Mathematics Enhancement Programme

TEACHING SUPPORT: Year 5

SOLUTIONS TO EXERCISES

1. *Question and Solution*

You have these number cards. $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{0}$ $\boxed{0}$ $\boxed{0}$

Use them to make, where possible, two different 6-digit numbers which are:

a) divisible by 10: 234 000, 324 000, etc. (36)

b) divisible by 10, but not by 100: 23 040, 24 030, etc. (18)

c) divisible by 100, but not by 10: impossible

d) not divisible by 10: 230 004, 240 003, etc. (24)

(p3, Q4)

Notes

For a), b) and d), there are many possibilities. An extension question for able students is to ask how many possible answers there are (36 in a), 18 in b) and 24 in d)). There are no possible solutions for c).

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2. *Question and Solution*

Write the units of measure that you know in the correct place in the table.

Number of times, or the fraction of, the basic unit	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
Units of length	kilometre km			metre (m)		cm	mm
Units of mass	kg			gram (g)			
Units of capacity				litre (ℓ)		cl	ml

(p4, Q1)

Notes

This clarifies the units that have already been covered in Year 4:

- for length (kilometre, metre, centimetre and millimetre)
- for mass (kilogram, gram)
- for capacity (litre, centilitre, millilitre)

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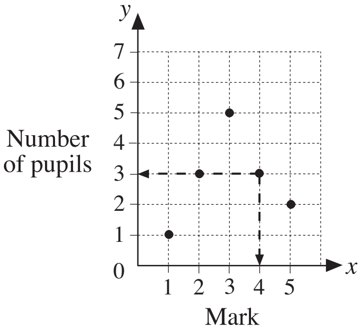
3. *Question and Solution*

The graph shows the marks scored by a class of 14 pupils in a test which had 5 marks in total.

For example, 3 pupils scored 4 marks, or 4 marks were scored by 3 pupils.

So this data point has coordinates (4, 3).



a) Complete the table.

Mark	1	2	3	4	5
Number of pupils	1	3	5	3	2

b) i) Which mark did most pupils score? This is the **mode**. $\boxed{3}$

ii) How many pupils scored it? $\boxed{5}$

c) List the marks of every pupil in increasing order in your exercise book.

1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5

d) Calculate the **mean** in your exercise book and write it here. $\boxed{3\frac{1}{7}}$

(p6, Q2)

Notes

Here we use coordinate axes to illustrate the data.

Part a) is straightforward.

In part b), the mode is the most frequently occurring number (or item); here that number is 3, as there are 5 pupils who score 3 marks.

In part c), the marks are put into increasing numerical order, that is,

1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5

For d), we find the mean by adding up the numbers,

$$1 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 4 + 4 + 4 + 5 + 5 = 44$$

and divide by the total number of pupils, 14, to give

$$44 \div 14 \Rightarrow \frac{44}{14} = \frac{22}{7} = 3\frac{1}{7}$$

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4. *Question and Solution*

Do these multiplications in a clever way in your exercise book.

a) $3 \times 4 \times 25 = 3 \times (4 \times 25)$

$= 3 \times 100$

$= 300$

b) $5 \times 63 \times 20 = 63 \times 5 \times 20$

$= 63 \times 100$

$= 0$

c) $63 \times 77 \times 0 = 0$

d) $1 \times 2 \times 4 \times 8 = 8 \times 8$
$= 64$

e) $1 \times 2 \times 3 \times 4 \times 5 \times 6 = (2 \times 5) \times (4 \times 6) \times 3$
$= 10 \times 24 \times 3$
$= 240 \times 3$
$= 720$

f) $5 \times 2 \times 7 \times 2 \times 7 \times 5 = (5 \times 2) \times 7 \times 7 \times (2 \times 5)$
$= 10 \times 49 \times 10$
$= 100 \times 49$
$= 4900$

g) $2 \times 8 \times 125 \times 4 = 8 \times (2 \times 125) \times 4$
$= 8 \times 250 \times 4$
$= 8 \times 1000$
$= 8000$

(p11, Q4)

Notes

There are different ways to tackle these questions but whenever possible, try to find combinations that make 10 (e.g 2×5) or 100 (4×25).

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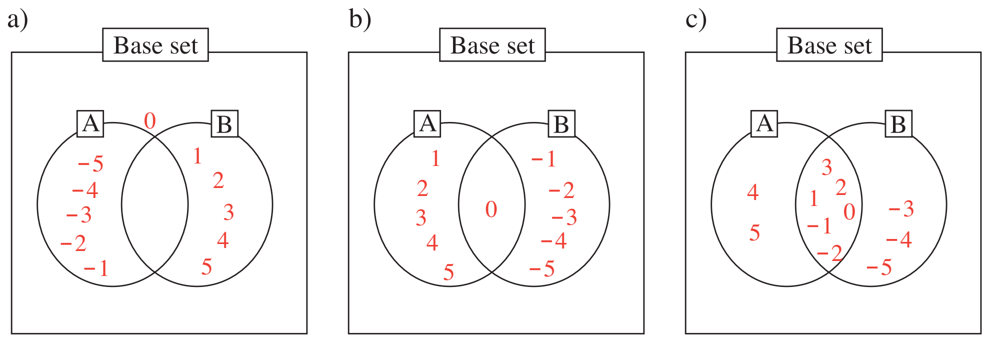
5. *Question and Solution*

The base set is : $U = \{-5,\ -4,\ -3,\ -2,\ -3,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\}$

Write the numbers in the Venn diagrams.



A = {negative numbers} B = {positive numbers}

A = {at least zero} B = {at most zero}

A = {more than – 3} B = {less than 4}

(p22, Q3)

Notes

This is a straightforward question if you know the definitions. Note that in a), the number zero is the outside region and *not* in the intersection as zero is neither positive nor negative.

