Singapore Pre-service Secondary Mathematics Teachers' Content Knowledge: Findings from an International Comparative Study

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Abstract.
In this article, we explore the mathematical content knowledge of one entire cohort of pre-service teachers (N = 107) through analysing their performance in a Secondary Mathematics Audit that was developed for the International Comparative Studies in Mathematics Teacher Training that was initiated by the University of Plymouth. We study how their mathematical content knowledge evolved during their one-year postgraduate teacher education programme by using a pre and post-course test scheme. An interview was also conducted with five selected pre-service teachers to identify how their responses in the post-course audit were influenced by the pedagogical training. The pre-service teachers' responses to the audits and interviews with selected pre-service teachers show that engagement with pedagogical training in the teacher education course improved the pre-service teachers' mathematical content knowledge. This study also reveals the questions that the pre-service teachers had not performed well or shown much improvement during the one-year programme. These questions were beyond the scope of the school mathematics curriculum.

Keywords: Pre-service teachers, mathematical content knowledge

1. Introduction

The research reported in this article is part of an international comparative study on mathematics teacher education including several countries across Asia and Europe. This research involved collecting Singapore pre-service secondary mathematics teachers' responses to a Secondary Mathematics Audit before and after their one-year teacher education programme in Singapore. The Secondary Mathematics Audit was developed by the Centre of Innovation in Mathematics Teaching (CIMT) of the University of Plymouth. An entire cohort of the Singapore pre-service mathematics teachers (N = 107; initially 108 pre-service teachers participated but one participant decided to withdraw from the study) from 2009 participated in this study. The pre-service teachers' performance in the Audit was analysed by their overall performance and performance in the individual items. As the pre-service teachers went through the teacher education programme, it would be expected that they would have widened their exposure to school mathematics, hence perform better in post-course audit compared to pre-course audit. It would, however, be useful information for teacher educators to identify potential areas of difficulties of school mathematics even after their teacher education programme. This article identifies the types of questions (and the topics) that the pre-service teachers had not performed well in the pre-course audit and did not show significant improvement in from the post-course audit.

2. Background

In Singapore, as in other countries, teachers are expected to have a 'deep understanding' of mathematical content knowledge. There has been a concern among teacher educators that pre-service mathematics teachers might not have acquired an acceptable standard of mathematical content knowledge when they are first deployed in schools (Toh, Chua & Yap, 2007). The main criterion for admission to the Singapore National Institute of Education, the sole teacher training institute in Singapore, for the one-year postgraduate diploma in education specializing in teaching mathematics at the secondary level (hereafter abbreviated as PGDE(Sec)) is a good pass at Pre-university Mathematics and Further Mathematics (Burghes, 2011, p 41). The candidates need not be graduates specialising in mathematics.

Recruited pre-service secondary mathematics teachers need to complete the PGDE(Sec) programme before they are deployed in secondary schools. The programme covers modules on Educational Studies, Curriculum Studies on teaching secondary school mathematics and one other subject, and ten weeks of teaching practice in a Singapore secondary school.

This paper reports how the pre-service teachers' performance in a mathematics audit on mathematical content knowledge changed from their admission to the PGDE(Sec) programme to their graduation. This study was part of the International Comparative Study on Mathematics Teacher Training (ICSMTT) initiated by the Centre of Innovation in Mathematics Teaching (CIMT). Teachers' mathematical content knowledge was one area of study of the ICSMTT to identify the good practices of mathematics teacher education around the world (Burghes, 2011)

Mathematics Teachers' Content Knowledge

Teachers' knowledge required for teaching has been described as complex (Chick, Baker, Pham & Cheng, 2006). There are diverse views on what teachers need to know for teaching (Cooney, 1999; Elbaz, 1983; Graeber, 1999; Kilpatrick, 2001; Schulman, 1986). Schulman identified two main categories of knowledge: content knowledge and pedagogical content knowledge. Content knowledge refers to the "amount and organisation of knowledge in the mind of the teacher" (Schulman, 1998, p.9); pedagogical content knowledge involves knowledge of how teachers represent and formulate their content knowledge when teaching (Schulman, 1998).

According to Eraut (1994), teachers' content knowledge is not a static entity but evolves; teachers go through the following phases of subject knowledge: (1) professional traditions; (2) practical wisdom; and (3) deliberate reflection. Thus the pedagogical content knowledge affects how teachers think about their subject matter knowledge.

The importance of teachers' mathematical content knowledge for effective teaching of mathematics has long been recognized (Ball, 1991; Chapman, 2005; Hutchinson, 1997; Mohr, 2006; Toh, Chua & Yap, 2007; Usiskin, 2001). The literature abounds with reports by researchers on teachers' mathematical content knowledge (for example, An Kulm & Wu, 2004; Menon, 2009).

Researchers have described teachers' mathematical content knowledge as thorough understanding of mathematics which has breadth, depth, connectedness, and thoroughness (Ma, 1999). Knowing school mathematics in depth and breadth is

recognized as an important dimension that proficient mathematics teachers need (Schoenfeld & Kilpatrick, 2008).

Recent studies have distinguished between *Specialised Content Knowledge* and *Common Content Knowledge* as two of the three sub-categories of mathematical content knowledge (Ball, Thames & Phelps, 2008). While specialised content knowledge is mathematical knowledge that is unique to teaching, common content knowledge refers to that knowledge held by an individual who can solve a particular mathematical problem (Ball, Thames & Phelps, 2008, p 399). Thus, this points to the fact that mathematics teachers need to know "a great deal of mathematics" (Usiskin, 2001) compared to other individuals.

