

STRAND: FINANCE

Unit 3 *Loans and Mortgages*

TEXT

Contents

Section

- 3.1 Annual Percentage Rate (APR)
- 3.2 APR for Repayment of Loans
- 3.3 Credit Purchases
- 3.4 Mortgage Repayments

3 Loans and Mortgages

3.1 Annual Percentage Rate (APR)

Most banks and building societies calculate interest more than once a year - often on a day-to-day basis. This may involve using a variety of interest rates which can be calculated from one another. There is much confusion at this point, however, and it was to help consumers that laws were introduced several years ago requiring all banks, finance houses and other institutions offering loan facilities to state in their advertisements the true **APR (annual percentage rate)** being charged by way of interest.

The APR is the interest rate figure that indicates the *total* cost of borrowing, including any charges. This information helps the borrower to compare like with like when considering various options.



Worked Example 1

The interest charges for borrowing on a major credit card are 1.85% per month. What is the APR?



Solution

Interest of 1.85% gives a multiplier of 1.0185 for each month.

So the multiplier for 12 months is $1.0185^{12} = 1.2460\dots$

Thus the APR is 24.6%.



Note

This is more than $12 \times 1.85\% = 22.2\%$ and although the difference is small, it becomes more significant with higher interest charges.



Worked Example 2

The interest charge for borrowing on Twengo is 4.95% per month. What is the APR?



Solution

Here the monthly multiplier is 1.0495, so for 12 months it is

$$(1.0495)^{12} = 1.7856\dots$$

So the APR is 78.6% which is considerably more than $12 \times 4.95\% = 59.4\%$

The situation is even more extreme with a daily rate of interest.



Worked Example 3

The interest charge for borrowing on Twengo is 0.5% per day.

- What is the APR?
- How much would you owe after 2 weeks if you borrowed £100 on these terms?



Solution

- (a) The daily multiplier is 1.005, so for a year (365 days), it is

$$(1.005)^{365} = 6.1746...$$

So the APR is 517.5%.

- (b) To calculate how much you would owe after 2 weeks, you use

$$\begin{aligned}\pounds 100 \times (\text{daily multiplier})^{14} &= \pounds 100 \times (1.005)^{14} \\ &= \pounds 107.23\end{aligned}$$



Note

Note that such loans (sometimes referred to as 'payday loans', usually for between £80 and £1000 and designed to keep your finances afloat until you receive your monthly wages), can be helpful for short term loans of just a few days or maybe weeks, but can lead to severe financial problems in the longer term. Most payday loan UK companies charge rates of interest into the thousands, with an option to roll on debt from month to month - a very bad idea considering the amount of interest you would be paying.

New rules for payday lenders, designed to protect vulnerable borrowers from spiralling costs, were introduced in January 2015 and will be reviewed in 2017. Payday loans continue to be an expensive and potentially problematic option for obtaining money.



Worked Example 4

What **weekly** interest rate corresponds to 26% APR?



Solution

If the yearly multiplier is 1.26, the weekly multiplier is $1.26^{\frac{1}{52}} = 1.0045...$

So the weekly interest rate is 0.45% per week.



Worked Example 5

A finance company charges interest at 26% per year calculated **weekly**. What APR is it charging?



Solution

26% per year is equivalent to $\frac{26}{52}\% = 0.5\%$ per week.

So the multiplier for 1 week is 1.005.

The multiplier for 52 weeks is 1.005^{52} which is 1.2960.

So the true APR is 29.6% .

You should bear in mind that real-life calculations are often not this simple. A credit card company, for example, may charge interest from the date of purchase, or from the date of the statement or from the end of the month, or from some other date. The company may even waive interest altogether as long as the whole balance is repaid within a certain

period. Details will be given in the 'Conditions of Use' issued to all cardholders; it is important that you read these very carefully.

Again, banks and building societies are normally required by law to deduct income tax at the current basic rate before paying interest to individual depositors, except in some cases when the depositors themselves are not liable for income tax. For this reason, banks and building societies usually quote **net** interest after allowing for the tax deducted: you should be very careful if you have to compare these rates with the **gross** rates (i.e., before deduction of tax) quoted by many other institutions. This is dealt with in Unit 5.