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6. *Question and Solution*

Fill in the missing numbers.

a)	8 is more than 0 by $\boxed{8}$	$8 - 0 = \boxed{8}$	$\boxed{8} + 0 = 8$
b)	-8 is less than $\boxed{0}$ by 8	$-8 - 0 = \boxed{-8}$	$\boxed{-8} + 0 = -8$
c)	8 is more than 2 by $\boxed{6}$	$8 - 2 = \boxed{6}$	$\boxed{6} + 2 = 8$
d)	8 is more than -3 by $\boxed{11}$	$8 - (-3) = \boxed{11}$	$\boxed{11} + (-3) = 8$
e)	-3 is more than -7 by $\boxed{4}$	$-3 - (-7) = \boxed{4}$	$\boxed{4} + (-7) = -3$
f)	4 is less than $\boxed{13}$ by 9	$4 - 13 = \boxed{-9}$	$\boxed{-9} + 13 = 4$
g)	-2 is less than 3 by $\boxed{5}$	$-2 - 3 = \boxed{-5}$	$\boxed{1} + (-3) = -2$

(p25, Q3)

Notes

Note that there is a pattern throughout parts a) to f) but part g) does not follow the pattern!

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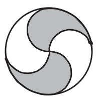
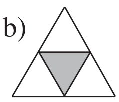
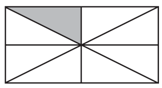
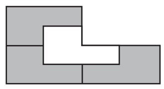
7. *Question and Solution*

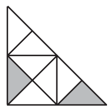
What part of the shapes are shaded?

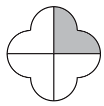
a)  $\frac{1}{2}$

b)  $\frac{1}{4}$

c)  $\frac{1}{8}$

d)  $\frac{3}{4}$

e)  $\frac{1}{4}$

f)  $\frac{1}{4}$

(p26, Q1)

Notes

This is a straightforward exercise in determining fractions of whole parts. Note that in e), the shaded part is 2 out of 8, i.e. $\frac{2}{8}$, and this is equal to $\frac{1}{4}$.

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8. *Question and Solution*

Write these numbers as decimals. Do necessary calculations in your exercise book.

a) $\frac{35}{10} = 3.5$	b) $\frac{7}{100} = 0.07$	c) $\frac{1003}{100} = 1.003$
d) $\frac{1003}{10} = 10.03$	e) $\frac{89}{10} = 8.9$	f) $83 + \frac{7}{10} = 83.7$
g) $\frac{3}{100} = 0.03$	h) $\frac{68}{100} = 0.68$	i) $\frac{527}{100} = 5.27$
j) $1 + \frac{1}{2} = 1.5$	k) $15 + \frac{2}{5} = 15.4$	l) $\frac{1}{4} = 0.25$
m) $\frac{6}{20} = 0.3$	n) $143 + \frac{17}{50} = 143.34$	o) $2\frac{3}{4} = 2.75$

(p28, Q2)

Notes

This question illustrates the important concept of changing fractions into decimals. Parts a) to i) are straightforward. It is only in the final six parts that more complex conversions are given.

In j), note that $\frac{1}{2} = \frac{1}{2} \times \frac{5}{5} = \frac{5}{10} = 0.5$, so $1 + \frac{1}{2} = 1.5$.

So the method is to find the equivalent fractions in which the denominator is 10 or 100, etc, as you can then form the decimal equivalent. For example,

k) $15 + \frac{2}{5} = 15 + \frac{2}{5} \times \frac{2}{2} = 15 + \frac{4}{10} = 15.4$

l) $\frac{1}{4} = \frac{1}{4} \times \frac{25}{25} = \frac{25}{100} = 0.25$

m) $\frac{6}{20} = \frac{6}{20} \times \frac{5}{5} = \frac{30}{100}$

n) $143 + \frac{17}{50} = 143 + \frac{17}{50} \times \frac{2}{2} = 143 + \frac{34}{100} = 143.34$

o) $2\frac{3}{4} = 2 + \frac{3}{4} = 2 + \frac{3}{4} \times \frac{25}{25} = 2 + \frac{75}{100} = 2.75$

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9. *Question and Solution*

The graph shows the variation in temperature over one day.

a) What temperature was it at 10.00 am?

15°C

b) At what time of day was it hottest?

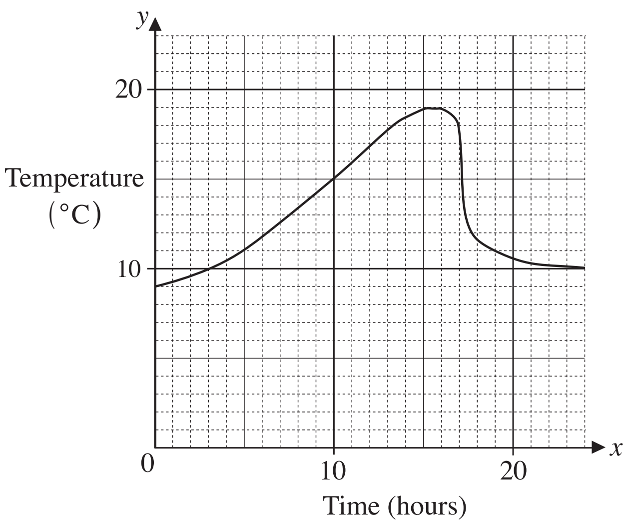
3.00 pm - 4.00 pm

c) During which times was the temperature rising?

0 - 15 hours

d) There was a downpour during the day. When do you think that it happened?

4.00 pm - 6.00 pm



(p34, Q1)

Notes

Note that the time goes from 0 (midnight) to 24 (midnight) so that in part b), the hottest time of the day is at about $15\frac{1}{2}$ or between 15 and 16, i.e, 3.00 pm and 4.00 pm.

In part d), the downpour corresponds to the lowering of the temperature, i.e. from 16 to 18 hours, or 4.00 pm to 6.00 pm.

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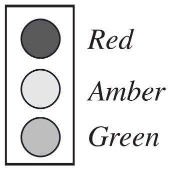
10. *Question and Solution*



Traffic lights light up in the order: R, RA, G, A, R
What other possible combinations could be used?

