

# Unit 1 *Algebraic Manipulation*

## Introduction

### *Learning objectives*

This unit develops the theme of algebraic concepts, assuming that you are familiar with basic algebraic manipulation, including solving linear equations. After completing this unit you should be able to

- solve simultaneous equations
- fully expand brackets
- factorise expressions by taking out single terms and linear terms
- factorise quadratic expressions into two linear terms.

### *Notes*

The word 'algebra' comes from the title of a book

*Al - jabr w'al muqabala*

written in 830 AD by the Arabic astronomer Al - Khwarizmi. The exact translation of the title is disputed: 'al - jabr' means something like "restoring and balancing" (which refers to the idea of shifting things from one side of an equation to the other), and 'w'al muqabala' means something like "cancelling and simplifying". These two ideas reflect the central art of algebra - namely that of 'rearranging and simplifying expressions'.

Full-blooded elementary algebra, in which, for example, the general quadratic equation can be written as

$$ax^2 + bx + c = 0$$

and its solution given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

was only developed after 1600 AD. The effect was dramatic. After waiting 3500 years for an effective language, the next century saw an explosive growth - with the rise of coordinate geometry (Descartes 1637) and calculus (Newton, Leibniz 1660 - 1684). Around 1600 Galileo observed that "The Book of Nature is written in the language of mathematics". It soon became clear that the language of all mathematics is algebra!

### *Key points*

- Simultaneous linear equations are of the form

$$ax + by = c$$

$$dx + ey = f$$

when  $a, b, c, d, e$  and  $f$  are constants.

- Simultaneous linear equations normally have unique solutions, but not when one equation is the multiple of another. For example, when

$$x + y = 1$$

$$2x + 2y = 2$$

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there is no unique solution. Also when there are two inconsistent equations, for example,

$$x + y = 1$$

$$x + y = 3$$

there are NO solutions.

- Having obtained solutions for  $x$  and  $y$  for simultaneous linear equations, you should then always check this solution for  $x$  and  $y$  in the original equations.

- When expanding brackets, ensure that all terms are considered. For example,

$$\begin{aligned} 2x(x + 3) &= 2x \times x + 2x \times 3 \\ &= 2x^2 + 6x \end{aligned}$$

- When taking out simple factors from an equation, ensure that you take out all possible factors.

$$\begin{aligned} \text{For example, } 2x^2 + 6x &= 2(x^2 + 3x) && \text{(taking out the constant term 2)} \\ &= 2x(x + 3) && \text{(taking out the factor } x) \end{aligned}$$

- For quadratic factorisation of  $x^2 + ax + b$ , if factors of the form  $(x + c)(x + d)$  exist, then

$$a = c + d, \quad b = cd$$

***Facts to remember***

- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- If  $x^2 + ax + b = (x + p)(x + q)$ , then

$$a = p + q \text{ and } b = pq$$

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## *Glossary of Terms*

### *Simultaneous linear equations*

These are of the form

$$ax + by = c$$

$$dx + ey = f$$

when  $a, b, c, d, e$  and  $f$  are constants.

### *Expanding brackets*

Expressions such as  $a(x + b)$  are expanded out.

For example,

$$2(x + 3) = 2x + 6$$

$$x(4 - x) = 4x - x^2$$

$$2x(1 + 3x) = 2x + 6x^2$$

$$a(x + b) = ax + ab$$

### *Linear factorisation*

Single factors are taken out of an expression.

For example,

$$3x + 9 = 3(x + 3)$$

$$4x^2 + x = x(4x + 1)$$

$$5x^2 + 10x = 5x(x + 2)$$

### *Quadratic factorisation*

Quadratic expressions are factorised into linear factors.

For example,

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

$$x^2 - 4 = (x + 2)(x - 2)$$

$$(2x - 1)(x + 2) = 2x^2 + 3x - 2$$