

STRAND: NUMBER

Unit 1 *Scientific Notation*

TEXT

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1 Scientific Notation

1.1 Standard Form

Standard form is a convenient way to write very large or very small numbers. It is particularly useful when working with a scientific calculator.

In standard form a number is written as

$$a \times 10^n \text{ where } 1 \leq a < 10 \text{ and } n \text{ is an integer}$$

That is, a is a number greater than or equal to 1 and less than 10, and n is a positive or negative whole number.



Note

Because of the way that the powers of 10 are used in standard form, it is important to remember that:

$$\begin{array}{lll} 10^4 = 10\,000 & \text{and} & 10^{-4} = 0.0001 \\ 10^3 = 1000 & & 10^{-3} = 0.001 \\ 10^2 = 100 & & 10^{-2} = 0.01 \\ 10^1 = 10 & & 10^{-1} = 0.1 \\ 10^0 = 1 & & \end{array}$$



Worked Example 1

Write each of the following numbers in standard form.

(a) 346 000 000 (b) 2710 (c) 0.000 543



Solution

$$\begin{aligned} \text{(a)} \quad 346\,000\,000 &= 3.46 \times 100\,000\,000 \\ &= 3.46 \times 10^8 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 2710 &= 2.71 \times 1000 \\ &= 2.71 \times 10^3 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 0.000\,543 &= 5.43 \times 0.0001 \\ &= 5.43 \times 10^{-4} \end{aligned}$$



Worked Example 2

Find

$$\begin{array}{ll} \text{(a)} \quad 4.21 \times 10^2 = & \text{(b)} \quad 3.1 \times 10^5 = \\ \text{(c)} \quad 3.6 \times 10^{-2} = & \text{(d)} \quad 4.7 \times 10^{-3} = \end{array}$$

1.1

**Solution**

$$\begin{aligned} \text{(a)} \quad 4.21 \times 10^2 &= 4.21 \times 100 \\ &= 421 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3.1 \times 10^5 &= 3.1 \times 100\,000 \\ &= 310\,000 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 3.6 \times 10^{-2} &= 3.6 \times 0.01 \\ &= 0.036 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 4.7 \times 10^{-3} &= 4.7 \times 0.001 \\ &= 0.0047 \end{aligned}$$

**Worked Example 3**

Write each of the following numbers in normal decimal form.

$$\text{(a)} \quad 3.217 \times 10^3 \qquad \text{(b)} \quad 3.68 \times 10^5 \qquad \text{(c)} \quad 4.7 \times 10^{-4}$$

**Solution**

$$\begin{aligned} \text{(a)} \quad 3.217 \times 10^3 &= 3.217 \times 1000 \\ &= 3217 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3.68 \times 10^5 &= 3.68 \times 100\,000 \\ &= 368\,000 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 4.7 \times 10^{-4} &= 4.7 \times 0.0001 \\ &= 0.00047 \end{aligned}$$

Most calculators can work with numbers in standard form. On a calculator display 3.01×10^{17} would look like

**Worked Example 4**

Calculate $366 \div 0.0012$ and write the answer

$$\text{(a)} \quad \text{exactly,} \qquad \text{(b)} \quad \text{in standard form.}$$

1.1

**Solution**

$$\begin{aligned}
 \text{(a)} \quad 366 \div 0.0012 &= \frac{366}{0.0012} \\
 &= \frac{3\,660\,000}{12} \\
 &= 305\,000
 \end{aligned}$$

$$\text{(b)} \quad 305\,000 = 3.05 \times 10^5 \text{ in standard form.}$$

**Exercises**

1. Write each of the following numbers in standard form.

- | | | |
|-------------|--------------------|-------------------|
| (a) 47 000 | (b) 52 100 | (c) 32 000 000 |
| (d) 324 100 | (e) 420 | (f) 81 000 |
| (g) 5000 | (h) 47 000 000 000 | (i) 3 200 000 000 |
| (j) 0.00062 | (k) 0.0571 | (l) 0.0000002 |
| (m) 0.124 | (n) 0.0371 | (o) 0.00021 |
| (p) 0.00007 | (q) 0.471 | (r) 0.0003 |

2. Write each of these numbers in standard form.

- | | | |
|------------------|-----------------|-----------------|
| (a) 1 million | (b) 15 thousand | (c) 6.4 million |
| (d) 30.4 million | (e) 4 million | (f) 0.4 million |

3. Write each of the following numbers using normal decimal notation.

- | | | |
|--------------------------|----------------------------|---------------------------|
| (a) 6×10^5 | (b) 4.31×10^2 | (c) 5.86×10^7 |
| (d) 8.3×10^{-4} | (e) 4.172×10^3 | (f) 6.42×10^{-5} |
| (g) 4.7×10^1 | (h) 3.2×10^{-1} | (i) 8.47×10^{-4} |
| (j) 3.34×10^8 | (k) 3.471×10^{-4} | (l) 8.421×10^2 |
| (m) 1.675×10^1 | (n) 8.4×10^{-6} | (o) 7.12×10^{-4} |

4. For each of the numbers below state whether or not it is in standard form. If it is not in standard form, write it in standard form.

