

Unit 6 *Binomial Theorem*

Introduction

Learning objectives

In this unit we generalise algebraic expressions of the form $(a + b)^n$. After completing the unit you should be able to

- understand how to calculate expressions of the form $(a + b)^n$ for positive integers, n
- use binomial coefficients, including factorials, in the binomial expansion
- apply factorials to determine the number of arrangements of letters
- understand and use the Binomial Theorem for $(a + x)^n$.

Notes

This unit shows that practical problems can be generalised using factorials and binomial coefficients. Although the binomial coefficient has applications for finding the number of arrangement of sets of letters, the real application of the theorem is in its use in Statistics with the binomial distribution. This is covered in the Statistics Strand.

Key points

- ${}^nC_r$ is the number of different combinations of n objects, r of one type, $(n - r)$ of another.
- $n! = n(n - 1)(n - 2) \dots 1$
- ${}^nC_r = \frac{n!}{r!(n - r)!} = \binom{n}{r}$
- The number of arrangements of an n -letter word, with ONE letter repeated p times is $\left(\frac{n!}{p!}\right)$
- The Binomial Theorem is given by

$$(a + x)^n = a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \dots + \binom{n}{r}a^{n-r}x^r + \dots x^n$$

where a and x are numbers and n is a positive integer.

Facts to remember

- $n! = n(n - 1)(n - 2) \dots 1$
- ${}^nC_r = \frac{n!}{(n - r)!r!} = \binom{n}{r}$
- $1! = 1$
- $0! = 1$

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Glossary of terms

Binomial coefficients are the coefficients of x in expansions of the form $(1 + x)^n$ where n is a positive integer. For example,

$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$$

So the binomial coefficients are 1, 4, 6, 4 and 1.

Factorials $n! = n(n - 1)(n - 2) \dots 1$