

Unit 1 *Probability of One Event*

Introduction

Learning objectives

This is a really important unit, the first of three that deal with *probability*. After completing this unit you should

- understand how to classify probabilities using words such as CERTAIN, LIKELY, UNLIKELY, IMPOSSIBLE and illustrate probabilities on a probability line
- be able to calculate simple probabilities
- be able to make estimates of probabilities using relative frequency
- be confident in determining the probability of an outcome when all possibilities are equally likely to occur.

Introduction

Probability owes its origins to gambling but is now defined as 'the chance or likelihood of something happening' and is of real importance in today's commercial world.

Gambling is an activity that has been practised by humankind for many centuries. Archeologists have found evidence of games of chance being played in Egypt as early as 3500 BC, and six-faced dice dating from about 3000 BC have been found in Iraq and India.

However, it was not until about 1500 AD that any real thought was given to the probabilities which underlie any game of chance, and the work done was very fragmented – more in the nature of speculation than reasoned argument. The generally accepted view of the beginnings of probability are these:

In about 1650, the *Chevalier de Méré*, who was an able and experienced French gambler, found that his own reasoning on the subject did not agree with his observations. Realising that it must be his reasoning which was wrong, he wrote to *Blaise Pascal* (1623-1662), a leading French mathematician at that time, and explained his dilemma. They considered the problems:

"Are you more likely to obtain one six in 4 tosses of a die than to obtain at least one double six in 24 tosses of two dice?"

The now famous exchange of correspondence between Pascal and another French mathematician, *Pierre de Fermat* (1601-1665) took place: these letters are now accepted as being the beginning of the theory of probability. One of the earlier (and easier) problems they dealt with was,

"How many times must one throw a pair of dice before expecting to get a double six?"

Blaise Pascal



Pierre de Fermat

Other problems considered at this time included,

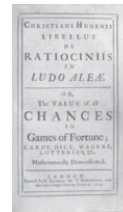
"Are you more likely to obtain a total of 9 when three fair dice are tossed, than a total of 10?" (*Galileo and Duke of Tuscany*)

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"Which is more likely: one 6 when six dice are tossed or two 6s when 12 dice are tossed?" (*Pepys to Newton*)

Christiaan Huygens (1629-1695), a Dutch mathematician, on a visit to Paris, learnt of the correspondence and, as a result, in 1657 published his own book. 'De Ratiociniis in Ludo Aleae' (On reasoning in dice games) – the first published work of importance on probability.



The cover and title page of Huygens' publication

More recently, probability forms the basis of risk-taking, which concerns all large companies in their business transactions. In the decade before the financial crisis of 2008, many banks and money lenders had taken excessive risks and when the financial markets lost confidence and share prices around the world tumbled, these companies were either forced to cease trading or were 'bailed out' (with financial help) by their governments. Many of these organisations had ignored the advice given by their statisticians. It was not that the probability analyses were incorrect but simply that human greed to make greater and greater profits caused people to ignore the warnings.

In addition to being the basis of risk analysis, probability is also fundamental to Genetics, especially in recent developments in genetic fingerprinting in criminal cases and paternity disputes and in the process of comparing different medical treatments.

Key points

- The probability of any outcome, p , must satisfy $0 \leq p \leq 1$.
- The sum of the probabilities of all outcomes must equal 1.
- Probabilities can be illustrated on a probability line: 

Facts to remember

- For finding probabilities by experiment:

$$\text{probability of event} = \frac{\text{frequency of event}}{\text{total frequency}}$$

- For equally likely outcomes:

$$\text{probability of particular event} = \frac{\text{number of ways of obtaining event}}{\text{total no. of possible outcomes}}$$

For example, when throwing a fair die,

$$p(6) = \frac{1}{6}, \quad p(\text{prime number}) = p(2, 3 \text{ or } 5) = \frac{3}{6} = \frac{1}{2}$$

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Glossary of terms

<i>Outcomes</i>	events that can occur after an experiment (for example, obtaining a '6' when throwing a die)
<i>Sample space</i>	for an experiment is the set of all possible outcomes (for example, {1, 2, 3, 4, 5, 6} when throwing a die)
<i>Event</i>	subject of the sample space (for example, obtaining even numbers when throwing a die)
<i>Relative frequency</i>	the frequency of an event divided by the total frequency; this is used as an estimate for the probabilities of that event
<i>Independent events</i>	when the result of one event happening does not affect the probability of the other
<i>Fair or unbiased die</i> (coin, spinner, etc.)	- every face (side) has an equal chance
<i>Biased die</i> (coin, spinner, etc.)	- all outcomes are not equally likely to occur. For example, a weighted die.

(Note that the word '*dice*' is the plural of '*die*', but is often used as a singular.)