- | | | |
|-------------------------|--------------------------|---------------------------|
| (a) 3.2×10^8 | (b) 43.2×10^2 | (c) 15.6×10^{-8} |
| (d) 0.4×10^3 | (e) 1.3×10^{-8} | (f) 0.7×10^{-4} |
| (g) 5.471×10^2 | (h) 54.71×10^3 | (i) 8.21×10^6 |

1.1

5. Give the answers to the following calculations in standard form.
- (a) $2000 \times 30 =$ (b) $4000^2 =$ (c) $50^3 =$
 (d) $\frac{4}{1000} =$ (e) $\frac{6}{3000} =$ (f) $0.04^2 =$
 (g) $0.004 \times 0.7 =$ (h) $22 \times 400 =$ (i) $\frac{18}{20000} =$
 (j) $30^2 =$ (k) $0.02^2 =$ (l) $100^2 =$
6. There are 1000 m in 1 km. Convert the following distances to metres, giving your answers in standard form.
- (a) 50 km (b) 620 km (c) 1456 km
7. Find the number of:
- (a) hours in a year (b) minutes in a week (c) seconds in a day,
 giving your answers in standard form.
8. The radius of the earth is 6.37×10^3 km.
- (a) Write this in a normal decimal form.
 (b) Find the radius of the earth in metres and express it in both decimal form and standard form.
 (c) Find the circumference of the earth in metres, giving the answer in standard form.
9. The mass of the earth is 5.9×10^{24} kg. Write this as a decimal number.
10. The width of a thin strip of metal is $\frac{3}{100}$ mm. Write this in standard form.
11. Scientists estimate the mass of a newly discovered planet as 482 000 000 kg. Write this in standard form.
12. The distance of the earth from the sun varies between 1.53×10^8 km and 1.47×10^8 km.
- (a) Write these numbers in a decimal format.
 (b) Convert both distances to metres and write them in standard form.
13. (a) Using a calculator, or otherwise, determine the exact value of
- (i) $2.3^2 + 4.1^2$ (ii) $\frac{0.18}{0.6} - 0.003$ (iii) $\frac{3\frac{1}{3} - 2\frac{3}{5}}{2\frac{1}{5}}$
- (b) (i) Write your answer in part (a) (i) correct to one significant figure.
 (ii) Write your answer in part (a) (ii) in standard form.

1.2 Calculations with Standard Form

When using standard form it is possible to multiply and divide numbers, taking advantage of the form in which they are written.

Note that we can use the rules

$$10^a \times 10^b = 10^{a+b}$$

$$10^a \div 10^b = 10^{a-b}$$

where a and b are numbers.



Worked Example 1

Find $4 \times 10^{18} \times 3 \times 10^4$



Solution

To do this calculation, you multiply together the 4 and the 3 and then multiply together the 10^{18} and the 10^4 .

$$\begin{aligned} 4 \times 10^{18} \times 3 \times 10^4 &= 4 \times 3 \times (10^{18} \times 10^4) \\ &= 12 \times 10^{22} \end{aligned}$$

This result is not in standard form so the final stage is to convert the result to standard form.

$$\begin{aligned} 12 \times 10^{22} &= 1.2 \times 10 \times 10^{22} \\ &= 1.2 \times 10^{23} \end{aligned}$$

$$4 \times 10^{18} \times 3 \times 10^4 = 1.2 \times 10^{23}$$



Worked Example 2

Find

(a) $3.2 \times 10^4 \times 5 \times 10^{-3}$

(b) $(6 \times 10^8) \div (3 \times 10^4)$

(c) $(7.2 \times 10^3) \div (6 \times 10^4)$



Solution

(a) Multiply together the 3.2 and the 5 and then multiply together the 10^4 and the 10^{-3} .