R, A, G, A, R, R, AG, G, A, R R, RA, G, GA, R

(p40, Q5)

Notes

Here you have to go through a sequence of lights from $R \rightarrow G \rightarrow R$ so this gives three more possibilities.

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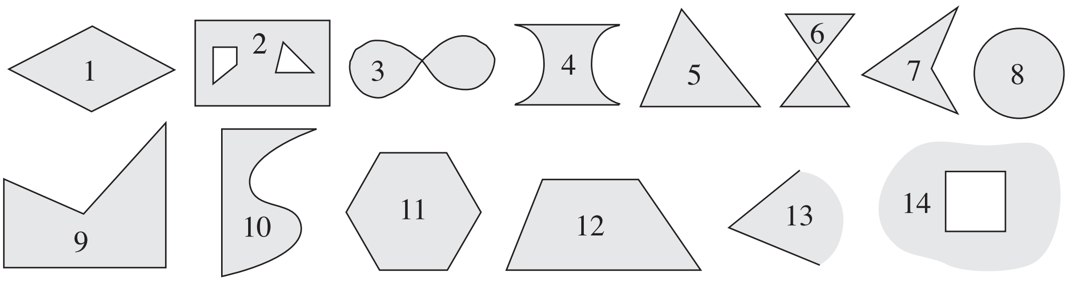
11. *Question and Solution*

List the numbers of the plane shapes which match the descriptions.



a) It is enclosed only by straight lines. 1, 2, 5, 6, 7, 9, 11, 12

b) It is enclosed by straight and curved lines. 4, 10

c) It is enclosed only by curved lines. 3, 8

d) It is not enclosed. 13, 14

e) It has parallel sides. 1, 2, 4, 6, 9, 11, 12, 14

f) It has perpendicular sides. 2, 9, 10. 14

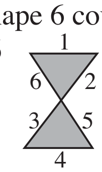
g) It has exactly 4 straight sides 1, 7, 12, (6)

h) It has exactly 6 vertices. 11

(p42, Q1)

Notes

The answers here are straightforward but be very careful to check each shape when answering the questions.

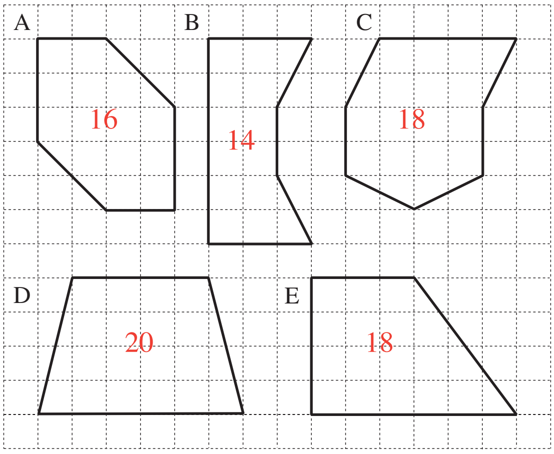
There is an issue with g) in which shape 6 could be thought of as having 4 straight sides but it could also be interpreted as having 6 . It is not, though, a quadrilateral.

Many of the shapes should be known by their name:

1 - rhombus; 2 - rectangle (but with a quadrilateral and a triangle cut out);
5 - triangle; 7 - deltoid (adjacent sides equal); 8 - circle; 9 - pentagon (irregular);
11 - hexagon (regular); 12 - trapezium (quadrilateral with two sides parallel).

12. *Question and Solution*

a) Write inside each polygon its area in unit squares.



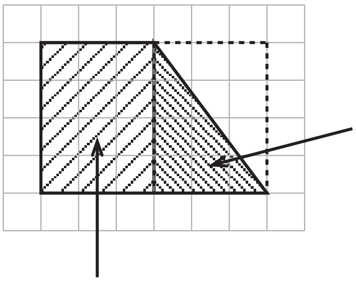
b) Which polygon has: i) the greatest area $\boxed{D}$ ii) the smallest area? $\boxed{B}$

c) Which polygons have equal areas? C and E

(p45 ,Q4)

Notes

For A, B, C and D, it is easy to combine shapes to form whole squares but for E, this is not so obvious.



This is $\frac{1}{2}$ area of rectangle $= \frac{1}{2} \times 12 = 6$ squares

$3 \times 4 = 12$ sq units

So total $= 12 + 6 = 18$ sq units

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13. *Question and Solution*

Fill in the missing differences. Continue drawing the graphs.

a)

$(+4) - (+6) = \boxed{-2}$

$(+4) - (+5) = \boxed{-1}$

$(+4) - (+4) = 0$

$(+4) - (+3) = +1$

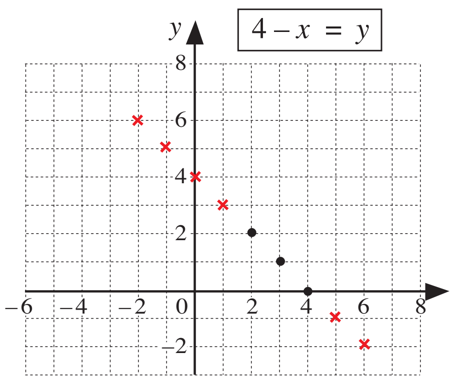
$(+4) - (+2) = +2$

$(+4) - (+1) = +3$

$(+4) - 0 = +4$

$(+4) - (-1) = \boxed{5}$

$(+4) - (-2) = \boxed{6}$



b) $(-4)-(+2) = \boxed{-6}$

$(-4)-(+1) = \boxed{-5}$

$(-4)-0 = -4$

$(-4)-(-1) = -3$

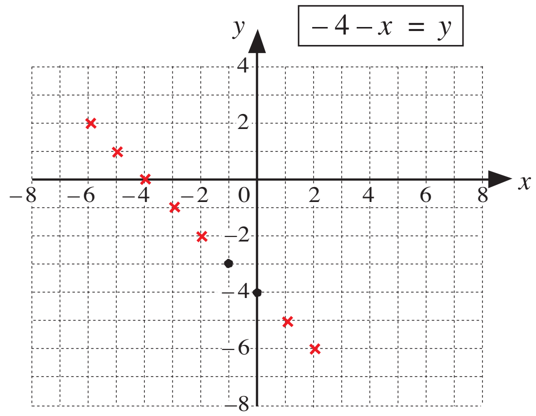
$(-4)-(-2) = -2$

$(-4)-(-3) = \boxed{-1}$

$(-4)-(-4) =$

$(-4)-(-5) = \boxed{1}$

$(-4)-(-6) = \boxed{2}$



(p59, Q1)

Notes

Here we illustrate the concept of the equation of a straight line and its graph, $y = 4 - x$ and $y = -4 - x$.

