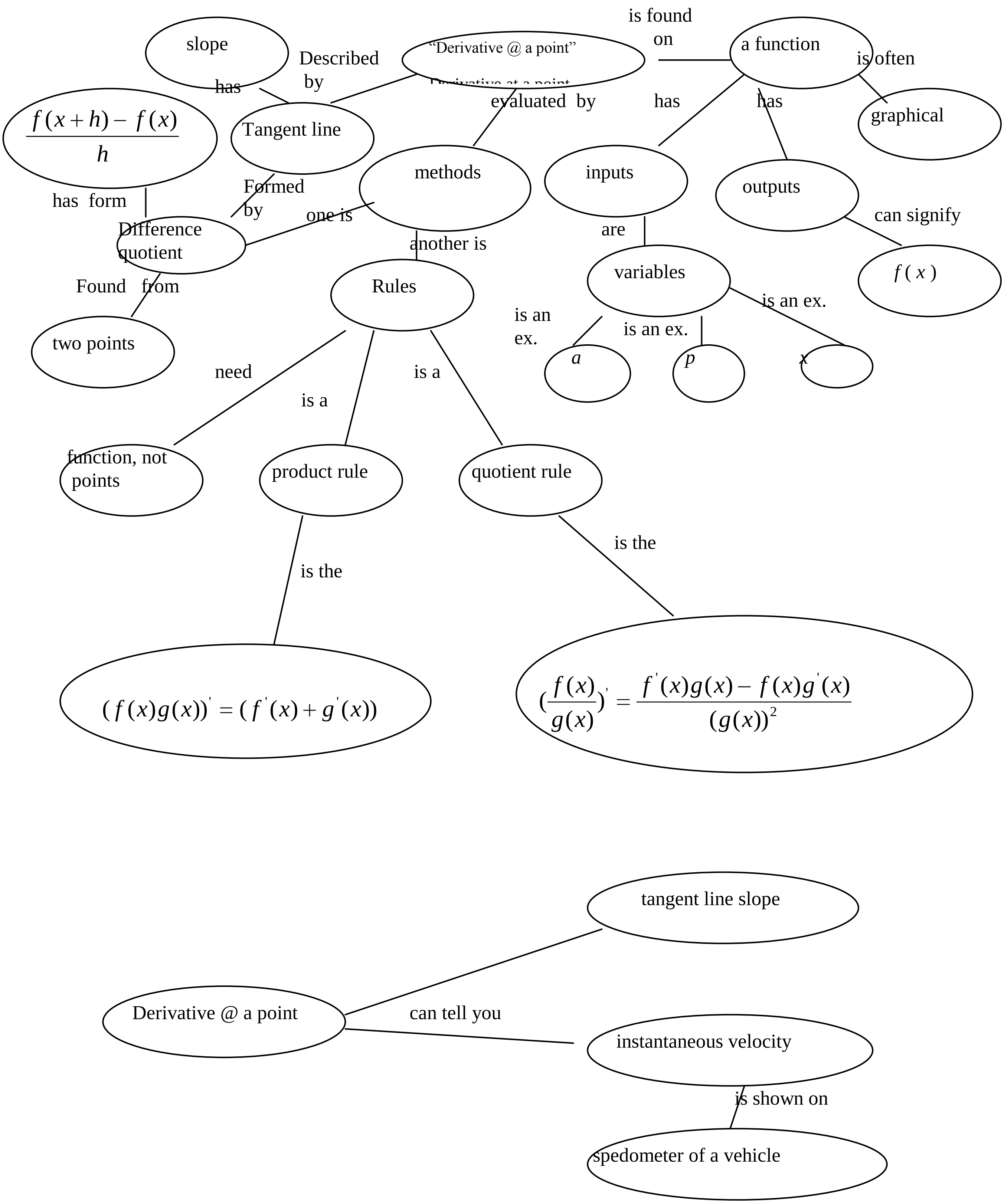

Figure 3. A traditional student concept map second sample

