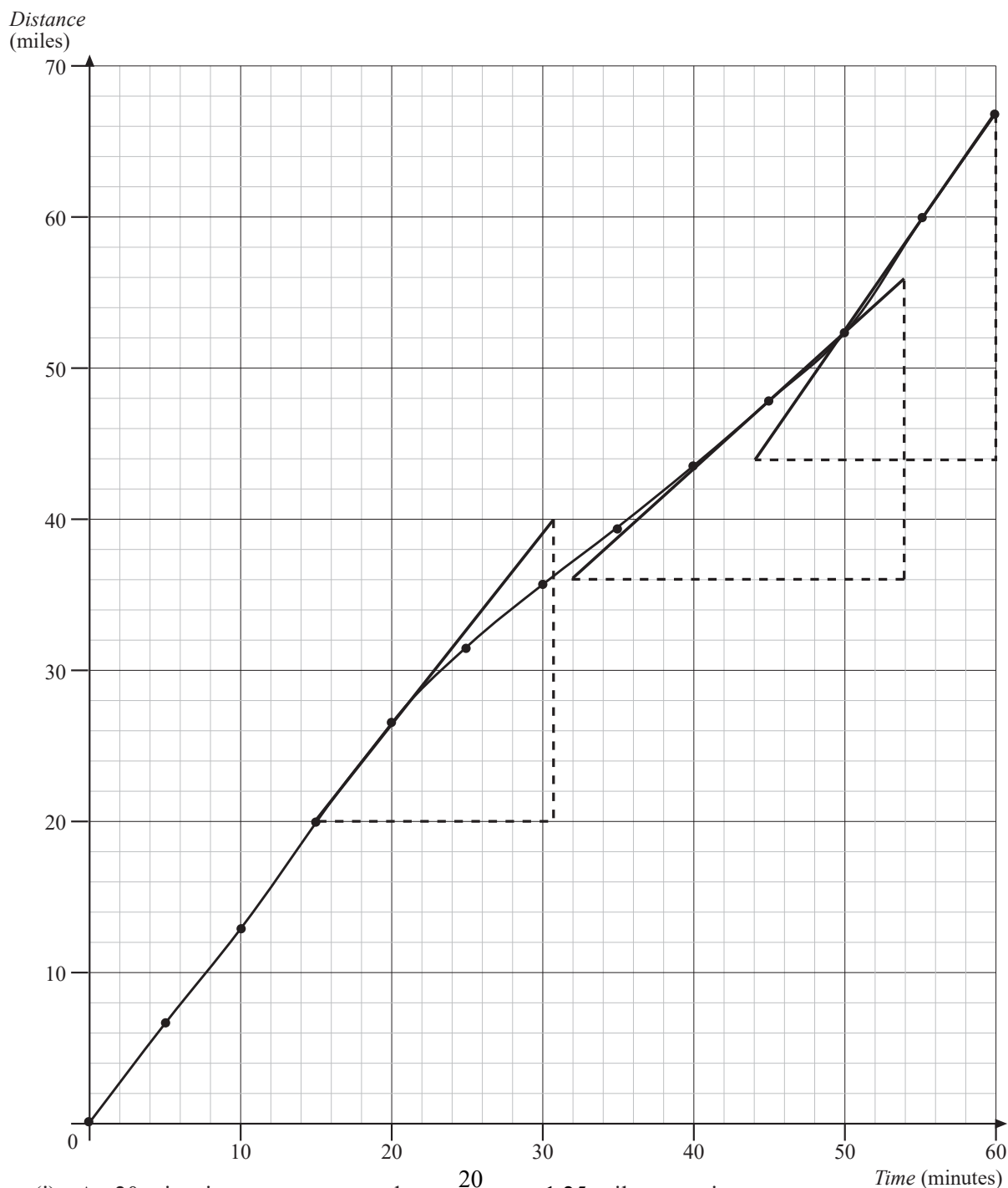


Unit 4 *Rates of Change*

Text Answers

4.2 Instantaneous Speed

1. (a)



(i) At 20 mins, instantaneous speed = $\frac{20}{31 - 15} \approx 1.25$ miles per min
(≈ 75 mph)

(ii) At 40 mins, instantaneous speed = $\frac{56 - 36}{54 - 32} \approx 0.91$ miles per min
(≈ 55 mph)