Note that the graph of the second equation is the same as the first but moved down by 8 units. You can see this from

$$y_1 = 4 - x$$

$$y_2 = -4 - x$$

Subtracting,
$$\begin{aligned} y_1 - y_2 &= 4 - x - (-4 - x) \\ &= 4 - x + 4 + x \\ &= 4 + 4 - x + x \\ &= 8 + 0 \\ &= 8 \end{aligned}$$

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14. *Question and Solution*

Fill in the products and notice how they change.

$5 \times 3 = \boxed{15}$

$5 \times 2 = \boxed{10}$

$5 \times 1 = \boxed{5}$

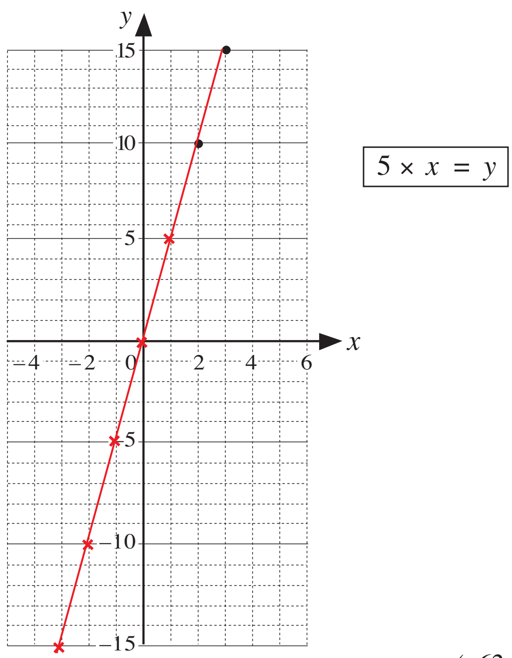
$5 \times 0 = \boxed{0}$

$5 \times (-1) = \boxed{-5}$

$5 \times (-2) = \boxed{-10}$

$5 \times (-3) = \boxed{-15}$

Complete the graph.



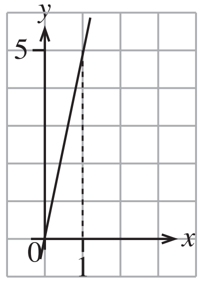
(p62, Q1)

Notes

Here is another graph, $y = 5 \times x$, which we write as $y = 5x$.

Note that the graph shows that y increases 5 times as fast as x.

The gradient or slope of the line is 5.



$$\text{gradient} = \frac{\text{increase in } y}{\text{increase in } x} = \frac{5}{1} = 5$$

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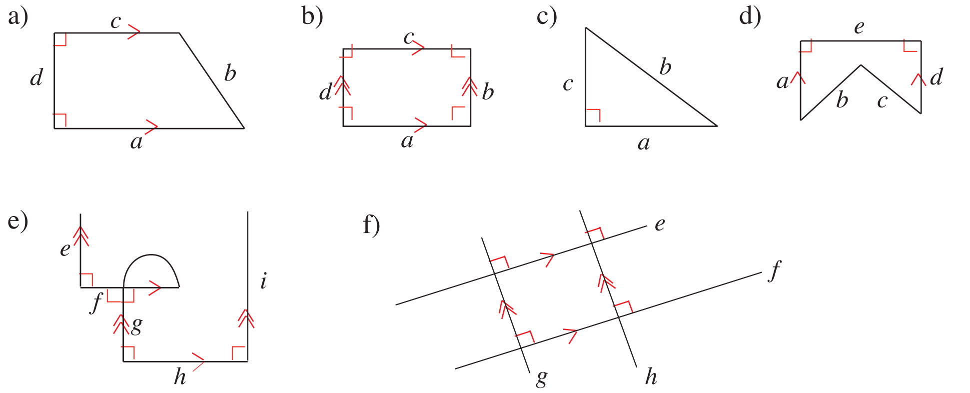
15. *Question and Solution*

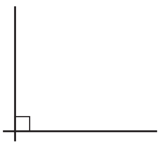
Mark on the diagrams, or list by their letters, the **perpendicular** and **parallel** lines.

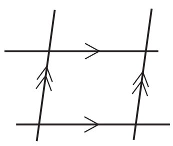


(p73, Q5)

Notes

This question illustrates two important concepts,

right angle  when two lines are perpendicular to one another (the right angle is denoted as shown here)

parallel lines  denoted by arrowheads drawn on lines which are parallel to one another

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16. *Question and Solution*

Decide whether the statements are true or false. Write a ✓ or a **X** .

a) Every rectangle is a trapezium. ✓

b) Every trapezium is a rectangle. X

c) Every rhombus is a parallelogram. ✓

d) Every parallelogram is a rhombus. X

e) A parallelogram can be a trapezium. ✓

f) All parallelograms are trapeziums. ✓

g) Not all parallelograms are trapeziums. X

h) A trapezium can be concave. X

i) A trapezium need not be a quadrilateral. X

j) There is no rhombus which is concave. ✓

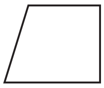
k) All rhombi are convex. ✓

l) Not every parallelogram is a rhombus. ✓

(p76, Q3)

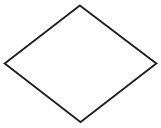
Notes

You must make sure that your students are clear about the definitions of shapes (see the *Facts to Know and Remember* section). To show that a statement is false, you just need to give a counter example such as, for b)



trapezium, but clearly not a rectangle (see definitions)

In c) note that

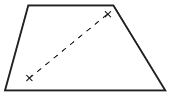


all *rhombi* (a rhombus has 4 equal sides) are also parallelograms. Note that a rhombus with 4 right angles is a square.