Exercises

Calculate:

1. the true APR equivalent to 2% per calendar month;
2. the true APR equivalent to 10% per week;
3. the monthly rate equivalent to a true APR of 9% p.a.;
4. the true APR equivalent to 12% p.a. calculated weekly;
5. the true APR equivalent to 14% p.a. calculated daily.

3.2 APR for Repayment of Loans

Here we consider the reverse problem to that considered in the previous section, that is, repayment of a sum of money, say C , borrowed on a certain date and repaid after time n years. We need to determine the APR charged. Note that the APR, say i , is given by

$$A = C(1 + i)^n$$

where A is the repayment sum.



Worked Example 1

A loan of £2000 is repaid by a payment of £2500 made 4 years later. What is the APR for this loan?



Solution

Using the formula above,

$$2500 = 2000(1 + i)^4$$

Hence
$$(1 + i)^4 = \frac{2500}{2000} = 1.25$$

and

$$(1 + i) = (1.25)^{\frac{1}{4}}$$

$$i = 0.05737...$$

So the APR is approximately 5.7%.



Worked Example 2

A borrower is loaned £4000 on 1st January 2015 and agrees to repay with a single payment of £5000 on 21st March 2016.

What is the APR for this loan.



Solution

Here $A = £5000$ and $C = £4000$ but the time interval, n , is needed in years.

$$\begin{aligned} n &= 1 \text{ year} + 3 \text{ months} \\ &= \left\{ 1 + \frac{(31 + 29 + 31)}{366} \right\} \text{ years} \\ &= \left(1 + \frac{91}{366} \right) \text{ years} \\ n &= 1.2486... \text{ years} \end{aligned}$$

Thus the APR is given, approximately, by

$$\begin{aligned} 5000 &= 4000(1 + i)^{1.2486} \\ (1 + i)^{1.2486} &= \frac{5000}{4000} \\ &= 1.25 \end{aligned}$$

Hence

$$\begin{aligned} (1 + i) &= (1.25)^{\frac{1}{1.2486}} \\ &= 1.19568... \end{aligned}$$

giving $i = 0.19568...$

So the APR is approximately 19.6%.



Exercises

1. Find the APR for each of the following loans.
 - (a) £1000 repaid by single repayment of £1100 made after 1 year.
 - (b) £1000 repaid by a single repayment of £1400 made after 4 years.
 - (c) £4500 repaid by single repayment of £5000 made after $2\frac{1}{2}$ years.
 - (d) £25 000 repaid by a single repayment of £30 000 after 10 years.

2. Find the APR for each of the loans shown in the table below.

	<i>Amount borrowed</i>	<i>Date of loan</i>	<i>Amount repaid</i>	<i>Date of repayment</i>
(a)	£1000	1/12/2016	£1100	31/3/17
(b)	£5000	1/1/15	£6000	30/9/17
(c)	£25 000	1/4/14	£32 500	1/8/16

3.3 Credit Purchases

In reality, many loans are repaid not in a lump sum at the end of the loan period but in a series of weekly, monthly or annual payments. Very few people buying a house can afford to pay for it in cash, for example: instead, the usual procedure is to obtain a loan from a bank or building society and to repay this by monthly instalments (with interest, of course) over a period of 25 years or so. The calculations are quite complicated because the outstanding balance on which interest is charged varies from month to month as repayments are made.

One method of finding how long it takes to repay a loan is to calculate the position at the end of each repayment period. The next worked example shows this method.



Worked Example 1

A friend has agreed to lend you £1000 at a rate of 5% per annum and you have agreed to pay back £150 per year.

- (a) How long will it take you to pay off the loan?
 (b) How much do you repay in total?



Solution

You can set out the calculations in a table:

	Interest (£)	Repayment (£)	Balance (£)
At the start			1000.00
After 1 year	+ 50.00	– 150.00	900.00
After 2 years	+ 45.00	– 150.00	795.00
After 3 years	+ 39.75	– 150.00	684.75
After 4 years	+ 34.24	– 150.00	568.99
After 5 years	+ 28.45	– 150.00	447.44
After 6 years	+ 22.37	– 150.00	319.81
After 7 years	+ 15.99	– 150.00	185.80
After 8 years	+ 9.30	– 150.00	45.10
After 9 years	+ 2.26	– 47.36	0.00

- (a) So it takes you 9 years to repay the loan.
 (b) You repay £1247.36 in total.