Unit 4 *Rates of Change*

Text Answers

4.2

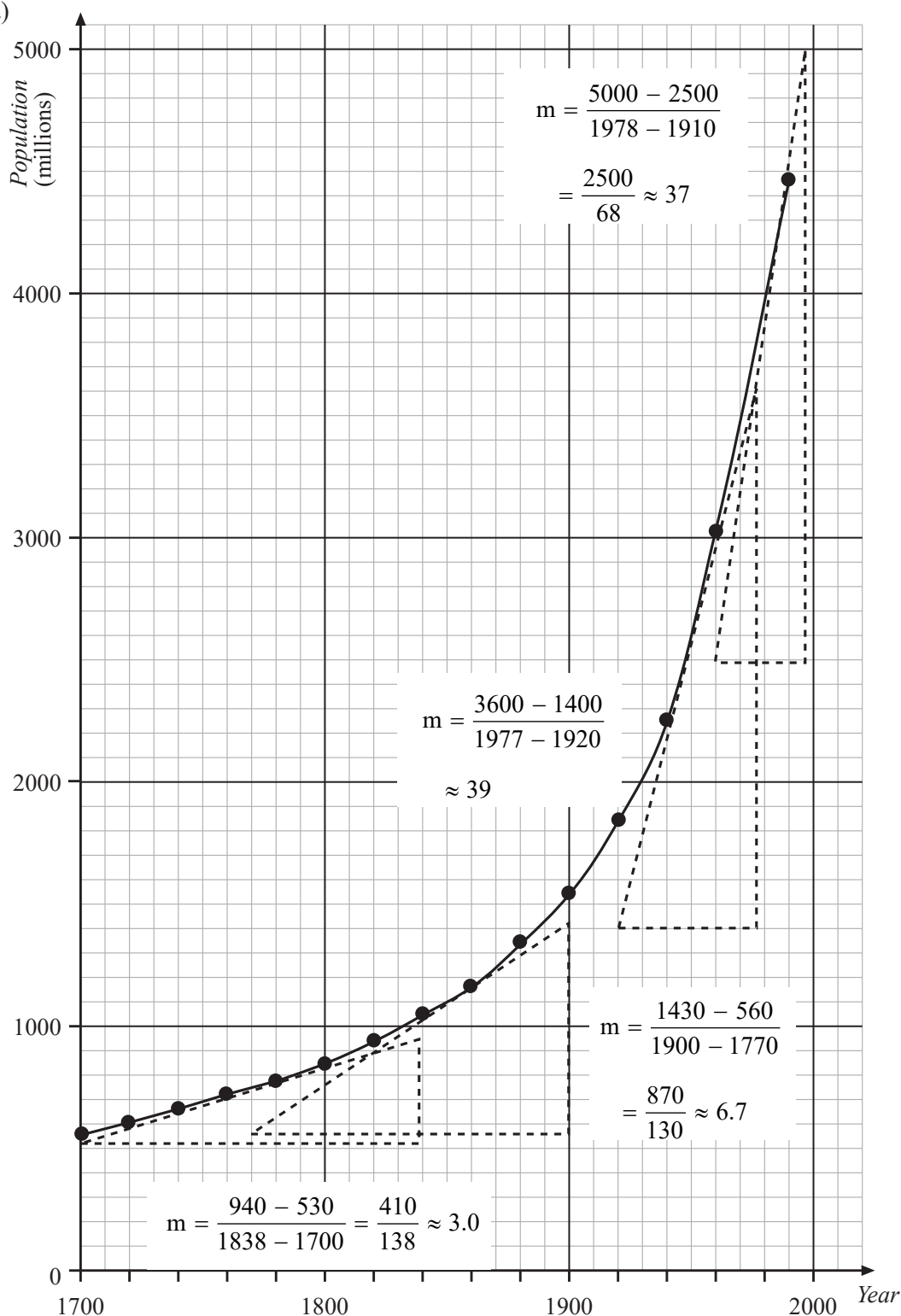
(iii) At 55 mins, instantaneous speed $= \frac{67 - 44}{60 - 44} = \frac{23}{16} \approx 1.4$ miles per min

(≈ 85 mph)

(b) Up to about 20 mins, the speed limit is being broken, and again from time 50 mins.

2.

(a)



Unit 4 *Rates of Change*

Text Answers

4.2

1750 : about 3 million per year

1850 : about 7 million per year

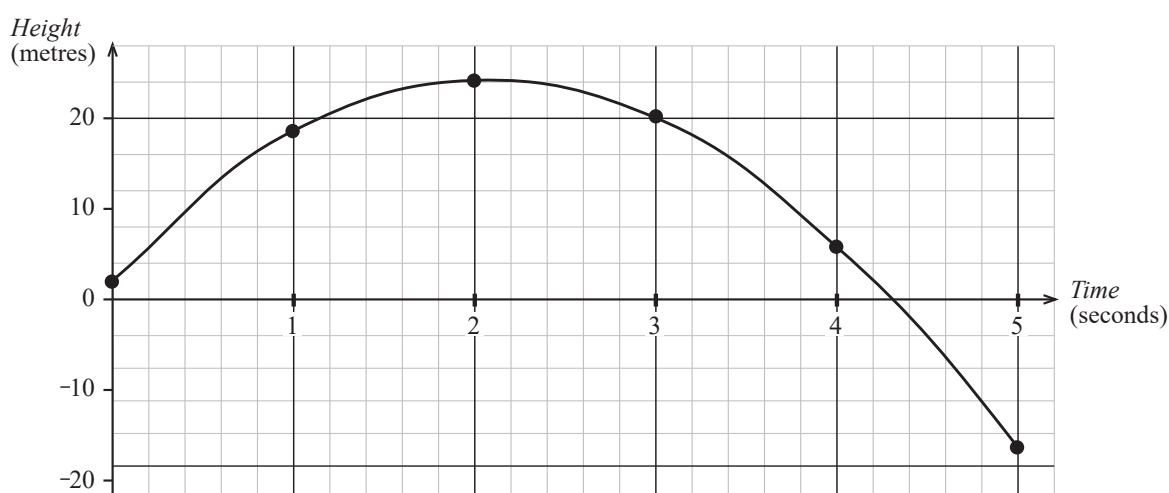
1950 : about 40 million per year

(b) The rate of increase of the world's populations is increasing so that the population is growing in an exponential way.