A counter example for d) is shown:  this parallelogram is clearly not a rhombus.

In g), all parallelograms are trapeziums, so the statement is false.

In h), you can always join any two points by a straight line *inside* the shape, that is, trapeziums are always convex.



In i), every trapezium is a quadrilateral.

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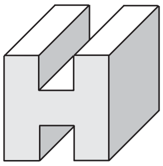
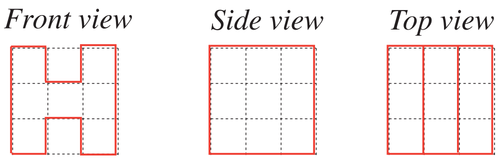
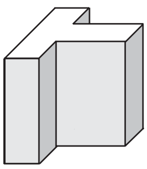
17. *Question and Solution*

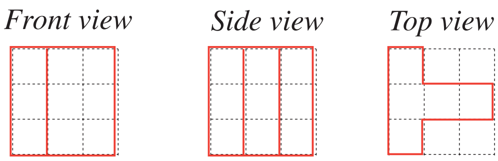
Each solid was cut from a cube with edges 3 units long. Draw how you would see it from the front, from the side and from above. Calculate its volume.

a) 



Volume = $3 \times 3 \times 3 - 2 \times 3 = 27 - 6 = 21$ (unit cubes)

b) 

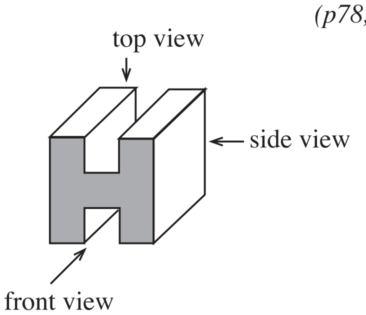


Volume = $5 \times 3 = 15$ (unit cubes) (5 columns of 3 cubes)

(p78, Q2)

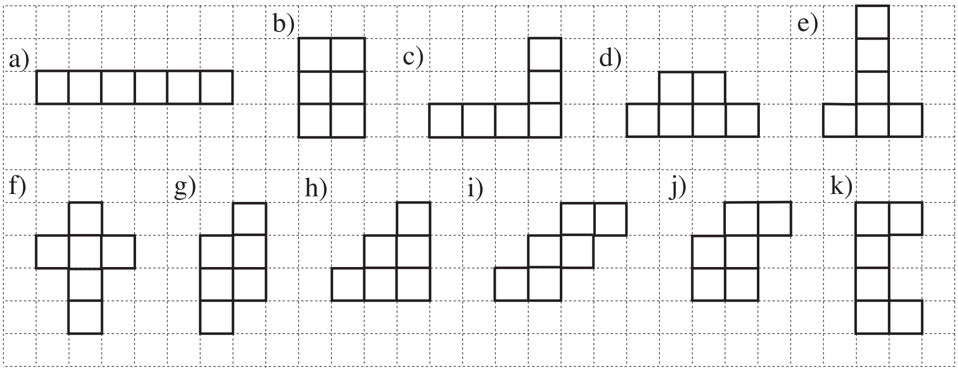
Notes

Here another concept is illustrated; you can illustrate any 3-dimensional shape with the front view, side view and top view.