Mathematical Content Knowledge and Teaching

There is clear evidence on the relationship between teachers' mathematical content knowledge and their ability to teach well in classrooms (e.g., Ball, Hill & Bass, 2005; Chapman, 2005). Research in the United States has shown that the quality and the rigour of the mathematics curriculum are strongly correlated to the mathematical content knowledge of the teachers (Schmidt, 2002). The existing literature also contains anecdotes of teachers who wanted to do a good job in teaching mathematics but faced many problems which were largely due to their lack of adequate preparation in school mathematics content knowledge (for example, Hutchinson, 1997).

Why exactly is mathematical content knowledge so important in teaching? First, teachers must be able to understand why a particular content is taught and how the content should be developed. Not only that, teachers must be able to use their mathematical knowledge in teaching for identifying a range of solutions and mathematical connections when they are teaching students, planning lessons and evaluating students' work (Ball, Thames, Bass Sleep, Lewis, Phelps, 2009; Ball et al, 2008). Further, teachers must be able to tap on a wide range of knowledge such as procedural knowledge and fluency, concepts and connections (Ball & Bass, 2003).

Why should there be a concern over pre-service teachers' mastery of secondary school mathematical content knowledge since they have already acquired the knowledge when they were students, and they must have some significant exposure to university mathematics when they were undergraduates? There is perhaps the issue of the time lapse since they were studying mathematics at schools. If this is the only reason for concern, then enabling the pre-service teachers to wider exposure to school mathematics during their one-year teacher education programme would suffice. However, a greater concern according to the existing literature is that the knowledge acquired during their school days as students could be limited because it is based mainly on their limited *experience as students* (Jaworski & Gellert, 2003). It was found that pre-service teachers usually enter teacher education programme with narrow conceptions of mathematics as a set of rules and conventions (eg, Ball, 1990a; Taylor, 2002; Wilson and Ball, 2002). Specific areas of weaknesses in pre-service teachers' mathematical content knowledge have been identified by various mathematics education researchers worldwide, which deserve teacher educators' attention (see, for example, Ball, 1990b; Even, 1993). Studies have also shown that generally teachers were unable to see the connection between university mathematics and school mathematics (Ma et al, 2008).

3. Methodology

This study used basic content analysis to analyse the pre-service teachers' responses (N = 107) to the items in the pre and post-course Secondary School Mathematics Audit. The pre-course audit was administered prior to the commencement of the teacher education programme and the post-course audit was administered after they had completed the entire programme just before their posting to schools. The scores of the individual test item in the audit and the pre-service teachers' total scores for the pre-course and post-course audits were then compared and analysed using simple statistical paired t-test of significance. To improve reliability of the test, the pre-service teachers were given venues suitable for university examinations to complete the test during the morning.

3.1 Instrument

The audit used in this study was designed by the CIMT (see Appendix A). The audit consisted of two parts: 15 questions in Part A on basic secondary school mathematical concepts and computation; 16 questions in Part B on more advanced secondary mathematical concepts. Most of the questions were within the scope of the Singapore Secondary Mathematics curriculum, except one question in Part A (Pigeonhole Principle) and two questions in Part B (Arithmetic and Geometric Progressions). The audit was sufficiently long and the items of the audit were selected from a wide range of topics from school mathematics and agreed by the participating countries of the international comparative study as relevant to their mathematics teachers.

3.2 Participants

The participants for this study were formed by the entire cohort of secondary mathematics pre-service teachers from one entire PGDE(Sec) cohort in 2009 (N = 107; 55% male; 45% female for the pre-audit; N = 107). This group of participants includes *all* pre-service teachers within that year (except one who opted out). This cohort was comparable to all the earlier cohorts of PGDE(Sec) and the subsequent cohorts (2007 to 2011) in terms of their academic qualification and ultimate performance in the PGDE(Sec) course.

3.3 Administration of the audit

The audit was administered prior to their admission to the programme. The same audit was administered to them again just after their completion of the programme. In the post-course audit, the researchers checked with the 107 participants whether they had solved the problems before. The pre-service teachers were asked to indicate their impression on the cover page of the audit if they could remember they had seen or solved the problems before. It was verified that they had forgotten the questions in the pre-course audit when they were working on the post-course audit. The researchers further randomly spoke to 20 of the participants to confirm that they had forgotten the problems in the audit.

3.4 Interviews

The audit only comprises questions requiring single answer responses; participants were not required to show their working or solution to the problems. Thus, it was difficult to have a deep understanding of the participants' understanding of the mathematical concepts for the questions. In view of this, five participants were selected for individual

interviews on five separate occasions after the researchers had analysed the participants' responses to the post-course audit. These interviews were conducted to understand how their participation in the one-year education course had impacted their responses to the audit. The use of such qualitative approach was not targeted at verification or generalization, but "discovery, a description that is not necessarily typical, but unique and individual" (Turlejska, 1998, p. 86). The detail (gender, qualification of their first degree, pre-course and post-course audit scores) of the five interviewees is shown in Table 1.

Table 1.
Particulars of the five interviewees.

Name	**Gender**	**Qualification**	**Pre**	**Post**
Andrew	Male	Engineer	32	36
Beatrice	Female	Math	30	28
Charles	Male	Comp Sc	28	35
Dennis	Male	Engineer	34	37
Emily	Female	Math	28	35

The interviews were conducted at five separate occasions one week prior to their actual posting to schools after their graduation from the PGDE(Sec) programme. Due to time and manpower constraint, only five interviews could be conducted. Thus, these five interviewees were selected based on their responses to the questions with the best performance and the weakest performance.

3.5 Data Collection

The participants' responses to the pre and post-audits were checked against the correct answer and given a score of one if the answer was correct and zero otherwise. No mark was deducted for incorrect response; partial marks were not awarded for intermediate steps or partially correct response. The participants' scripts were scanned for record and analysis purposes.

Field notes were taken during the interview with the five interviewees. Specific questions targeted at their individual responses and general questions on how they had managed to keep updated with the school mathematical content knowledge or otherwise were asked during the interview.

