

STRAND: FINANCE

Unit 4 *Inflation Indices*

TEXT

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4 Inflation Indices

4.1 Weighted Averages

You will be familiar with the mean value of a set of n numbers, which is defined as

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

– in this case each number is given *equal* weighting, that is, they are all equally valued.

But, for example, if x_1 is valued twice as much as the other numbers, the *weighted* mean is given by

$$\frac{2x_1 + x_2 + \dots + x_n}{(n + 1)}$$

Effectively, we are counting x_1 twice; hence the division by $(n + 1)$.

In general, the weighted average of n values $x_1, x_2, \dots, x_n$, repeated in the ratio $a_1, a_2, \dots, a_n$ is given by

$$\frac{a_1 x_1 + a_2 x_2 + \dots + a_n x_n}{(a_1 + a_2 + \dots + a_n)}$$

Note that, if all the x_i 's are equal to x , say, then the weighted average is given by

$$\frac{a_1 x + a_2 x + \dots + a_n x}{(a_1 + a_2 + \dots + a_n)} = \frac{(a_1 + a_2 + \dots + a_n)x}{(a_1 + a_2 + \dots + a_n)} = x$$

as you would expect. In the following example, you will see how to use this concept .



Worked Example 1

Find the weighted average of the numbers 4, 5 and 6 when

- (a) they are equally weighted, (b) the first number is double weighted,
- (c) the first number is triple weighted, (d) they are weighted in the ratio 3 : 2 : 1.

Explain the significance of the different values obtained in (b), (c), and (d) in comparison with (a).



Solution

$$(a) \quad \text{Mean average} = \frac{(4 + 5 + 6)}{3} = 5$$

$$(b) \quad \text{Weighted average} = \frac{(2 \times 4 + 1 \times 5 + 1 \times 6)}{(2 + 1 + 1)} = \frac{19}{4} = 4.75$$

$$(c) \quad \text{Weighted average} = \frac{(3 \times 4 + 1 \times 5 + 1 \times 6)}{(3 + 1 + 1)} = \frac{23}{5} = 4.6$$

$$(d) \quad \text{Weighted average} = \frac{(3 \times 4 + 2 \times 5 + 1 \times 6)}{(3 + 2 + 1)} = \frac{28}{6} \approx 4.67$$

In (b), the weighted average is reduced since the smallest number, 4, is effectively being counted *twice*; in (c) it is further reduced as the 4 is being counted *three* times; in (d) there is a slight increase from (c) as the number 5 is now counted twice.



Worked Example 2

The weighted average of the numbers 4 and 6 is 4.5.

What weightings are used in this average?



Solution

If 2 numbers, x_1 and x_2 , are weighted in the ratio $a : b$, then the weighted average is given by

$$\frac{ax_1 + bx_2}{a + b}$$

So here we have

$$4.5 = \frac{4a + 6b}{(a + b)}$$

or

$$9(a + b) = 2(4a + 6b)$$

$$a = 3b$$

Hence

$$a : b = 3 : 1$$

$$\text{Check:} \quad \frac{3 \times 4 + 1 \times 6}{(3 + 1)} = \frac{12 + 6}{4} = 4.5$$



Worked Example 3

In a cricket season, two county batsmen, Steve and Russell, scored the following numbers of runs in their first 5 innings; the most recent scores are shown on the right of the list.

Steve : 25, 15, 39, 36, 25

Russell: 76, 23, 37, 2, 4

- What are the average scores for each batsman?
- Using the weightings 1, 2, 3, 4, 5, what are their averages now?
- Which batsman would you pick to play for England?



Solution

$$(a) \quad \text{Steve's average score:} \quad \frac{(25 + 15 + 39 + 36 + 25)}{5} = \frac{140}{5} = 28$$

$$\text{Russell's average score:} \quad \frac{(76 + 23 + 37 + 2 + 4)}{5} = \frac{142}{5} = 28.4$$

- (b) Using the weighting 1, 2, 3, 4, 5, we get the weighted averages:
Steve's weighted average:

$$\frac{(25 \times 1 + 15 \times 2 + 39 \times 3 + 36 \times 4 + 25 \times 5)}{(1 + 2 + 3 + 4 + 5)} = \frac{441}{15} = 29.4$$

Russell's weighted average:

$$\frac{(76 \times 1 + 23 \times 2 + 37 \times 3 + 2 \times 4 + 4 \times 5)}{(1 + 2 + 3 + 4 + 5)} = \frac{261}{15} = 17.4$$

- (c) Looking at the data above, it would appear that Steve is more consistent and that Russell is losing his touch. On the other hand, Russell seems more capable of making high scores!