This long and repetitive calculation is tedious when done by hand but can be processed almost instantly using a calculator or spreadsheet.



Exercises

For each of the following loans, calculate

- (i) the repayment period and (ii) the actual total amount paid.
- £500 borrowed at 9% interest and repaid at £200 per year.
 - £1200 borrowed at 14% interest and repaid at £350 per year.
 - £200 borrowed at 25% interest and repaid at £80 per year.
 - £250 borrowed at 1.5% per month and repaid at £50 per month.

3.4 Mortgage Repayments

The method of calculation in Section 3.3 is far from ideal. Apart from the time taken, its main disadvantage is that it can be used only when the size of the repayment is already known. More commonly, it is the **period** of the loan that is known and the **repayment** which has to be calculated, and for this reason a different technique is needed. You will need to have studied both geometric series and logarithms to understand what follows.

First, take the problem in Worked Example 1 of Section 3.3, and consider it in a slightly different way. You borrow £1000 at 5% interest and pay it back at £150 per year.

How long will this take?

Balance, in £, at ...

$$0 \text{ years} = 1000$$

$$1 \text{ year} = 1000 \times 1.05 - 150$$

$$2 \text{ years} = (1000 \times 1.05 - 150) \times 1.05 - 150$$

(because interest is calculated as 5% of the previous balance and then a payment is deducted).

$$3 \text{ years} = (1000 \times 1.05^2 - 150 \times 1.05 - 150) \times 1.05 - 150$$

$$= 1000 \times 1.05^3 - 150 \times 1.05^2 - 150 \times 1.05 - 150$$

This should suggest a fairly obvious pattern:

Balance,,, in £, at

$$\begin{aligned} n \text{ years} &= 1000 \times 1.05^n - 150 \times 1.05^{n-1} - 150 \times 1.05^{n-2} - \dots - 1.05 \times 1.05 - 150 \\ &= 1000 \times 1.05^n - 150 (1.05^{n-1} + 1.05^{n-2} + \dots + 1.05 + 1) \end{aligned}$$

Now the terms in brackets form a geometric series with n terms. It is convenient to read this series backwards, so that its 'first' term is 1 and its common ratio 1.05; the usual

formula then gives its sum as $\frac{1(1 - 1.05^n)}{(1 - 1.05)}$.

$$\begin{aligned} \text{So balance} &= 1000 \times 1.05^n - \frac{150(1 - 1.05^n)}{(1 - 1.05)} \\ &= 1000 \times 1.05^n - 150 \frac{(1 - 1.05^n)}{(-0.05)} \\ &= 1000 \times 1.05^n + 3000(1 - 1.05^n) \\ &= 1000 \times 1.05^n + 3000 - 3000 \times 1.05^n \\ &= 3000 - 2000 \times 1.05^n \end{aligned}$$

The loan is repaid when this balance reaches zero.

$$\begin{aligned} 0 &= 3000 - 2000 \times 1.05^n \\ \Rightarrow 2000 \times 1.05^n &= 3000 \\ \Rightarrow 1.05^n &= 1.5 \\ \Rightarrow n \ln 1.05 &= \ln 1.5 \\ \Rightarrow n &= \ln 1.5 \div \ln 1.05 \\ \Rightarrow n &= 8.31... \end{aligned}$$

So the loan is repaid after about 8.3 years, which agrees with the answer already obtained. This method may seem very long and complicated in comparison with the earlier one, but it becomes easier with practice and it can be used for long periods without any difficulty.



Worked Example 1

Andrew and Alison borrow £45 000 from the building society to buy a house. (They would probably say that they get a mortgage, although from a lawyer's point of view they actually give the building society a **mortgage** - a legal charge on the house - in exchange for a loan.) The effective interest they will pay after allowing for tax relief is 9% per annum, calculated monthly, and they plan to repay £380 per month.

How long will it take them to pay off the loan?



