

Unit 5 *Sequences and Series*

Introduction

Learning objectives

This unit extends the algebraic themes met in earlier units, to sequences and series. After completing this unit you should

- understand the concept of a general geometric sequence
- be able to sum the first n terms of a geometric series and consider the convergence of this sum as $n \rightarrow \infty$
- understand the concept of a general arithmetic sequence
- be able to sum the first n terms of an arithmetic series
- understand and use the Σ notation of summing series.

Notes

This is a topic that has been met at GCSE level but here the concepts are extended. It is important to recognise the difference between geometric sequences and arithmetic sequences, which sum to produce geometric and arithmetic series. There are some key results, particularly the sum of n terms of both geometric and arithmetic series.

Key points

- The general form of a geometric sequence of n terms is

$$a, ar, ar^2, \dots, ar^{n-1}$$

where a is a constant and r , the ratio of consecutive terms, is called the *common ratio*.

- The sum of n terms of a geometrical series is given by

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \quad (r \neq 1)$$

- If $|r| < 1$, the sum to infinity of a geometric series is given by

$$S_n \rightarrow \frac{a}{(1 - r)} \quad \text{as } n \rightarrow \infty$$

- The general arithmetic sequence is of the form

$$a, a + d, a + 2d, a + 3d, \dots$$

where a and d are constants.

- The sum of n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2} (2a + (n - 1)d)$$

If l is the last term, then $l = a + (n - 1)d$ and

$$S_n = \frac{n}{2} (a + l)$$

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- Sigma notation can be used to write a sum of series.

For example, $2 + 4 + 8 + \dots + 2^{12} = \sum_{r=1}^{12} 2^r$

- Important results are

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Facts to remember

- The sum of n terms of a geometrical series is given by $S_n = \frac{a(r^n - 1)}{(r - 1)}$
- If $|r| < 1$, $S_n \rightarrow \frac{a}{1 - r}$ as $n \rightarrow \infty$
- The sum of n terms of an arithmetic series is given by

$$S_n = \frac{n}{2}(2a + (n - 1)d) = \frac{n}{2}(a + l)$$

- $1 + 2 + 3 + \dots + n = \sum_{r=1}^n r = \frac{1}{2}n(n + 1)$

Glossary of terms

A *geometric sequence* is of the form $a, ar, ar^2, ar^3, \dots$ where a and r are constants and r is the common ratio.

An *arithmetic sequence* is of the form $a, a + d, a + 2d, a + 3d, \dots$ where a and d are constants.