3.6 Errors for their Learning

We believe that the pre-service teachers' errors can be used positively by the pre-service teachers to re-organise their own mathematical understanding. A copy of the feedback containing the descriptors of the general misconceptions, was given to the individual pre-service teachers after they had completed the PGDE(Sec) programme.

3.7 Limitations

A limitation of the use of the mathematics audit is that all the questions require single-answer responses. Scanned copies of candidates' solution were checked against their answers. The researchers adjusted the scores of the candidates who had apparently used incorrect mathematical steps to obtain the correct answer. Students who had given the

correct answers (even without showing any working) were awarded the marks, unless the working clearly demonstrated incorrect reasoning.

The interviews conducted in this study provided useful insights and allowed the researchers to have an in-depth understanding of the pre-service teachers' content knowledge. Due to the constraint of time factor, only five candidates were selected for the interview.

4. Results

The pre-service teachers generally showed an improvement in the pre-service teachers' performance from the pre-audit to the post-audit, in terms of the total score and the scores of the individual items for most items. In the following section, the performance of the items demonstrated will be given in the pre and post-test scores and the *p*-values obtained by doing a paired-sample t-test.

There were seven items which showed a significant improvement (5% level of significance) in the pre-service teachers' performance. The overall performance (calculated from the total score of the audit) showed a significant improvement among the Singapore pre-service teachers in the overall performance in the audit in the one year programme.

Part A Question 1: *Simplify as far as possible* $\frac{\sqrt{147}}{\sqrt{3}}$. (Pre–post = -0.13084, SD = 0.4362, $p = 0.02$)

Pre-service teachers have the misconception of the square root symbol (Toh, 2007). All the incorrect responses were due to identifying this positive square root symbol as denoting both the positive and negative roots (Figure 1).

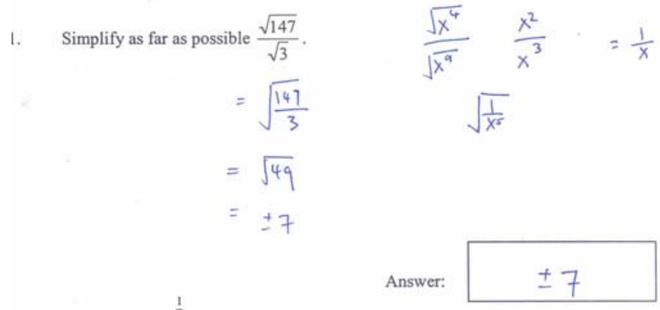


Figure 1. *Sample of pre-service teachers' errors in Part A Question 1.*

Andrew, Charles, Dennis and Emily made this mistake in the pre-audit but provided correct responses in the post-audit; Beatrice gave incorrect response for both the pre and post-course audits. An extract of their interview with the researcher's commentary is included in Table 2.

Table 2.
Interview with the participants on the mistake of Question 1.

	Speech Segment	Commentary
Andrew:	*This was pointed out by my PGDE tutor … it is important point for teaching.*	These four participants had acquired the knowledge of the meaning of the symbol during their PGDE(Sec). Emily had resolved the conflict of the square root notation through familiarising herself with the school curriculum requirement during PGDE(Sec).
Charles:	*When I studied for my SMMT*[1] *…*	
Dennis:	*….since my sec[ondary school] days, I thought this was correct. The green book points out that…*	
Emily:	*I remember some university books used the radical symbol to refer to all the roots ah even the complex roots…… O-Level syllabus notation was different …*	
Beatrice:	*I thought this [radical] symbol refers to both positive and negative root since secondary school. …but I think there are some square root questions did not really try them.*	Beatrice had relied on her prior secondary math knowledge in answering this question.

Part A Question 8: *Calculate* $\left(4.2\times10^{-3}\right)\div\left(0.7\times10^{2}\right)$, *giving your answer as a decimal.* (Pre – Post = -0.11215, SD = 0.48227, $p = 0.018$)

Little information could be elicited from the participants' response to this question, since the errors appear more evident to be careless computation rather than conceptual error.

Part A Question 12b: *Mark each of the following statements using the letters A, S and N where*

A: always true *S: sometimes true* *N: never true*

A square is a rectangle. (Pre – Post = -0.16822, SD=0.46553, $p < 0.001$)

The pre-service teachers Beatrice, Charles and Emily provided the correct responses for both pre and post-audits. Andrew and Dennis gave incorrect responses during the pre-audit but correct responses for the post-audit.

[1] School Mathematics Mastery Test. It is a test compulsory for all PGDE(Sec) mathematics pre-service teachers at the end of the first Semester of the course. Toh, Chua and Yap (2007) gave the detail of the conceptualization and implementation of the test.

Table 3.
Part of interview segment on Question 12b.

	Speech Segment	Commentary
Andrew:	*All the while, I thought a square is a square and a rectangle [is] a rectangle…they are not the same. In PGDE class my tutor got me to see the relationship using GSP* [2] *and property. So manage[d] to get it correct.*	Both pre-service teachers attributed their awareness of the relationship between these geometrical objects during their pre-service course.
Dennis:	*Learn this in teaching geometry my tutor show[ed] this by Venn Diagram. You have a circle for rectangle, a circle for square………*	

Apparently, the PGDE(Sec) programme provided the instructional space to lead the pre-service teachers to better understand the relation between various geometrical shapes and the intricate thinking processes behind it. –From the interview segment above, it is apparent that the use of Van Hiele's Theory in teacher education in the PGDE(Sec) programme (Lee & Lee, 2009, p120 – 123) has contributed to the pre-service teachers' deep thinking into the relation between geometrical shapes.

Part A Question 13a. *Are these statements true or false? Write T or F in the box.*
If the result of squaring a number is 49, the original number must be 7. (Pre – Post = -0.10280, SD=0.43295, p = 0.016)

Andrew, Beatrice, Dennis and Emily provided correct responses for both pre and post-course audits; only Charles gave incorrect responses for pre-course audit but gave correct response for the post-course audit.

