

Unit 4 *Risk*

Text Answers

4.1 Expected Values

1. (a) $\frac{1}{4} \times 10 = 2\frac{1}{2}$

(b)	Correct	Incorrect	Mark
	10	0	10
	9	1	$8\frac{1}{2}$
	8	2	7

	7	3	$5\frac{1}{2}$

So need

$$p(X \geq 8) = p(X = 8) + p(X = 9) + p(X = 10)$$

when $X \sim \text{B in } \left(10, \frac{1}{4}\right)$ as predictability of guessing correctly is $\frac{1}{4}$ for each question.

$$p(X \geq 8) = {}^{10}C_8 \left(\frac{1}{4}\right)^8 \left(\frac{3}{4}\right)^2 + {}^{10}C_9 \left(\frac{1}{4}\right)^9 \left(\frac{3}{4}\right)^1 + \left(\frac{1}{4}\right)^{10}$$

$$\approx 0.000336$$

2. Outcomes:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$p(\text{odd}) = \frac{18}{36} = \frac{1}{2} \qquad p(\text{even}) = \frac{18}{36} = \frac{1}{2}$$

So the game is 'neutral'. In the long run, profit will be close to zero, so there is no real incentive for you to play this game.

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3. (a) Outcomes:

	1	2	3	4	5	6	Outcomes mostly 2 or 5 or both
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	2
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	6
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	2
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	2
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	6
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	2
Total =							20

So of the 36 possible outcomes, 20 are 2 or 5 (or both).

$$\text{Hence, } p(\text{Option 1}) = \frac{20}{36} = \frac{5}{9} \text{ and } p(\text{Option 2}) = \frac{4}{9}$$

So you should choose *Option 1*.

$$\begin{aligned} \text{(b) Expected winnings} &= 10 \times \left(\frac{5}{9} \times £5 \right) \\ &= \frac{£250}{9} \\ &\approx £27.78 \end{aligned}$$

4.2 Gambling

1. (a) $6 \times \frac{1}{2^4} = \frac{3}{8}$ (Note that you could draw all the possible winning routes and all the non-winning ones in order to calculate the probability.)

- (b) You can summarise the number of successful routes from each square in the first table on the right.

Finish	①	①
①	②	③
①	③	⑥

This generalises to a 4×4 square, as in the second table on the right.

$$\text{Hence } p(\text{winning}) = 20 \times \frac{1}{2^6} = \frac{5}{16}$$

Finish	①	①	①
①	②	③	④
①	③	⑥	⑩
①	④	⑩	⑳

2. $p(\text{at least 2 with birthdays on same day}) = 1 - \left(\frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \dots \times \frac{346}{365} \right)$
 ≈ 0.4114

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3. *Option A:* $p(6) = \frac{1}{6}$ when throwing one dice.

So if X = no. of sixes and $n = 6$, $X \sim B \text{ in } \left(6, \frac{1}{6}\right)$

$$\begin{aligned} p(X \geq 1) &= 1 - p(X = 0) \\ &= 1 - \left(\frac{5}{6}\right)^6 \\ &\approx 0.665 \end{aligned}$$

Option B: Again let X = no. of sixes and $n = 12$, $X \sim B \text{ in } \left(12, \frac{1}{6}\right)$

$$\begin{aligned} p(X \geq 2) &= 1 - (p(x = 1) + p(x = 0)) \\ &= 1 - \left(12 \times \left(\frac{5}{6}\right)^{11} \times \frac{1}{6} + \left(\frac{5}{6}\right)^{12}\right) \\ &= 0.619 \end{aligned}$$

So *Option A* is the better bet!

4. (a)	<i>Symbol</i>	<i>Dial 1</i>	<i>Dial 2</i>	<i>Dial 3</i>
	BAR	2	1	1
	ORANGE	1	8	7
	PINEAPPLE	7	2	3
	BELL	8	2	4
	CHERRIES	2	7	0
	SEVEN	0	0	5

(b) (i) 2 (ii) $20^3 = 8000$ (iii) $p = \frac{1}{4000}$

(c) 3 ORANGES $p = \frac{56}{8000} = \frac{7}{1000}$

3 PINEAPPLES $p = \frac{42}{8000} = \frac{21}{4000}$

3 BELLS $p = \frac{64}{8000} = \frac{1}{125}$

2 BARS $p = \frac{(2 \times 1 \times 19 + 2 \times 19 \times 1 + 18 \times 1 \times 1)}{8000} = \frac{94}{8000} = \frac{47}{4000}$

2 CHERRIES $p = \frac{280}{8000} = \frac{7}{200}$

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$$\begin{aligned} \text{(d) Expected winnings} &= 40 \times \frac{1}{4000} + 5 \times \frac{7}{1000} + 5 \times \frac{21}{4000} + 5 \times \frac{1}{125} + 20 \times \frac{47}{4000} + 5 \times \frac{7}{200} \\ &= \frac{417}{800} \text{ (10 p's)} \end{aligned}$$

$$\begin{aligned} \text{So average expected gain} &= \left(\frac{417}{800} - 1 \right) \times 10\text{p} \\ &= -4.7875\text{p} \quad (\approx -5\text{p}) \end{aligned}$$