Solution

The monthly interest rate is $9\% \div 12 = 0.75\%$, so the monthly multiplier is 1.0075.

Owing, in £, at

$$0 \text{ months} = 45\,000$$

$$1 \text{ month} = 45\,000 \times 1.0075 - 380$$

$$2 \text{ months} = (45\,000 \times 1.0075 - 380) \times 1.0075 - 380$$

$$= 45\,000 \times 1.0075^2 - 380 \times 1.0075 - 380$$

and so on ...

$$n \text{ months} = 45\,000 \times 1.0075^n - 380 \times 1.0075^{n-1} - \dots - 380$$

$$= 45\,000 \times 1.0075^n - \frac{380(1 - 1.0075^n)}{(1 - 1.0075)}, \text{ summing the arithmetic progression}$$

$$= 45\,000 \times 1.0075^n + 50\,666.67(1 - 1.0075^n)$$

$$= 45\,000 \times 1.0075^n + 50\,666.67 - 50\,666.67 \times 1.0075^n$$

$$= 50\,666.67 - 5\,666.67 \times 1.0075^n$$

The loan is repaid when this balance is zero, which happens when

$$5\,666.67 \times 1.0075^n = 50\,666.67$$

$$\Rightarrow 1.0075^n = 8.9411\dots$$

$$\Rightarrow n \log 1.0075 = \log 8.9411\dots$$

$$\Rightarrow n = 293.18\dots$$

So the period of the loan is 293 months (just under 25 years).

The real advantage of this method over the simpler step-by-step method is that it can be used to calculate the repayments needed to pay off the loan within a given period, as we will see in the next worked example.



Worked Example 2

Suppose your brother borrows £400 from you to go on a sports weekend. He agrees to pay it back (with interest at 10% p.a. calculated monthly) by twelve monthly instalments. How much should each instalment be?



Solution

Suppose each payment is £ P .

The monthly interest rate is $10\% \div 12 = 0.833\dots\%$.

Balance at

$$0 \text{ months} = 400$$

$$1 \text{ month} = 400 \times 1.00833 - P$$

$$\begin{aligned} 2 \text{ months} &= (400 \times 1.00833 - P) \times 1.00833 - P \\ &= 400 \times 1.00833^2 - P \times 1.0083 - P \end{aligned}$$

and so on ...

$$\begin{aligned} 12 \text{ months} &= 400 \times 1.00833^{12} - P(1.00833^{11} + \dots + 1) \\ &= 400 \times 1.00833^{12} - \frac{P(1 - 1.00833^{12})}{(1 - 1.00833)} \\ &= 441.87 - 12.57P \end{aligned}$$

But this balance is to be zero, giving

$$\begin{aligned} P &= 441.87 \div 12.57 \\ &= 35.152... \end{aligned}$$

He should pay back £35.15 each month.



Exercises

- Sam and Sheila borrow £50 000 from a building society at a net interest rate of 10% p.a., added at the beginning of each year to the balance then outstanding. Each year, Sam and Sheila make 12 monthly repayments of £450 each, totalling £5400. How long will it be before they have paid off the loan completely?
- Repeat Question 1, with the building society calculating the interest monthly at a rate equivalent to 10% p.a., and Sam and Sheila still repaying £450 per month.
- John and Julie borrow £40 000 from a building society to buy a house. The rate of interest (which is added to the balance at the beginning of each year) is 10.5% p.a., and John and Julie want to pay off the mortgage over a period of 25 years. What should their monthly repayment be?
- Repeat Question 1, assuming that interest is added monthly. (Remember that 10.5% is not the APR.)
- The manager of an electrical shop wants to offer her customers new credit terms. A laptop computer costing £239 is to be offered for twenty-four equal monthly payments so that the APR is 15%. How much should each payment be?
- A used car is advertised at a cash price of £6674, but the garage offers credit terms with a quoted APR of 17%. These terms call for a £99 deposit followed by 48 monthly payments of £185.75. Check this calculation and comment on your result.
- Angela buys a computer and pays by instalments. The cash price of the computer would be £550, but Angela pays £250 down, £200 after six months, and £150 after twelve months. Work out the half-yearly interest rate and the APR.