

Unit 4 *Rates of Change*

Introduction

Learning objectives

In this unit we begin the development of an important branch of mathematics, namely calculus, based on consideration of finding rates of change. After completing this unit you should

- be able to find the gradient of curves by measurement
- understand the difference between instantaneous speed and average speed
- be able to use straightforward differentiation to find gradients of curves
- be able to solve optimisation problems based on differentiation.

Notes

Calculus, known in early history as *infinitesimal calculus*, is based on finding limits to determine the rates of change of functions. Both Isaac Newton and Gottfried Leibniz, building on previous studies, worked independently on developing calculus. However, in a dispute that continued until the end of their lives, each claimed that the other stole had stolen his work.

Here we concentrate on the application of finding the slope of graphs to determine the speed and acceleration of moving objects.

Facts to remember

- Average speed = $\frac{\text{total distance}}{\text{time taken}}$
- Instantaneous speed is given by the gradient of a distance-time graph at a particular time
- The gradient of the quadratic function

$$y = ax^2 + bx + c \text{ is given by } 2ax + b$$

- Using calculus notation, if $y = ax^2 + bx + c$, then its differential is

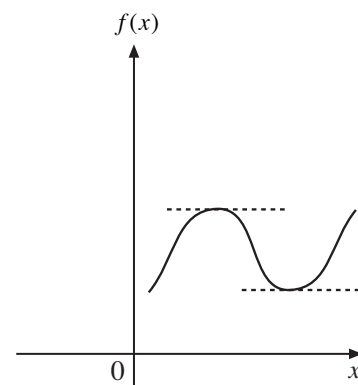
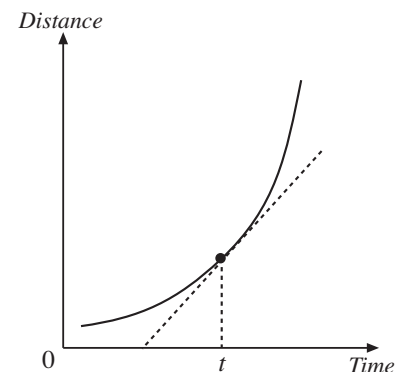
$$\frac{dy}{dx} = 2ax + b$$

- If $y = x^3$, $\frac{dy}{dx} = 3x^2$

- If $y = \frac{1}{x}$, $\frac{dy}{dx} = -\frac{1}{x^2}$

- The maximum (or minimum) of a function $f(x)$ is given by

$$\frac{df}{dx} = 0 \text{ (i.e. gradient is zero)}$$



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Glossary of Terms

- *Instantaneous speed:* the gradient of a time-distance graph at a particular time
- *Derivative:* the derivation of a function $f(x)$ is the gradient of function, denoted by $\frac{df}{dx}$.
 $\frac{df}{dx}$ is known as the differential of the function $f(x)$.