Table 4.
Part of interview segment on Question 13a.

	Speech Segment	Commentary
Charles:	*Not that I do not know negative numbers. I have missed out, you see taking square root of 49 is 7, you forget about negative seven. In the [PGDE] class we talked about teaching negative numbers and the product of negative numbers. So become more careful in solving this question.*	The pedagogical training in PGDE(Sec) programme had made Charles more careful in the procedure of finding square roots of a positive number, although he had already known the concept of negative numbers.

The PGDE(Sec) programme heightens the pre-service teachers' awareness of the importance and pedagogical implications of teaching negative numbers to lower secondary school students (Lee & Lee, 2009, pp. 3 – 16). This awareness had most likely raised the pre-service teachers' scope of thinking in solving a mathematical problem related to numbers and hence rendered them more careful in mathematical procedures.

[2] A dynamic geometry software that is used in PGDE(Sec) training.

Part A Question 13b. *Are these statements true or false? Write T or F in the box.*
All prime numbers are odd numbers. (Pre – Post = -0.07477, SD = 0.28118, p = 0.045)

All the five interviewees provided incorrect responses in the pre-audit; Andrew and Charles gave the correct response while Beatrice, Dennis and Emily still gave incorrect responses in the post-audit.

The pre-service teachers had not thought through much about the properties of prime numbers (in this case, the parity of prime numbers), which is not emphasized in the secondary school mathematics curriculum. Through the PGDE(Sec) programme such knowledge is emphasized to the pre-service teachers.

Table 5.

Part of interview segment on Question 13b.

	Speech Segment	**Commentary**
Andrew:	*Most of the time we think of prime numbers we think of numbers like 3, 5, 7 you know. Miss out the number 2. I remember in my discussion in class ah! Then I recall 2 is the only prime that is even.*	Andrew and Charles were sensitized to that 2 is a prime number during the PGDE(Sec) classes by giving them "problem solving" activities involving prime numbers.
Charles:	*I know….but just miss out [overlooked]. My tutor gave us a few problem solving type questions…use the fact 2 is the only even prime…..now we get used to this type of question.*	
Beatrice:	*Why? Prime numbers are odd.*	Although Beatrice was a mathematics graduate, her knowledge of prime numbers was acquired through PGDE(Sec) programme instead of the undergraduate mathematics course.
Researcher:	What do you know about prime numbers?	
Beatrice:	*Divisible by 1 and itself. Cannot be 1 by Fundamental Theorem of Arithmetic*	
Researcher:	Where did you first hear about Fundamental Theorem of Arithmetic?	
Beatrices:	*In the green book [course book for PGDE]. But dunno what exactly it is about. It is because of Fundamental Theorem of Arithmetic that 1 is not a prime number.*	

The interview segments for Dennis and Emily are not presented here. Summarily, they were also not sensitized to the fact that 2 is the only even prime in both audits.

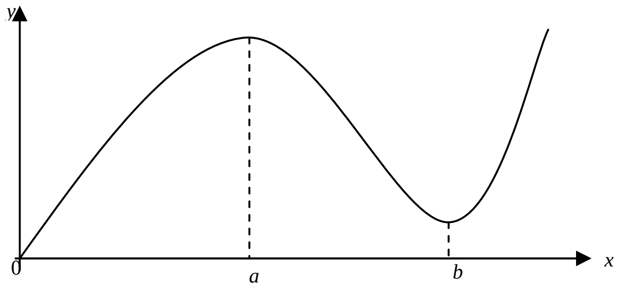
Part B Question 11. *Simplify* $\log_3\left(3^4\right)$. (Pre – Post = -0.05607, SD=0.23115, p = 0.014)

The mistakes made during the pre-course audit were mainly the lack of attention to the use of parenthesis:

$$\log_3\left(3^4\right) = \left(\log_3 3\right)^4 = 1.*$$

It was apparent that the pre-service teachers became more conscious of the appropriate use of parenthesis in mathematical statements in the post-audit.

Part B Question 14c. *The diagram shows the graph $y = f(x)$.*



Is each statement below true or false? *At $x = b$,* $\frac{d^2y}{dx^2} > 0$ (Pre – Post =-0.14953, SD=0.45149, p = 0.001)

Both students and teachers might not have understood graphical interpretation of the second derivative of a graph in terms of its concavity (Toh, 2009a; 2009b). The significant improvement in the pre-service teachers' performance in this question suggests that they had managed to acquire this calculus concept on concavity during the PGDE(Sec) prorgramme. Out of the five interviewees, only Emily did not provide the correct response for both pre and post-audits. Charles gave the correct response for both; Andrew, Beatrice and Dennis gave correct responses for post-audit only.

Table 6.
Part of interview segment on Question 14c.

	Speech Segment	**Commentary**
Andrew:	*d^2 y/dx^2 is the rate of change of the gradient. Postive means [the slope] getting steeper and steeper so a smiling curve.*	In PGDE(Sec), the pre-service teachers were aided to recall the secondary school concepts they had forgotten. Even though the focus was on pedagogy, school mathematics concepts were revised.
Researcher:	How do you know this?	
Andrew:	*Actually in A-Levels we learnt it already. Just forgotten about it.*	
Researcher:	What about university math?	
Andrew:	*Didn't learn this thing. Until in the PGDE course the tutor show[ed] use again how to teach this ah this revise[d] my conceps.*	

Overall performance

There is a significant improvement (Pre = 32.69; Post = 33.50, Pre – Post = -0.80514, SD = 2.85313, p = 0.004) in the pre-service teachers' performance in the overall audit after

passing through the one-year teacher training. They have apparently improved in their school mathematics content knowledge passing through the PGDE(Sec) programme. The pre-service mathematics teachers have acquired conceptual understanding of some secondary school mathematics content that they might have slipped in their earlier years as students.

