

Unit 2 *Indices and Factors*

Introduction

Learning objectives

The work in this unit focuses on the use of indices to simplify algebraic expression and its applications for expressing numbers in standard form (sometimes known as scientific notation).

After completing this unit, you should

- be familiar with the notation for square, square root, cube and cube root
- be able to extend this concept to negative and fractional powers
- understand and be able to use the rules of indices
- understand what is meant by a factor
- be able to express any positive integer as a product of prime factors
- be able to find the highest common factor (HCF) and lowest common multiple (LCM) of two positive integers.

Key points

- Laws of indices $\left(a^m \times a^n = a^{m+n}, a^m \div a^n = a^{m-n}, (a^n)^m = a^{nm}\right)$
- Factors and prime factors; any positive integer can be written uniquely as a product of its prime factors.
- Highest common factor (HCF) and lowest common multiple (LCM) of positive integers.

Facts to remember

- $a^n = a \times a \times a \times \dots \times a$ (n times)
- $a^1 = a$
- $a^0 = 1$
- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^n)^m = a^{nm}$
- $a^{-n} = \frac{1}{a^n}$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$

Unit 2 *Indices and Factors*

Introduction

Glossary of Terms

Squares and cubes

The square of x is x^2 ($= x \times x$) For example, $5^2 = 25$

The cube of x is x^3 ($= x \times x \times x$) For example, $5^3 = 125$

Square roots and cube roots

The square root of x , denoted by $\sqrt{x}$, is such that $\sqrt{x} \times \sqrt{x} = x$

For example, $4 \times 4 = 16$, so $\sqrt{16} = 4$

The cube root of x , denoted by $\sqrt[3]{x}$, is such that $\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = x$

For example, $2 \times 2 \times 2 = 8$, so $\sqrt[3]{8} = 2$

Index notation

We denote $\underbrace{x \times x \times \dots \times x}_{n \text{ terms}}$ by x^n and call this " x to the power of n ".

For example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$ and $5^3 = 5 \times 5 \times 5 = 125$

Factors

The factor of a positive whole number is any positive whole number that will divide exactly into it.

For example, the factors of 18 are 1, 3, 6, 9 and 18.

Prime numbers

Any positive whole number with exactly two factors; 1 and itself.

For example, 7 ($= 1 \times 7$) is only divisible exactly by 1 and 7.

Prime factors

Every factor that is a prime number is a prime factor. Any positive whole number can be written as the product of prime factors.

For example, $60 = 2 \times 2 \times 3 \times 5$

Coprime

Two or more numbers are *coprime* if they have no factors in common other than 1.

For example, 6 and 35 are coprime, but 6 and 27 are *not* coprime because they are both divisible by 3.

The number 1 is coprime to every integer.

Unit 2 *Indices and Factors*

Introduction

Highest common factor (HCF)

The highest common factor of two positive integers is the largest number which is a factor of both.

For example, the factors of 12 are 1, 2, 3, 4, 6, 12. The factors of 15 are 1, 3, 5, 15.

1 and 3 are the only common factors; the HCF of 12 and 15 is 3.

Lowest common multiple (LCM)

The lowest common multiple of two positive integers is the lowest number which both the numbers can divide into exactly.

For example, the LCM of 5 and 7 is 35; the LCM of 15 and 12 is 60.

Reciprocal

We write $a^{-1} = \frac{1}{a}$ and this is the reciprocal of a .

For example, $4^{-1} = \frac{1}{4}$, $\left(\frac{1}{3}\right)^{-1} = 3$

Negative powers

Similarly, $a^{-n} = \frac{1}{a^n}$

For example, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, $\left(\frac{1}{5}\right)^{-2} = 5^2 = 25$