$$\begin{aligned} 3.2 \times 10^4 \times 5 \times 10^{-3} &= 3.2 \times 5 \times 10^4 \times 10^{-3} \\ &= 16.0 \times 10^1 \end{aligned}$$

This number is not in standard form so converting gives

$$\begin{aligned} 16.0 \times 10^1 &= 1.6 \times 10 \times 10^1 \\ &= 1.6 \times 10^2 \end{aligned}$$

1.2

- (b) Division follows a similar approach to multiplication. First divide 6 by 3 and then divide 10^8 by 10^4 .

$$\begin{aligned}(6 \times 10^8) \div (3 \times 10^4) &= (6 \div 3) \times (10^8 \div 10^4) \\ &= 2 \times 10^4\end{aligned}$$

This result is in standard form so no further work is required.

- (c) First divide 7.2 by 6 and then divide 10^3 by 10^4 .

$$\begin{aligned}(7.2 \times 10^3) \div (6 \times 10^4) &= (7.2 \div 6) \times (10^3 \div 10^4) \\ &= 1.2 \times 10^{-1}\end{aligned}$$

This result is in standard form.

Problems can be done directly on a calculator, or by entering numbers using the EE or EXP keys.



Exercises

1. Do the following calculations, making sure that your answer is in standard form.
Do not use a calculator.

(a) $3 \times 10^8 \times 2 \times 10^4 =$	(b) $2 \times 10^5 \times 4 \times 10^3 =$
(c) $9 \times 10^6 \times 1 \times 10^{10} =$	(d) $5 \times 10^3 \times 4 \times 10^8 =$
(e) $6 \times 10^3 \times 4 \times 10^{11} =$	(f) $3 \times 10^{-2} \times 4 \times 10^8 =$
(g) $1.2 \times 10^6 \times 2.4 \times 10^5 =$	(h) $1.1 \times 10^6 \times 2 \times 10^{-4} =$
(i) $8.1 \times 10^8 \times 7.2 \times 10^{-2} =$	(j) $5.2 \times 10^3 \times 1.3 \times 10^{-7} =$
(k) $6.2 \times 10^{-3} \times 2.1 \times 10^{-6} =$	(l) $1.8 \times 10^{-4} \times 2.5 \times 10^{-9} =$

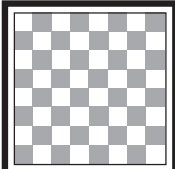
2. Give the answers to the following calculations in standard form.
Do not use a calculator.

(a) $(8 \times 10^6) \div (2 \times 10^2) =$	(b) $(9 \times 10^5) \div (3 \times 10^2) =$
(c) $(8 \times 10^4) \div (4 \times 10^2) =$	(d) $(1.6 \times 10^5) \div (2 \times 10^2) =$
(e) $(3.6 \times 10^8) \div (3 \times 10^2) =$	(f) $(4.8 \times 10^{12}) \div (4 \times 10^3) =$
(g) $(8.1 \times 10^4) \div (3 \times 10^5) =$	(h) $(4.5 \times 10^3) \div (9 \times 10^{-5}) =$
(i) $(1.64 \times 10^8) \div (4 \times 10^{-12}) =$	(j) $(1.32 \times 10^5) \div (1.2 \times 10^{-3}) =$
(k) $(9.6 \times 10^4) \div (3.2 \times 10^{-5}) =$	(l) $(1.21 \times 10^{-4}) \div (1.1 \times 10^6) =$