The following section presents the questions which show either a significant decrease in the mean score from the pre-audit to the post-audit, or overall low mean score.

Part B Question 13. *If* $\frac{dy}{dx} = 4y$, *what is y as a function of x? Choose from the statements below.*

A: $y = 4ke^{x}$ *B:* $y = ke^{x}$

C: $y = ke^{4x}$ *D:* $y = 4e^{x}$

(Pre = 0.841; Post = 0.748)

The number of correct responses from the pre-audit was higher than that in the post-audit. Emily provided the correct response in the pre-audit (no working was shown) but gave an incorrect response in the post-audit (Figure 2).

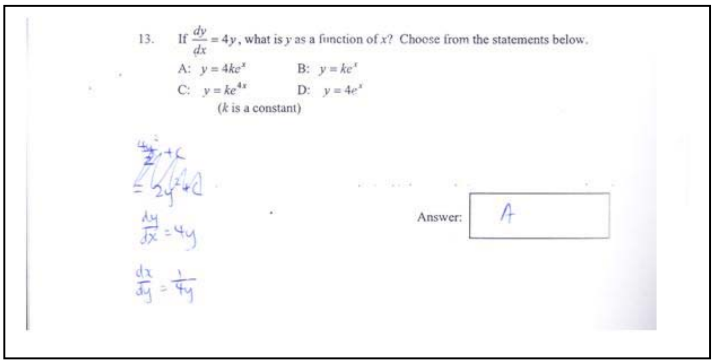


Figure 2. *Emily's response in the post-audit for Q13.*

This was further explored in the interview segment. In the pre-course audit, she had forgotten how to solve a differential equation; hence she used trial-and-error to check each of the choices. On the other hand, she tried solving the differential equation in post-audit as she had recalled that the equation given in Q13 is a first order differential equation. However, she was unable to solve the equation correctly as she had forgotten the procedure (see Figure 2).

When asked why she had not thought of verifying each of the choices as in the pre-audit, she mentioned that solving the equation was the "correct method" to solve this equation.

Apparently, she had become more conscious of using the "correct method" of solving a mathematical problem, instead of a more intuitive approach reflected in her pre-course audit (not shown here).

Part A Question11. *There is a large number of 5 different kinds of sweets in a bag. What is the least number you must take from the bag (with your eyes closed) to make sure that you get at least 3 of the same kind?* (Pre = 0.25; Post = 0.28)

The content of this question is based on basic counting principle (Pigeonhole Principle). All the five interviewees did not give the correct response for both pre and post-audits. The interview segments with Andrew and Emily are presented in Tables 7 and 8.

Table 7.
Part of interview segment with Andrew on Part A Question 11.

	Speech Segment	**Commentary**
Andrew:	*I dunno what the question is talking about.*	Andrew was not familiar with such discrete mathematics arguments. He had not been exposed to discrete mathematics during his engineering course back in university.
Researcher:	How then would you solve this problem?	
Andrew:	*Infinity ah can go on forever.*	
Researcher:	How many cases are there?	
Andrew:	*No… something like permutation and combination A-Levels but not really. Never see this in my university [engineering] math.*	

Table 8.
Part of interview segment with Emily on Part A Question 11.

	Speech Segment	**Commentary**
Emily:	*Doesn't make sense. It can go to infinity. Depends on whether you put back or not.*	Emily had exposure to discrete mathematics in university mathematics. She was not able to link this to her discrete mathematics knowledge.
Emily:	*Many, no limit…… Must consider all the cases…this will take a long time*	
Researcher:	…Have you learnt this before in university?	
Emily:	*No….I can't remember.*	
Researcher:	Have you learnt counting? Eh Pigeonhole principle?	
Emily:	*Oh yes…But how is this related to pigeonhole principle?*	

Andrew had not been exposed to the type of discrete mathematics argument of Q11. Emily, a mathematics graduate, had been exposed to such mathematical argument but was unable to apply her university mathematical knowledge to solve this problem.

Part A Question 12(a) *Mark each of the following statements using the letters A, S and N where*

A: always true *S: sometimes true* *N: never true*

Quadrilaterals tessellate. (Pre = 0.23; Post = 0.24)

The pre-service teachers were generally not proficient in tessellation. Charles provided correct responses for both the pre and post-course audits; the other four interviewees gave

incorrect responses for both audits. The abridged interview segment with Charles is in Table 9.

Table 9.
Part of interview segment with Charles on Part A Question 12(a).

	Speech Segment	**Commentary**
Charles:	*Actually I only learnt it [tessellation] through teaching my tuition kid [in primary level].*	Charles' knowledge of tessellation was acquired through his personal engagement of private tuition. This topic was not emphasized in PGDE(Sec) programme.
Charles:	*Never learnt it before…it is new [to me]…We didn't discuss tessellation in PGDE. I don't think it is in the syllabus. [Most likely] [w]e didn't discuss.*	

Tessellation was not taught in PGDE(Sec) programme since it is an isolated topic in the Singapore primary school mathematics curriculum only. The pre-service teachers might not be familiar with concepts involving tessellation. However, Charles was familiar with tessellation as he had been involved in giving private lessons to primary school students.

5. Discussion

Based on the data of the pre-service teachers' performance in the pre-course audit (part of the data of ICSMTT), Singapore teachers entered the one-year training programme with rather proficient mathematical content knowledge[3]. Singapore pre-service teachers are relatively competent in their mathematical content knowledge despite the fact that many pre-service teachers are recruited from disciplines other than mathematics.

According to Eraut (1994), teachers' content knowledge is a dynamic entity that evolves; teachers go through the following phases of subject knowledge: (1) professional traditions; (2) practical wisdom; and (3) deliberate reflection. Menon (2009) reported these categories as *traditional, pedagogical* and *reflective* phases. At the initial phase, their content knowledge is largely shaped by their own learning of the subject. It eventually evolves into the stage when teachers reflect on the actual content knowledge and how they use the content knowledge for their teaching. Thus it is understandable to observe in this study that their performance in mathematical content knowledge items have generally improved over the one-year PGDE(Sec) programme, signalling that they have picked up some areas of content knowledge which they have not previously picked up or have forgotten..

