

Unit 3 *Solving Equations*

Introduction

Learning objectives

This unit extends algebraic skills by solving more complicated equations. After completing this unit you should

- be able to manipulate algebraic fractions
- be able to solve quadratic equations that are derived from algebraic equations with fractions
- understand how to solve simultaneous equations where one is linear and one is quadratic.

Introduction

This is the third unit in this series of 6 units on *Algebra*. Here we build on the work and techniques introduced in the earlier units.

Key points

- Algebraic fractions are dealt with in a similar way to fractions.
- In solving simultaneous equations when one is linear and one is quadratic, substitute the linear equation with the quadratic, simplify the equation and then solve.

Facts to remember

- $\frac{1}{x+a} + \frac{1}{x-a} = \frac{2x}{x^2 - a^2}$

- When solving equations of the form

$$\frac{x+a}{x+b} + \frac{x+c}{x+d} = e$$

you multiply both sides by $(x+b)(x+d)$

- The solution of simultaneous equations, one linear and one quadratic, can have 0, 1 or 2 solutions: in this unit you will meet only simultaneous equations with 2 solutions.
- If $(x-a)(x-b) = 0$ then $x = a$ or $x = b$
- The formula for solving the quadratic equation

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Introduction

- The number of real roots of a quadratic is

$$2 \text{ if } b^2 > 4ac$$

$$1 \text{ if } b^2 = 4ac \text{ (repeated root)}$$

$$0 \text{ if } b^2 < 4ac$$

*Glossary of terms**Algebraic fractions*

These are of the form

$$\frac{ax + b}{cx + d}$$

when a , b , c , and d are constants.

For example, $\frac{x + 3}{3x - 1}$, $\frac{2y + 4}{3 - 2y}$, $\frac{3}{x + 2}$

Quadratic factorisation

This is when you can write a quadratic formula in the form

$$ax^2 + bx + c = a(x - p)(x - q)$$

Formula for solving quadratic equation

The formula for solving the quadratic equation

$$ax^2 + bx + c = 0$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the square

This is when we write the quadratic in the form

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$