(c) In 1980, the gradient is about 37 million per year so the population estimate for 1981 is $4480 + 37 = 4517$ million.

3. (a)

Time, t (seconds)	0	1	2	3	4	5
Height, h (metres)	2	16	24	20	6	-18



(b) Velocity estimates 10, 0, -10, -20 (m/s)

(c) Estimate is $18 + 10 \times 0.1 = 19$, actual value = 19.05

4.3 Finding the Gradient

1. (a) $(1 + h, (1 + h)^3)$ (b) $\frac{(1 + h)^3 - 1}{h} = 3 + 3h + h^2$

(c) gradient $\rightarrow 3$ (d) gradient = 3

2. (a) $(2 + h, (2 + h)^3)$ (b) $\frac{(2 + h)^3 - 2^3}{h} = 12 + 6h + h^2$

(c) gradient $\rightarrow 12$ (d) gradient = 12

Unit 4 *Rates of Change*

Text Answers

4.4 Gradient of Quadratics

- (a) $4x$ (b) $2x+1$ (c) $2t+4$ (d) $2x-1$
 (e) $12l$ (f) $10 - \frac{2x}{5}$ (g) $\frac{2y}{9} + 3$ (h) $\frac{2n-5}{2}$
 (i) $1-12v$ (j) $\frac{3}{2}x + \frac{1}{5}$
- (a) -7 (b) 5 (c) $23\frac{5}{6}$ (d) $\frac{9}{4}$ (e) 71
- (a) 16 ms^{-1} (b) -7 ms^{-1}

4.5 Differentiation

- (a) $3x^2 + 10x + 3$ (b) $18t^2 - 20t + 2$ (c) $10x - \frac{1}{x^2}$ (d) $3x^2 - 1$ (e) $\frac{3t^2 + 3}{5}$
- (a) $2(x+2)$ (b) $3x^2 - 1$ (c) $3s^2 + \frac{4}{3}s + \frac{1}{9}$
 (d) $\frac{8}{3}y^2 + \frac{2}{3}y$ (e) $3x^2 - 5 + \frac{1}{x^2}$
- (a) 9 (b) $\frac{13}{4}$ (c) $(2, 19)$ (d) $(-1, 18)$
 (e) $(2, 41), (1, 18)$
- (a) $\frac{1}{3}$ (b) $\frac{3}{16}$ (feet per year)

4.6 Optimisation

- $(-2, 55)$ max; $(6, -201)$ min
- (a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ min (b) $(3, 18)$ min, $(-3, -18)$ max
 (c) $(-7, -469)$ min, $(5, 395)$ max (d) $(2, 12)$ min
- (a) $U = 5Q + 180 + \frac{12500}{Q}$ (b) $50, 680$
- (a) 47.6 mph (b) These are the usual speeds for a car.

4.7 Real Problems

- 5 m^2
- (a) 13.33 m^2 (b) 7.59 m^2
- 20 m
- 120 m
- (a) $V = hx^2$ (b) $15000 = 4xh + 2x^2$
 $\Rightarrow h = \frac{3750}{x} - \frac{x}{2}$

Unit 4 *Rates of Change*

Text Answers

4.7

$$(c) \quad V = \left(\frac{3750}{x} - \frac{x}{2} \right) x^2 = 3750x - \frac{x^3}{2}$$

$$\frac{dV}{dx} = 3750 - \frac{3x^2}{2} = 0 \Rightarrow x^2 = 2500 \Rightarrow x = 50$$

$$\text{and } V = 125\,000 \text{ cm}^3 = 125 \text{ litres}$$

$$6. \quad 120, 239, 159 \text{ mm}$$

$$7. \quad S = 2\pi r^2 + 2\pi rh \text{ where } V = \pi r^2 h$$

$$\text{So } S = 2\pi r^2 + \frac{2V}{r}$$

$$\frac{dS}{dr} = 4\pi r - \frac{2V}{r^2} = 0 \Rightarrow r^3 = \frac{V}{2\pi} \Rightarrow r = \left(\frac{V}{2\pi} \right)^{\frac{1}{3}}$$

$$h = \frac{V}{\pi r^2} = \frac{V}{\pi} \left(\frac{V}{2\pi} \right)^{-\frac{2}{3}} = 2 \left(\frac{V}{\pi} \right)^{\frac{1}{3}} \Rightarrow h = 2r \quad (= \text{diameter})$$

$$8. \quad (a) \quad v = 3t^2 - 24t + 45$$

$$(b) \quad t = 3 \text{ s or } 5 \text{ s} \quad (c) \quad t = 4 \text{ s}$$

$$9. \quad 3$$