Although the PGDE(Sec) programme for mathematics teachers does not include an explicit mathematics content course, the pre-service teachers acquires from the programme many secondary school mathematical concepts. These mathematical concepts and procedures are infused into the mathematics pedagogy course.

This study further shows that the pre-service teachers did not perform well in questions testing mathematical content knowledge of concepts not directly taught in the secondary school mathematics in both pre and post-course audit. This includes

[3] From the data obtained from Burghes (2011), Singapore pre-service teachers' mathematical content knowledge was ranked fourth in this International Comparative Study – slightly behind Japan, Russia and China.

mathematical items from both primary and pre-university mathematics content. They did not generally show significant improvement in such questions even after going through the programme. As shown in the selected interviews in this study, even mathematics graduates might not relate to their university mathematical knowledge when solving problems on tessellation, arithmetic and geometric progression and a question on basic counting principle. They did not make any connection to their undergraduate mathematical knowledge.

6. Conclusion

There have been studies on elementary school teachers' mathematical content knowledge (eg., Livy & Vale, 2011; Ma, Millman & Wells, 2008; Norton, 2010; Ryan & McCrae, 2005/2006). This paper reports an exploratory study on secondary school teachers' mathematical content knowledge. The findings of this paper, especially on the items that the pre-service teachers have not performed well in both the pre and post-course audit, identifies the area of school mathematics that the pre-service teachers have not picked up even after the one-year teacher education programme. This adds to the knowledge of research on secondary mathematics teachers' content knowledge. Hopefully this can spur further interest in this area of research.

From a professional development perspective, knowledge of pre-service teachers' errors and misconceptions can be used to inform their own personal professional development or for the teacher training institute to implement appropriate intervention during the teacher preparation programme.

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Appendix A

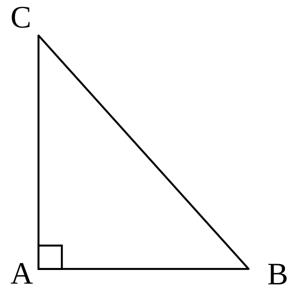
PART A

1. Simplify as far as possible $\frac{\sqrt{147}}{\sqrt{3}}$.

2. Calculate $(125)^{\frac{1}{3}}$.

3. Let a = 2, b = -1. Calculate the value of H when $\frac{1}{H} = \frac{1}{a} + \frac{1}{b}$.

4. Triangle ABC is a right angled triangle. BC = 12cm and AC = $6\sqrt{3}$ cm. What is the size of angle ABC?



5. A ball is dropped from a height of 12 metres. It bounces on the ground and reaches $\frac{3}{4}$ of its original height. It continues to bounce in this way, each time rising to $\frac{3}{4}$ of the previous height. What height does the ball reach after three bounces? Give your answer as a fraction.

6. Factorise $x^2 - 7x + 12$.
7. Wenhui, Siti and Puspa have a sum of $575 to be shared among them. They agree to divide it so that Wenhui gets $19 more than Siti, and Siti gets $17 more than Puspa. How much does Wenhui get?
8. Calculate $\left(4.2 \times 10^{-3}\right) \div \left(0.7 \times 10^{2}\right)$, giving your answer as a decimal
9. A bag contains 5 red counters, 4 blue counters and 3 white counters. Counters are taken out in succession and are **not** replaced. What is the probability of obtaining two red counters for your first two choices?
10. The length of each side of a cube is multiplied by 3. By what amount is the surface area of the cube multiplied?
11. There is a large number of 5 different kinds of sweets in a bag. What is the least number you must take from the bag (with your eyes closed) to make sure that you get at least 3 of the same kind?
12. Mark each of the following statements using the letters A, S and N where

    A: always true S: sometimes true N: never true

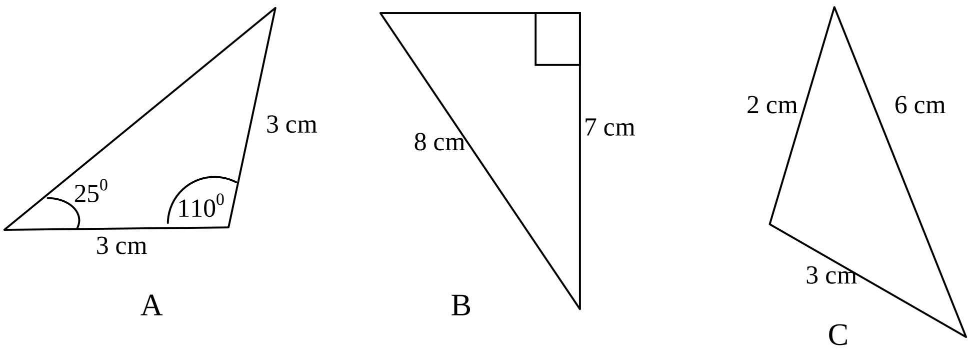
(a) Quadrilaterals tessellate.
(b) A square is a rectangle.
(c) A trapezium has at least one line of symmetry.

13. Are these statements true or false? Write T or F in the box.
(a) If the result of squaring a number is 49, the original number must be 7.
(b) All prime numbers are odd numbers.
(c) The lengths of the sides of a triangle are *a*, *b* and *c*.
(d) If $a^2 + b^2 = c^2$, then the triangle contains a right angle

14. The price of a television set was increased by 20%. In a sale, its new price was reduced by 20%. How does this price compare with the original price?

