

Unit 5 *Binomial Distribution*

Introduction

Learning objectives

This is an important extension to basic Probability. After studying this unit you should

- be able to recognise when to use the Binomial Distribution
- understand how to find the mean and variance of the distribution
- be able to apply the Binomial Distribution to a variety of problems.

Introduction

A random variable follows a Binomial Distribution when each trial has two possible outcomes, that is, success and failure. It was first studied in connection with games of chance.

The Swiss mathematician Jacob Bernoulli (1655 - 1705), in a proof published posthumously in 1713, determined that the probability of k outcomes in n repetitions is equal to the k^{th} term ($k = 0, 1, \dots, n$) in the expansion of the binomial expression $(p + q)^n$, where $q = 1 - p$ and p is the probability of success. The Binomial Distribution is applied to a wide range of contexts.

Key points

- The Binomial Distribution can be applied to finite trials of an experiment where the underlying probability of success remains constant
- The number of trials, usually denoted by n , is finite
- The underlying probability of success, p , remains constant throughout the trials
- If a random variable, X , follows a Binomial Distribution, then we write

$$X \sim \text{Bin}(n, p) \text{ or } X \sim B(n, p)$$

when n is the number of trials and p the underlying probability.

Facts to remember

- If $X \sim \text{Bin}(n, p)$, then

$$p(x = r) = \binom{n}{r} p^r q^{n-r}$$

where $q = 1 - p$ and $r = 0, 1, 2, \dots, n$

- $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
- $\binom{n}{0} = 1$, $\binom{n}{1} = n$, $\binom{n}{2} = \frac{n(n-1)}{2}$, etc and $\binom{n}{n} = 1$