Exercises

- Find the weighted average of the numbers 1, 5 and 9 when
 - they are equally weighted
 - the middle number is double weighted
 - the first number is triple weighted
 - the numbers are weighted in the ratio 2 : 1 : 2.
- Two numbers, 2 and 20, are weighted so that their weighted average is 4. What is the ratio of weighting used?
- If the numbers x_1 and x_2 are weighted in the ratio a to $1 - a$, what is the formula for the weighted average?
- This season, three batsmen have so far scored the following numbers of runs:

Batsman A : 10, 0, 56*, 93

Batsman B : 61, 47, 53, 51

Batsman C : 51*, 0, 15, 101*, 2

(* means 'not out' and is not counted as an innings completed.)

- Calculate each batsman's average score so far this season.
- Use weighting 1, 2, 3, ..., to find their average weighted scores. Who has the highest weighted average?
- If the weighting is changed to 1, 2, 4, 8, ..., will this make a difference to who is the highest-rated batsman?

4.2 Index Numbers

In this section we will see how the concept of *index numbers* is used to compare price increases over a period of time (usually years). The index is based on the price relative, that is, the new value based on an initial value of £100. This is illustrated in the worked example below.



Worked Example 1

The price of a holiday in 2010 and 2011 was

<i>Year</i>	<i>2010</i>	<i>2011</i>
<i>Price</i>	£310	£380

- Calculate the index number for the price of the holiday in 2011, taking 2010 as the base year with index number 100.
- Taking 2010 as the base year with an index of 100, the cost of a different holiday is divided into accommodation and travelling as shown in the table below.

<i>Price relative</i>	<i>2010</i>	<i>2011</i>	<i>Weight</i>
<i>Accommodation</i>	£100	£130	70%
<i>Travel</i>	£100	£120	30%

Calculate the weighted index for the holiday in 2011.



Solution

- Since 2010 corresponds to 100, then the index value at 2011 is given by

$$100 \times \frac{380}{310} \approx 123$$

- The index value now needs a weighted average. This is now given by

$$\begin{aligned} & \frac{0.7 \times 130 + 0.3 \times 120}{(0.7 + 0.3)} \\ &= \frac{91 + 36}{1} \\ &= 127 \end{aligned}$$



Worked Example 2

- In 2010 the average cost of a small flat was £ 70 000. In 2000 the same type of flat cost £ 40 000.

Taking the index for 2000 to be 100 the index for 2010 was calculated to be 175.

What does this tell you?

- Using the following information, calculate the weighted index number for the year 2010 based on the year 2000.

<i>Item</i>	<i>% increase in price from 2000 to 2010</i>	<i>Weight</i>
Food	20	5
Clothing	15	1
Entertainment	25	1
Rent	20	3



Solution

- (a) The cost of the flat has increased by 75% in the period 2000-2010.
 (b) Weighted percentage price increase is

$$\begin{aligned}
 & \frac{(5 \times 20) + (1 \times 15) + (1 \times 25) + (3 \times 20)}{(5 + 1 + 1 + 3)} \\
 &= \frac{200}{10} \\
 &= 20
 \end{aligned}$$

So the weighted index number is $100 + 20 = 120$.



Exercises

1. The cost of a camera and a tin of cocoa are shown for the years 2010 and 2011.

	<i>2010</i>	<i>2011</i>
<i>Camera</i>	£145	£160
<i>Cocoa</i>	90p	120p

- (a) Calculate the index number for the price of cocoa in 2011 using the 2010 base year index number as 100.

The 2012 index number for the camera price increase, using 2010 as the base year, was 120.

- (b) Find the index number for the increase in price of the camera between 2011 and 2012.
2. A small company produces three colours of paint: white, red and green, in 1 litre tins. The following data relate to the per unit production costs for each paint colour in 2010 and 2014.

<i>Colour of paint</i>	<i>Amount produced in 2014 (litres)</i>	<i>Production cost per litre</i>		<i>2014 price index 2010 = 100</i>
		<i>2010</i>	<i>2014</i>	
White	4000	1.40	2.50	178.6
Red	5000	1.90	3.30	
Green	3500	2.10	3.70	176.2

- (a) (i) Calculate to one decimal place the 2014 price index for the cost per litre of red paint.
- (ii) What do the price indices indicate about the manufacturing costs of each paint variety?
- (b) Obtain to one decimal place a weighted index number to show the production costs incurred by the company in 2014, using 2010 as base year.
3. The table below shows the price of a mountain bike and a racing bike in 2008 and 2010.