A: the same B: less C: more

15. Which of these triangles can actually be constructed?



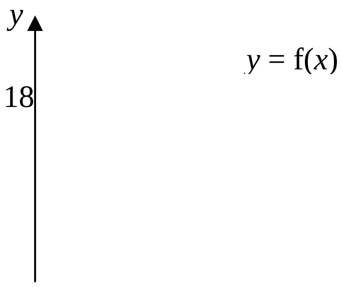
PART B

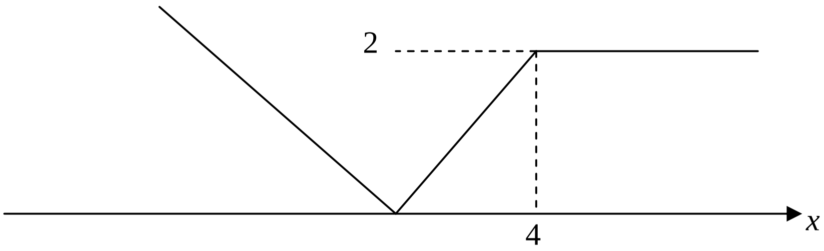
1. If $x^2 + 6x - 3 = (x+a)^2 + b$, calculate the values of *a* and *b*.
2. Determine the number of real solutions of this quadratic equation.

$$2x^2 - 6x + 9 = 0$$

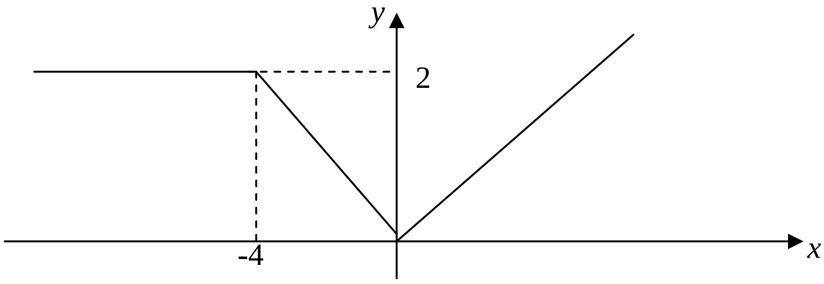
3. If $\dfrac{6}{2-\sqrt{3}} = p(2+\sqrt{3})$, determine the value of *p*.
4. What is the range of the function f(x) = x^4 +1? Choose from A, B, C, D or E.
A: f(x) > 1
B: f(x) ≥ 0
C: f(x) > 0
D: f(x) ≥ 1
E: f(x) > 2.

5. The graph of y = f(x) is shown below.





The graph is translated to give the graph below.



Which of these expressions is the equation of the new graph?

A: f(x) – 1
B: f(-x)
C: f(x) + 2
D: f(x – 2)
E: - f(x)
F: f(x + 2) – 1.

6. The equations of two lines are given below.

$y + 3x - 6 = 0$ and $y - 7x + 5 = 0$.

Which of the statements below is true?

A: The two lines are parallel.
B: The two lines are perpendicular.
C: The two lines both have *positive* gradients, but are *not* parallel.
D: The two lines both have *negative* gradients, but are *not* parallel.
E: *None* of the above is true.

7. An infinite geometric series begins

$$5 + 2.5 + 1.25 + 0.625 + \ldots\ldots..$$

Is the sum of this series finite? Write Yes or No
If Yes, what is the sum of the series?

8. An arithmetic series has 20 terms. The first term is 2 and the last term is 44. Calculate the sum of the series.

9. How many solutions does the equation below have in the interval $0 \le \theta \le 360^0$?

$$8 = 2 + 5 \sin 3\theta$$

10. Which of the statements below is true for all values of x?

A: $e^{-x} < 0$ B: $e^{-x} > 1$ C: $e^{-x} < e^{x}$
D: $e^{-x} < -1$ E: $e^{-x} > 0$

11. Simplify $\log_3(3^4)$.

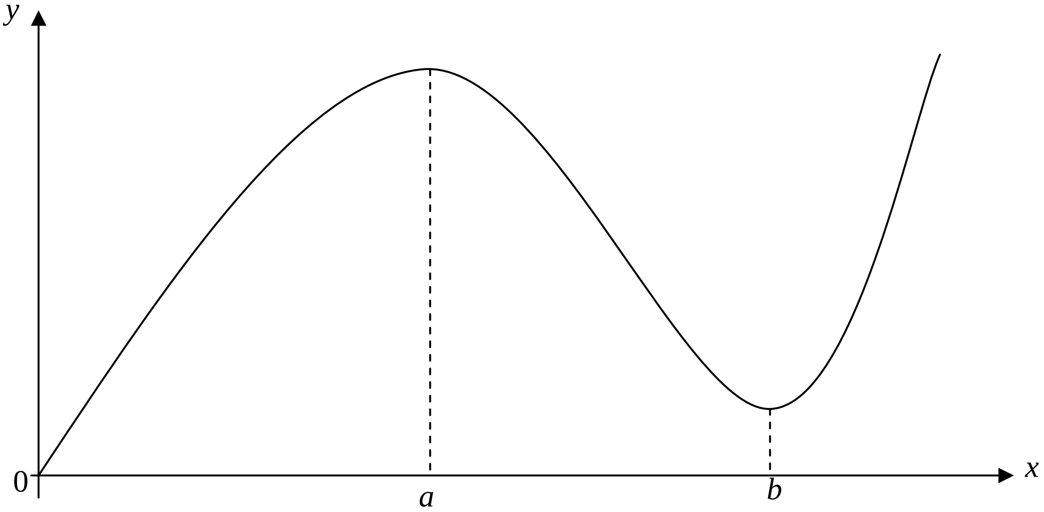
12. Evaluate 3^0.

13. If $\frac{dy}{dx} = 4y$, what is y as a function of x? Choose from the statements below.

A: $y = 4ke^x$ B: $y = ke^x$

C: $y = ke^{4x}$ D: $y = 4e^x$ (k is a constant)

14. The diagram shows the graph $y = f(x)$.



Is each statement below *true* or *false*?