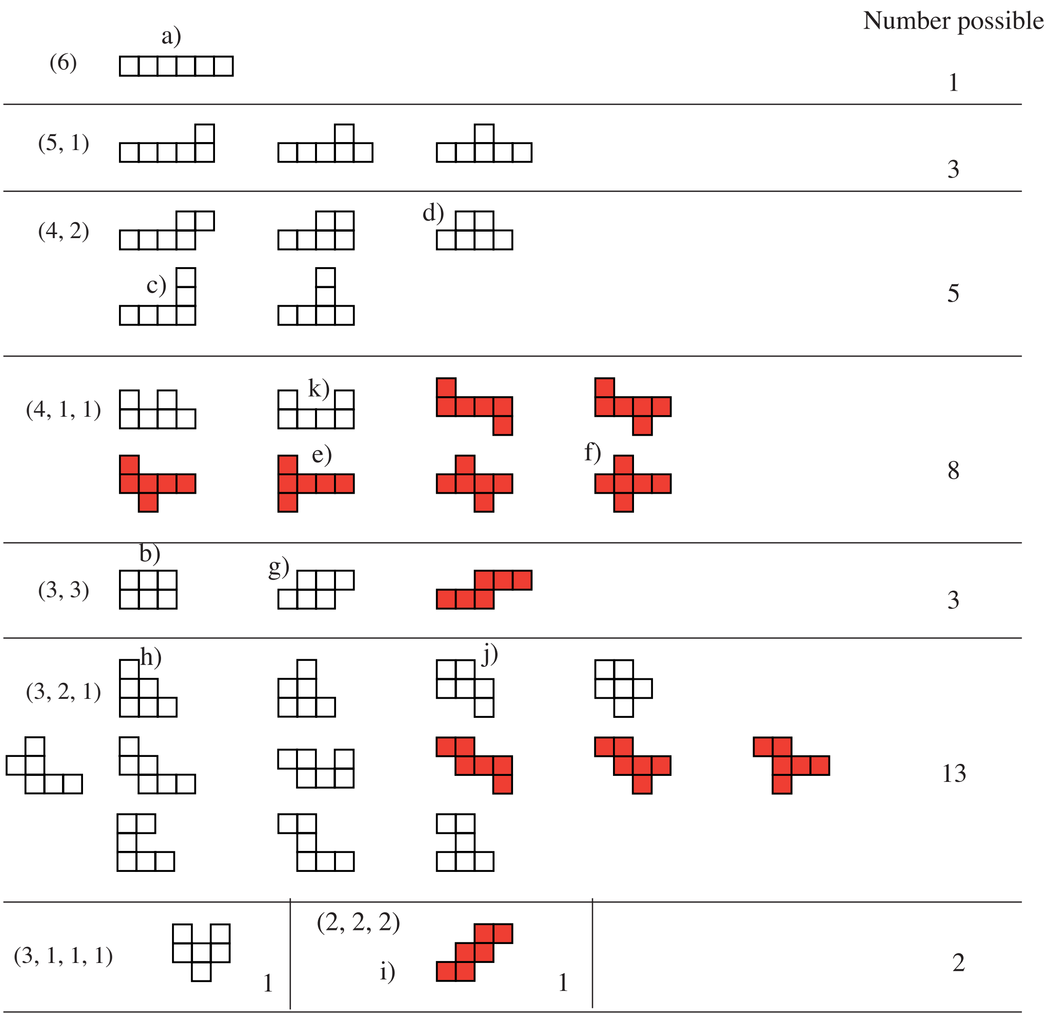
18. *Question and Solution*

Hexominoes are formed by connecting 6 squares along at least one side.
Here are 11 examples of different hexominoes.



i) In your exercise book, draw as many other different hexominoes as you can. How many hexominoes have you found altogether? 35 in total

ii) Colour the hexominoes in the diagram and in your exercise book which could be used as the net for a cube. How many did you colour? 11



Total: 35

(p80, Q3)

Notes

There are 35 different hexominoes but only 11 of them can form a net for a cube. Encourage a systematic strategy to find all the possibilities. The solution given shows squares joined in a row.

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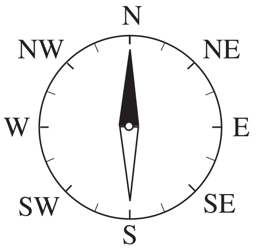
19. *Question and Solution*

Measure or calculate the angles between the given compass directions.



a)	S and W	90°	b)	S and NE	135°
c)	E and SW	135°	d)	N and SE	135°
e)	NW and SW	90°	f)	NW and E	135°
g)	SSW and SE	$67\frac{1}{2}°$	h)	SSW and NNE	180°

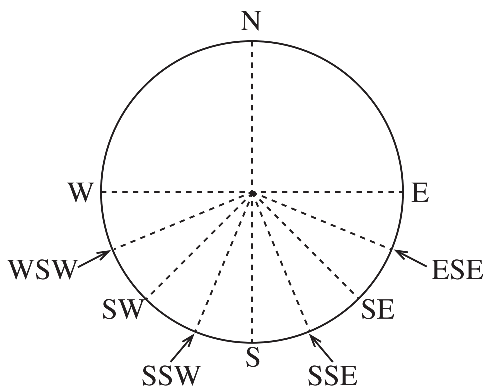
(p85, Q3)

Notes

This is straightforward, but note that a right angle is $90°$ (that is, the angle between, for example, N and E is $90°$) so the angle between N and NE is half a right angle, that is $45°$.

Similarly, the angle between SSW and S is a quarter of a right angle, that is $22\frac{1}{2}°$.

See the diagram opposite that clarifies the position of SSW (half way between SW and S), etc.



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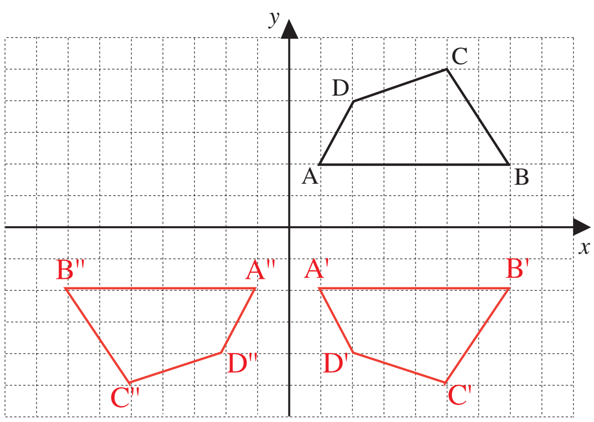
20. *Question and Solution*

Reflect the quadrilateral in the x-axis, then **reflect** its image in the y-axis.



Are these statements true or false?

a) BC = B'C' T

b) BC = B''C'' T

c) BC // B'C' F

d) BC // B''C'' T

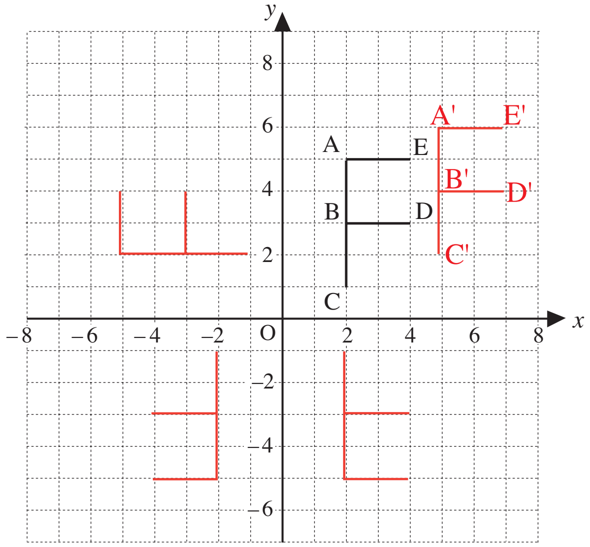
e) BC ⊥ B'C' F

(p90, Q4)

Notes

Students must take care to follow the instructions exactly. If they are successful with this question, they have fully understood reflection.

21. *Question and Solution*



a) **Translate** shape F so that the coordinates of point C' are (5, 2).

b) **Reflect** the original shape F in the x-axis.

c) **Rotate** the original shape F by $90°$ around point O.

d) **Rotate** the original shape F by $180°$ around point O.