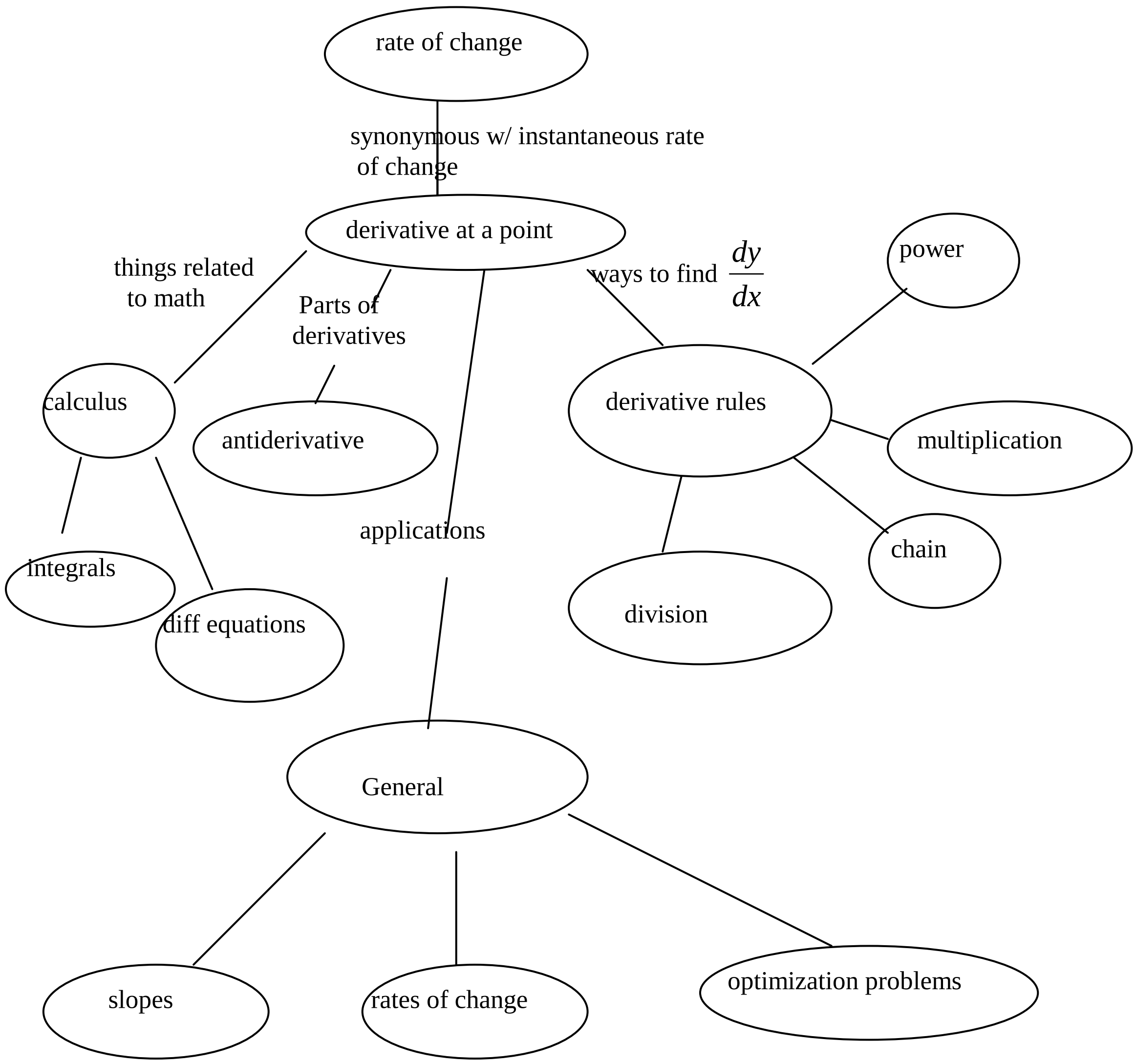

Figure 4. An experimental student concept map first sample