A: At $x = a$, $\frac{dy}{dx} = 0$

B: At $x = a$, $\frac{d^2y}{dx^2} > 0$

C: At $x = b$, $\frac{d^2y}{dx^2} > 0$

15. Differentiate $\ln(2x)$ with respect to x.

16. What is $\int_a^b e^{8x}dx$?

A: $8(e^a - e^b)$ B: $8(e^{8b} - e^{8a})$ C: $\frac{e^{8a} - e^{8b}}{8}$

D: $\frac{e^b - e^a}{8}$ E: $\frac{e^{8b} - e^{8a}}{8}$

Appendix B – Statistical Results of the Mathematics Audit

Paired Samples Test for Part A of the Audit

		Paired Differences					t	df	*p*-value
		Mean	Std. Dev	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Q1pre - Q1post	-.13084	.43620	.04217	-.21444	-.04724	-3.103	106	.002
Pair 3	Q3pre - Q3post	.00935	.37606	.03636	-.06273	.08142	.257	106	.798
Pair 4	Q4pre - Q4post	-.01869	.49490	.04784	-.11355	.07616	-.391	106	.697
Pair 5	Q5pre - Q5post	.02804	.50391	.04871	-.06854	.12462	.576	106	.566
Pair 6	Q6pre - Q6post	.00935	.21698	.02098	-.03224	.05093	.446	106	.657
Pair 7	Q7pre - Q7post	.07477	.40518	.03917	-.00289	.15242	1.909	106	.059
Pair 8	Q8pre - Q8post	-.11215	.48227	.04662	-.20458	-.01971	-2.405	106	.018
Pair 9	Q9pre - Q9post	-.03738	.49383	.04774	-.13203	.05727	-.783	106	.435
Pair 10	Q10pre - Q10post	.01869	.45519	.04400	-.06855	.10593	.425	106	.672
Pair 11	Q11pre - Q11post	-.04673	.37324	.03608	-.11827	.02481	-1.295	106	.198
Pair 12	Q12apre - Q12apost	-.00935	.50461	.04878	-.10606	.08737	-.192	106	.848
Pair 13	Q12bpre - Q12bpost	-.16822	.46553	.04500	-.25745	-.07900	-3.738	106	.000
Pair 14	Q12cpre - Q12cpost	-.09346	.55851	.05399	-.20051	.01359	-1.731	106	.086
Pair 15	Q13apre - Q13apost	-.10280	.43295	.04185	-.18578	-.01982	-2.456	106	.016
Pair 16	Q13bpre - Q13bpost	-.07477	.38118	.03685	-.14783	-.00171	-2.029	106	.045
Pair 17	Q13cpre - Q13cpost	-.04673	.34704	.03355	-.11324	.01979	-1.393	106	.167
Pair 18	Q13dpre - Q13dpost	.00000	.30715	.02969	-.05887	.05887	.000	106	1.000
Pair 19	Q14pre - Q14post	.02804	.21535	.02082	-.01324	.06931	1.347	106	.181
Pair 20	Q15pre - Q15post	-.10280	.54832	.05301	-.20790	.00229	-1.939	106	.055

Paired Samples Test for Part B of the Audit

		Paired Differences					t	df	*p*-value
		Mean	Std. Dev	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Q1apre - Q1apost	-.01869	.13607	.01315	-.04477	.00739	-1.421	106	.158
Pair 2	Q1bpre - Q1bpost	-.01869	.33594	.03248	-.08308	.04570	-.576	106	.566
Pair 3	Q2pre - Q2post	-.02804	.34907	.03375	-.09494	.03887	-.831	106	.408
Pair 4	Q3pre - Q3post	.00000	.30715	.02969	-.05887	.05887	.000	106	1.000
Pair 5	Q4pre - Q4post	.00000	.38851	.03756	-.07446	.07446	.000	106	1.000
Pair 6	Q5pre - Q5post	-.04673	.39771	.03845	-.12296	.02950	-1.215	106	.227
Pair 7	Q6pre - Q6post	.00935	.21698	.02098	-.03224	.05093	.446	106	.657
Pair 8	Q7apre - Q7apost	-.01869	.53166	.05140	-.12059	.08321	-.364	106	.717
Pair 9	Q7bpre - Q7bpost	.06542	.51891	.05016	-.03404	.16488	1.304	106	.195
Pair 10	Q8pre - Q8post	.03738	.51258	.04955	-.06086	.13563	.754	106	.452
Pair 11	Q9pre - Q9post	-.05607	.45208	.04370	-.14272	.03057	-1.283	106	.202
Pair 12	Q10pre - Q10post	-.04673	.44262	.04279	-.13156	.03811	-1.092	106	.277
Pair 13	Q11pre - Q11post	-.05607	.23115	.02235	-.10038	-.01177	-2.509	106	.014
Pair 15	Q13pre - Q13post	.09346	.46647	.04510	.00405	.18286	2.072	106	.041
Pair 16	Q14apre - Q14apost	-.00935	.16797	.01624	-.04154	.02285	-.576	106	.566
Pair 17	Q14bpre - Q14bpost	-.04673	.42076	.04068	-.12737	.03392	-1.149	106	.253
Pair 18	Q14cpre - Q14cpost	-.14953	.45149	.04365	-.23607	-.06300	-3.426	106	.001
Pair 19	Q15pre - Q15post	.01869	.54912	.05309	-.08656	.12394	.352	106	.725
Pair 20	Q16pre - Q16post	.00000	.38851	.03756	-.07446	.07446	.000	106	1.000

Paired Samples Test for the Total Score

		Paired Differences					t	df	*p*-value
		Mean	Std. Dev	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Totalpre - Totalpost	-.80374	2.85313	.27582	-1.35058	-.25689	-2.914	106	.004