(p94, Q1)

Notes

This question covers not only reflections but also translations and rotations. Note that for

- *translations* all lines are moved the same distance in the x and y (horizontal and vertical) directions

(so to get from C to C', you move 5 up (vertical) and 2 across (horizontal) and you do the same for each vertex; hence the new shape is identical to the original shape, but in a new position).

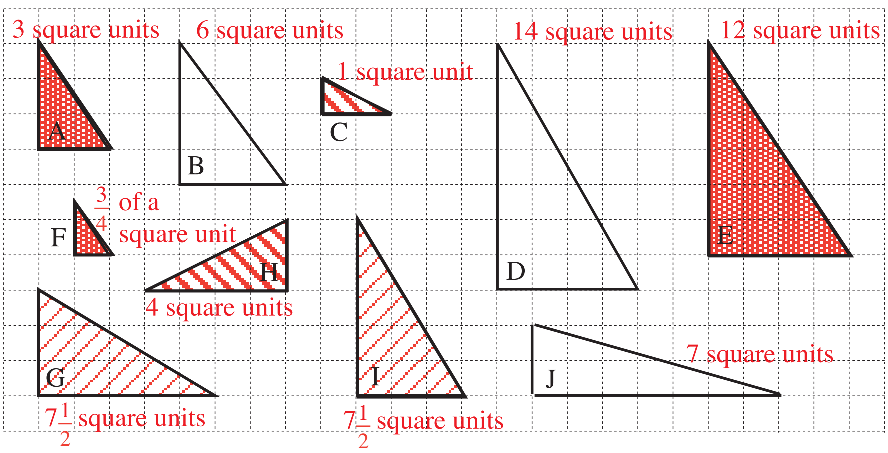
- *rotations* to define a rotation you need to specify the *angle and direction of rotation* and the *centre of rotation*. The only exception is a rotation of $180°$, which requires a centre of rotation but not an angle as $180°$ clockwise will give the same image as $180°$ anticlockwise.

(Using tracing paper will help to clarify the position of the new image, keeping it fixed at the centre of rotation and moving by the amount specified in the directions given.)

Note that we can use the words *object* and *image* for the original shape and the new shape.

22. *Question and Solution*

Colour **similar** triangles in the same colour. Calculate the area of each triangle..



(p95, Q2)

Notes

Note that to be similar, the ratio of the shortest sides (that is, the sides adjacent to the right angle) must be the same. For example,

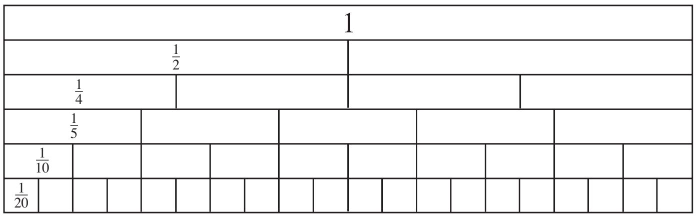
A	ratio	$3:2$		
F	ratio	$1\frac{1}{2}:1$	$\Rightarrow$	$3:2$
F	ratio	$6:4$	$\Rightarrow$	$3:2$

Note also that G and I are not only similar but actually congruent (the same) although their orientation is different.

---

23. *Question and Solution*

Write different forms of the same quantities from the diagram.



$1 = \frac{2}{2} = \frac{4}{4} = \frac{5}{5} = \frac{10}{10} = \frac{20}{20}$ $\quad \frac{1}{2} = \frac{2}{4} = \frac{5}{10} = \frac{10}{20}$ $\quad \frac{1}{4} = \frac{5}{20}$

$\frac{3}{4} = \frac{15}{20}$ $\quad \frac{1}{5} = \frac{2}{10} = \frac{4}{20}$ $\quad \frac{2}{5} = \frac{4}{10} = \frac{8}{20}$ $\quad \frac{3}{5} = \frac{6}{10} = \frac{12}{20}$

$\frac{4}{5} = \frac{8}{10} = \frac{16}{20}$ $\quad \frac{1}{10} = \frac{2}{20}$ $\quad \frac{3}{10} = \frac{6}{20}$ $\quad \frac{7}{10} = \frac{14}{20}$ $\quad \frac{9}{10} = \frac{18}{20}$

(p98, Q1)

Notes

This question covers all aspects of equivalent fractions, showing why, for example,

$$\frac{1}{2} = \frac{2}{4} = \frac{5}{10} = \frac{10}{20}$$

Note that, mathematically, you can get from $\frac{1}{2}$ to $\frac{10}{20}$ by multiplying both the numerator and denominator by 10; for example,

x 10

$$\frac{1}{2} = \frac{10}{20}$$

x10

In the same way, you can work in the reverse direction; for example,

÷ 2 ÷ 2

$$\frac{8}{20} = \frac{4}{10} = \frac{2}{5}$$

÷ 2 ÷ 2

÷ 4

$$\frac{8}{20} = \frac{2}{5}$$

÷ 4

24. *Question and Solution*

Calculate the sums and differences.

a) $\frac{3}{5} + \frac{3}{10} = \frac{9}{10}$ b) $\frac{7}{8} + \frac{1}{4} = \frac{9}{8}$ c) $\frac{1}{2} + \frac{1}{10} - \frac{2}{5} = \frac{2}{10} \left(= \frac{1}{5} \right)$

d) $\frac{4}{11} + \frac{5}{11} - \frac{2}{11} = \frac{7}{11}$ e) $\frac{7}{12} - \frac{1}{3} = \frac{3}{12} \left(= \frac{1}{4} \right)$ f) $\frac{5}{7} - \frac{5}{21} = \frac{10}{21}$

g) $\frac{2}{3} + \frac{2}{9} - \frac{3}{18} = \frac{13}{18}$ h) $\frac{1}{4} + \frac{3}{8} - \frac{5}{16} = \frac{5}{16}$ i) $1\frac{1}{5} - \frac{3}{10} = \frac{9}{10}$

(p105, Q4)

Notes

This illustrates how to add and subtract fractions, procedures which are simple when the denominators are the same, as in part d):

$$\frac{4}{11} + \frac{5}{11} - \frac{2}{11} = \frac{4+5-2}{11} = \frac{9-2}{11} = \frac{7}{11}$$

but which require more thought when they are different. In this case you need to form equivalent fractions, choosing a value that all the denominators can divide into exactly. For example, for part g), the numbers 3, 9 and 18 all divide exactly into 18. Hence we have

$$\frac{2}{3} + \frac{2}{9} - \frac{3}{18} = \frac{12}{18} + \frac{4}{18} - \frac{3}{18}$$

$$= \frac{12+4-3}{18}$$

$$= \frac{16-3}{18}$$

$$= \frac{13}{18}$$

In fact, we are finding the *lowest common multiple* of the denominators each time.