	<i>Price (£)</i> <i>2008</i>	<i>Price (£)</i> <i>2010</i>	<i>Price index</i> <i>(2010 relative to 2008)</i>
Mountain Bike	400	300	<i>X</i>
Racing Bike	200	300	<i>Y</i>

- (a) (i) Find the value of *X*, the price index of a Mountain Bike.
- (ii) Find the value of *Y*, the price index of a Racing Bike.
- (b) In 2005 the price index (relative to 2008) of a Mountain Bike was 100. What can you say about the 2005 price of a mountain bike?

4.3 Retail Price Index

The Retail Price Index (RPI) and Consumer Price Index (CPI) are used as estimates of consumers' spending patterns. For example, the CPI is based on the 12 items listed with their weights in the following table.

	<i>CPI weight (per cent)</i>
1 Food & non-alcoholic beverages	10.6
2 Alcohol & tobacco	4.6
3 Clothing & footwear	6.3
4 Housing & household services	10.5
5 Furniture & household goods	6.5
6 Health	2.4
7 Transport	14.8
8 Communication	2.5
9 Recreation & culture	15.1
10 Education	1.7
11 Restaurants & hotels	13.9
12 Miscellaneous goods & services	11.1

The following examples show how this concept can be used in practical situations.



Worked Example 1

A newspaper article gave the following data concerning the cost of living.

Item of expenditure	Percentage relatives		Average expenditure per £
	2010	2013	
Housing	100	80	40p
Food	100	121	25p
Heating	100	125	15p
Clothing	100	110	15p
Sundries	100	130	5p

- How does the change in the cost of housing differ from the other items?
- I paid £525 for heating during 2010. How much would I expect to pay in 2013?
- Calculate a retail price index for 2013.



Solution

- It has decreased, whilst all other categories have increased.

$$(b) \quad £525 \times \frac{125}{100} = £656.25$$

- Retail price index

$$\begin{aligned}
 &= \frac{(40 \times 80) + (25 \times 121) + (15 \times 125) + (15 \times 110) + (5 \times 130)}{(40 + 25 + 15 + 15 + 5)} \\
 &= \frac{10\,400}{100} \\
 &= 104
 \end{aligned}$$



Worked Example 2

The following data relate to the UK Index of Industrial Production.

Industries	Weight	Index September 1993 (1985 = 100)
Energy and Water Supply	309	88
MANUFACTURING		
1. Metals	26	121
2. Other Minerals	35	117
3. Chemicals	71	115
4. Engineering	295	118
5. Food, Drink and Tobacco	91	108
6. Textiles, Footwear and Clothing	47	94
7. Other Manufacturing	126	132

Source:
*Monthly
Digest of
Statistics*
(October
1993)

- (a) Calculate the combined Index of Industrial Production for all seven *manufacturing industries* relative to 1985.
- (b) The Index of Industrial Production for *all* industries in the previous table is 108.3. Suggest a reason why this differs from your answer to (a).
- (c) If the All Items Index of Industrial production for 1985 was 108.1 with 1980 = 100, explain briefly what these indices represent.



Solution

$$\begin{aligned}
 & (26 \times 121) + (35 \times 117) + (71 \times 115) + (295 \times 118) \\
 & \quad + (91 \times 108) + (47 \times 94) + (126 \times 132) \\
 \text{(a) Index} &= \frac{(26 + 35 + 71 + 295 + 91 + 47 + 126)}{81094} \\
 &= \frac{81094}{691} \\
 &\approx 117 \text{ (to nearest whole number)}
 \end{aligned}$$

- (b) The extra industry included (Energy and Water Supply) actually had a decrease in its index, and so the overall index will decrease from the value in (a).
- (c) The index of 108.1 for 1985 compared with 100 for 1980 shows that the index increased from 1980 to 1985 by about 8% and again from 1985 to 1993 by about 8%.



Exercises

1. The following table shows the percentage relatives and weight of certain commodities in 2005, taking 2003 as the base year.

	Percentage relatives		Weight
	2003	2005	
<i>Mortgage</i>	100	110	0.4
<i>Heat and Lighting</i>	100	130	0.2
<i>Clothing</i>	100	125	0.1
<i>Food</i>	100	115	0.25
<i>Other items</i>	100	120	0.05

- (a) Give *one* reason why a weighted average is sometimes more appropriate than the ordinary arithmetic average.
- (b) Given that the clothing bill in 2003 was £400, how much would it have been in 2005?
- (c) Use the information in the table above to calculate a retail price index for 2005.