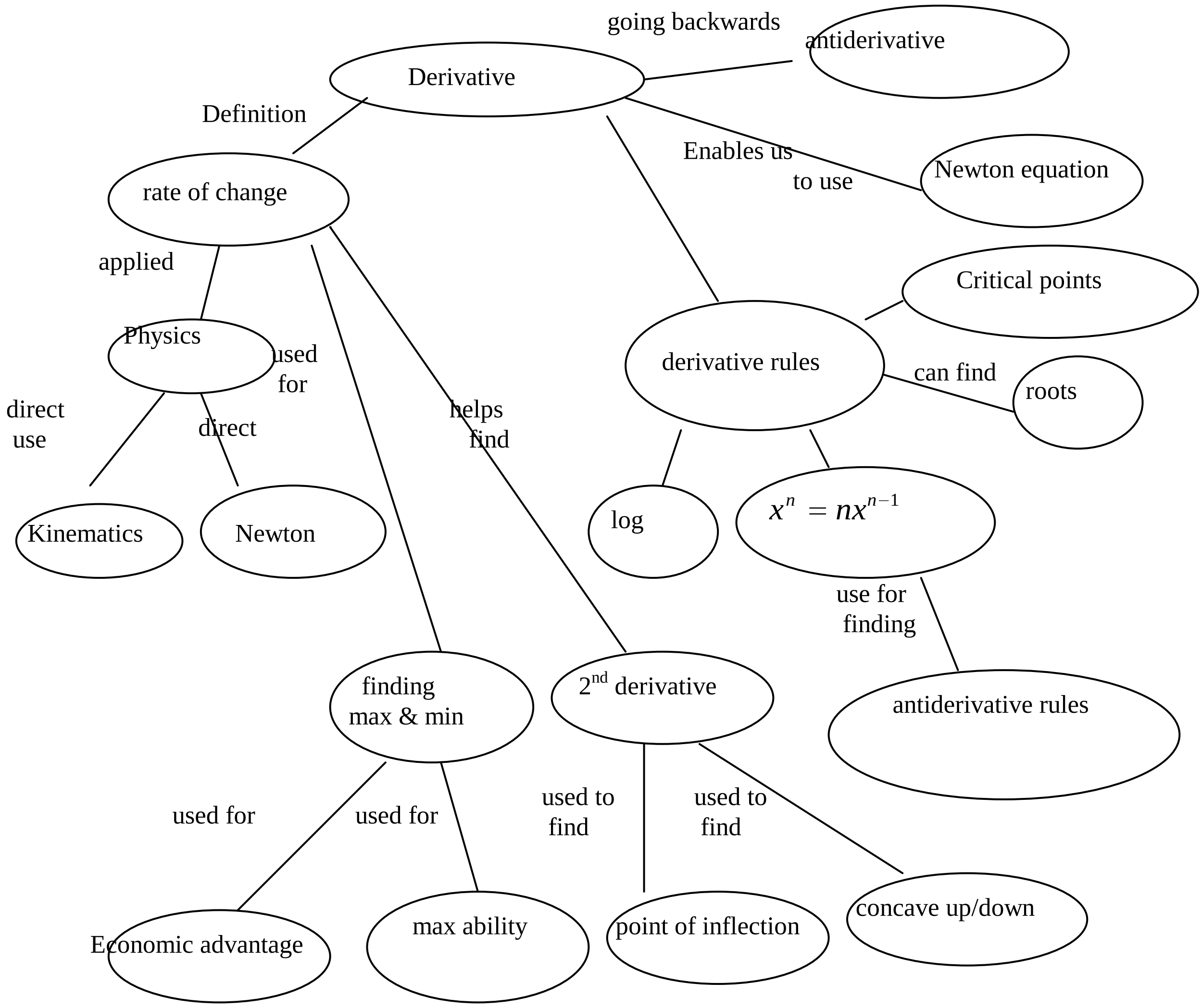

Figure 5. An experimental student concept map second sample