1.2

3. Do the following using a calculator, giving your answers in standard form.
- | | |
|---|--|
| (a) $(4.2 \times 10^6)^2 =$ | (b) $(3.7 \times 10^{-2})^2 =$ |
| (c) $(1.2 \times 10^{-5})^3 =$ | (d) $\sqrt{8.1 \times 10^8} =$ |
| (e) $6.2 \times 10^8 \times 1.2 \times 10^{14} =$ | (f) $3.8 \times 10^4 \times 4.1 \times 10^{-12} =$ |
| (g) $(1.84 \times 10^6) \div (1.92 \times 10^7) =$ | (h) $\frac{4.7 \times 10^8}{3.2 \times 10^5} =$ |
| (i) $\frac{1.62 \times 10^{-5}}{3.2 \times 10^4} =$ | (j) $\sqrt{\frac{3 \times 10^8}{5 \times 10^3}} =$ |
| (k) $4.8 \times 10^{11} + 3.2 \times 10^{10} =$ | (l) $6.8 \times 10^{12} - 4.7 \times 10^{10} =$ |
4. There are 8.64×10^4 seconds in one day. How many seconds are there in:
- | | | |
|-------------|------------|-------------|
| (a) 10 days | (b) 1 week | (c) 1 year? |
|-------------|------------|-------------|
5. The mass of an electron is 9.1×10^{-31} kg. Find the mass of:
- | | | |
|----------------------------------|----------------------------------|--------------------------------|
| (a) 3×10^{18} electrons | (b) 4×10^{32} electrons | (c) 7×10^8 electrons. |
|----------------------------------|----------------------------------|--------------------------------|
6. A rectangle has sides of length 3×10^5 mm and 4.2×10^6 mm. Find the area of the rectangle.
7. The speed of sound is 3.32×10^2 ms⁻¹.
- | |
|---|
| (a) How far would the noise from a 'bang' travel in: |
| (i) 10 seconds (ii) 3×10^3 seconds (iii) 4×10^{-2} seconds? |
| (b) How long would it take the noise from a 'bang' to travel: |
| (i) 10 metres (ii) 2×10^3 metres (iii) 2×10^{-2} metres? |
8. The speed of light is 3×10^8 ms⁻¹.
- | |
|--|
| (a) How far would light travel in 100 seconds? |
| (b) The mean distance of the earth from the sun is 1.5×10^{11} m.
How long does it take for light to travel from the sun to the earth? |
9. The distance from the earth to the moon is 3.84×10^5 km.
- | |
|---|
| (a) Find this distance in metres. |
| (b) How long would it take a spaceship to travel to the moon from earth if its average speed was 400 ms ⁻¹ ? |

1.2

10. The density of air is $1.3 \times 10^{-6} \text{ kg/cm}^3$.
- How many cubic centimetres are in one cubic metre?
 - Find the mass of 1 m^3 of air.
 - Find the volume of air that will have a mass of 3 grams.
 - The density of hydrogen is $9 \times 10^{-8} \text{ kg/cm}^3$.
Repeat (b) and (c) for hydrogen.
11. The population of the world was estimated to be 6.7×10^9 in 2008. If the population increases by 3% each year, estimate the population at the beginning of the year 2020.
12. (a) The approximate population of Jamaica in 2008 is given in standard form as 2.8×10^6 . Write this as an ordinary number.
- (b) The thickness of grade A paper is $6.0 \times 10^{-2} \text{ cm}$. Grade B paper is twice as thick as grade A.
Calculate, in centimetres, the thickness of grade B paper. Write your answer in standard form.
13. Between 1970 and 2005 the number of people living in towns and cities in a group of countries increased from 2.86×10^8 to 1.14×10^9 .
Calculate the increase in the number of people, giving your answer in standard form.
14.  Centuries ago, a man promised to give his wife some grains of rice. He took a chess board and placed one grain on the first square, two grains on the second square, four grains on the third square, eight grains on the fourth square, and so on.
If he had completed all 64 squares on the chess board he would have used approximately 1.845×10^{19} grains of rice.
One grain of rice weighs about 0.01 grams. Calculate an estimate of the weight of rice used. Give your answer in tonnes, correct to one significant figure.
[1 tonne = 1000 kg]
15. (a) Evaluate 1.2^8 .
- (b) Evaluate $(0.0009)^{\frac{1}{2}}$. Give your answer in standard form.
16. The mean distance of the earth from the sun is 149.6 million kilometres.
- (a) Write the number 149.6 million in standard index form.
The earth travels a distance, D km, in one day. The value of D is given by the formula
- $$D = \frac{2\pi \times \text{distance of earth from sun}}{365}$$
- (b) Calculate the value of D , giving your answer in standard index form.

1.2

17. The number 10^{100} is called a googol.

(a) Write the number, 50 googols, in standard index form.

A nanometre is 10^{-9} metres.

(b) Write 50 nanometres in metres.

Give your answer in standard index form



Challenge!

In astronomy, the distance between stars is measured in light years, which is the distance travelled by light in a year.

One light year = $3 \times 10^5 \times 60 \times 60 \times 24 \times 365$ km.

This is approximately 9 460 800 000 000 km.

How long would it take for light to travel from the Sun to the Earth if their distance apart is 1.5×10^8 km?



Investigation

Han Sin, a Chinese general, devised a method to count the number of soldiers that he had. First, he ordered his soldiers to form groups of 3, followed by groups of 5 and then groups of 7.

In each case he noted down the remainder. Using the three remainders, he was able to calculate the exact number of soldiers he had without doing the actual counting.

Do you know how he did it?