2. The following data have been taken from the General Index of Retail Prices and relate to September 1993.

<i>Group</i>	<i>Weight</i>	<i>Index</i>
1. Food	154	111.3
2. Catering	49	118.0
3. Alcoholic drink	83	114.7
4. Tobacco	36	106.4
5. Housing	175	138.0
6. Fuel and light	54	109.0
7. Household goods	71	110.0
8. Household services	41	113.0
9. Clothing and footwear	73	111.0
10. Personal goods and services	37	115.0
11. Motoring expenditure	128	120.0
12. Fares and other travel costs	23	126.0
13. Leisure goods	47	107.0
14. Leisure services	29	117.0

The 'All groups' index number for September 1993 is 117.79.

- Calculate, to one decimal place, a weighted index number to represent expenditure on motoring and travel (i.e. Groups 11 and 12).
 - Comment on the difference between the 'All groups' index and your answer in (a).
 - Included in Group 5 (Housing) is expenditure on mortgage interest payments which has a weighting of 60 and an index of 168.2.
 - Calculate, to two decimal places, an index number representing 'All groups' excluding expenditure on mortgage interest.
 - Explain briefly the effect this has had on the 'All groups' index.
3. A newspaper article gave the following data concerning the cost of living.

<i>Item of expenditure</i>	<i>-Price index</i>		<i>Average expenditure per £1</i>
	<i>1990</i>	<i>1993</i>	
Housing	100	80	40p
Food	100	121	25p
Heating	100	125	15p
Clothing	100	110	15p
Sundries	100	130	5p

- How does the change in the cost of housing differ from the other items?
- I paid £525 for heating during 1990.
How much would I expect to pay during 1993?
- Calculate a retail price index for 1993.

4. (a) Whilst out shopping, Mrs Gausden, a pensioner, buys a newspaper which she reads on the bus during her journey home. She cannot believe the headline . . .

- GOVERNMENT – KEEPING PRICES DOWN –
- ONLY 5 PER CENT INCREASE IN THE RETAIL PRICE INDEX FOR THE PAST 3 YEARS!

On arriving home she empties her shopping basket and decides to calculate her own price index using the items from her shopping basket. Her list with prices for 1989 and estimates for 1986, is displayed below along with her calculation of an index of retail prices.

<i>Item</i>	<i>Estimated price in 1986</i>	<i>Actual price in 1989</i>	<i>% increase</i>
Loaf of bread	40p	48p	20%
Jar of coffee	210p	256p	18%
Bottle of milk	20p	32p	38%
Packet of biscuits	45p	57p	18%
Salad cream	78p	80p	3%
Cans of beer	165p	185p	11%
Newspaper	10p	20p	50%

$$\text{Average price increase} = \frac{158}{7} = 23 \text{ per cent over 3 years.}$$

Mrs Gausden writes a letter of complaint, suggesting that she cannot be expected to live on salad cream! Your task is to reply to her letter.

- (i) Give THREE reasons why Mrs Gausden's index would NOT represent an Index of Retail Prices for an average household.
 - (ii) Where is the Retail Price Index used and why is it of value?
- (b) The indices of building costs for the years 1986 to 1989 taking 1988 as base are shown below.

1986	1987	1988	1989
72	90	100	106

- (i) What do the index numbers tell you?
- (ii) A house cost £80 000 to build in 1986. Calculate, to the nearest £1000, the cost of building an identical house in 1989.

5. During two consecutive months, a random sample of the school children in Town B had their spending habits monitored. The results are recorded in the table below. The figures are average amounts spent on each item by members of the sample.

<i>Item</i>	<i>Amount spent (pence)</i>	
	<i>Month 1</i>	<i>Month 2</i>
Entertainment	646	703
Sweets, Food, etc.	221	351
Books, Papers, etc.	306	295
Travel	441	474
Other	389	447

Using Month 1 as base, calculate for Month 2

- the price relative for Entertainment,
 - the price relative for Books, Papers, etc.
 - an index of total expenditure.
6. The table shows the index numbers and weights for five different items of household expenditure in 2003, 2007 and 2009.

<i>Item</i>	<i>Index Number</i>			<i>Weight</i>
	<i>2003</i>	<i>2007</i>	<i>2009</i>	
Food	100	104	106	0.2
Heat and Light	100	105	108	0.2
Clothing	100	100	97	0.15
Mortgage	100	106	106	0.4
Other	100	105	109	0.05

- The base year is 2003.
How do you know this from the figures in the table?
- What happened to household expenditure on mortgages from 2007 to 2009?
- Which is the only item to show a reduction in expenditure from 2003 to 2009?
- Explain what the weight for Mortgage expenditure shows.
- Heat and light expenditure for a typical household in 2007 was £630.
How much would this have been in 2009?
 - Find the actual increase in Heat and Light expenditure from 2003 to 2009.
- What is the overall index for household expenditure in 2009, based on 100 in 2003 ?