---

25. *Question and Solution*

The farmer had some chickens. He sold $\frac{5}{8}$ of them and had 180 chickens left.
How many chickens did the farmer have at first?

$\boxed{480}$ *(p110, Q3)*

Notes

Since $1 - \frac{5}{8} = \frac{3}{8}$, we have

$\frac{3}{8} \rightarrow$ 180 chickens,

so divide both sides by 3 to give

$$\frac{1}{8} \rightarrow \frac{180}{3} = 60 \text{ chickens.}$$

Now multiply both sides by 8 to give

$$8 \times \frac{1}{8} = 1 \rightarrow 8 \times 60 \text{ chickens } = 480 \text{ chickens}$$

---

26. *Question and Solution*

Fill in the missing numbers. Check that they make the statements true.

a) $\frac{2}{5} \times \boxed{2} = \frac{4}{5}$ b) $\boxed{3} \times \frac{5}{9} = \frac{15}{9}$ c) $\frac{3}{10} \times \boxed{10} = \frac{30}{10}$

d) $\frac{5}{8} \times \boxed{2} = \frac{5}{4}$ e) $\frac{\boxed{5}}{6} \times 4 = \frac{10}{3}$ f) $\frac{5}{\boxed{2}} \times \boxed{4} = 10$

(there are many possible solutions to f), e.g. $\frac{5}{11} \times 0 = 0$, as any number multiplied by zero is zero).

(p112, Q2)

Notes

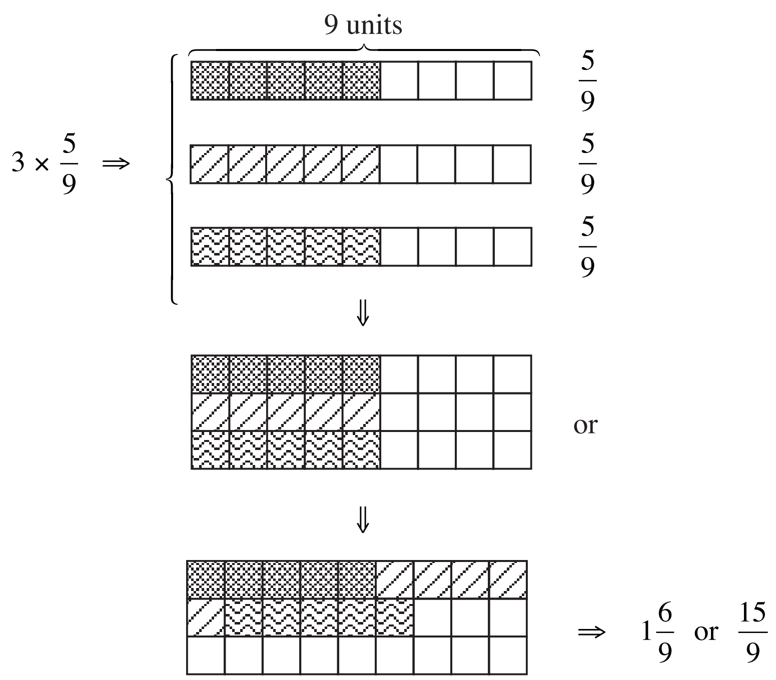
Here the topic is multiplying fractions and the rule that we are illustrating is

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}} \qquad (b,\, d \neq 0)$$

Multiplying first by whole numbers shows that this rule works as, for example, in part b),

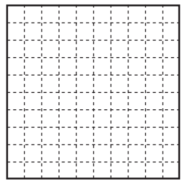
$$3 \times \frac{5}{9} = \frac{3}{1} \times \frac{5}{9} = \frac{3 \times 5}{1 \times 9} = \frac{15}{9}$$

We are not expecting your students use the rule yet but to move towards it. You can show this geometrically if they need more convincing; for example,



27. *Question and Solution*

Use the diagram to help you do this addition in different ways.



$$0.2 + \frac{1}{10} + \frac{37}{100} + 0.17 + \frac{3}{100}$$

Calculate using:

a) fractions: $\boxed{\frac{1}{5} + \frac{1}{10} + \frac{37}{100} + \frac{17}{100} + \frac{3}{100} = \frac{20}{100} + \frac{10}{100} + \frac{37}{100} + \frac{17}{100} + \frac{3}{100} = \frac{87}{100}}$

b) decimals: $\boxed{0.2 + 0.2 + 0.37 + 0.17 + 0.03 = 0.87}$

c) percentages: $\boxed{20\% + 10\% + 37\% + 17\% + 3\% = 87\%}$

(p125, Q1)

Notes

This question reinforces the equivalence of

$\boxed{\text{fractions, decimals and percentages}}$

---

28. *Question and Solution*

Answer the questions by writing a ratio or a fraction, as required.
In a group of students at a youth camp, 3 are *Americans*, 4 are *British* and 1 is *Greek*.

a) What part of the group is:

a) *American* $\boxed{\frac{3}{8}}$ *British* $\boxed{\frac{4}{8}}$ *Greek* $\boxed{\frac{1}{8}}$ *British* **or** *Greek*? $\boxed{\frac{5}{8}}$

b) What is the ratio in the group of:

i) *American* students to *British* student 3 : 4

ii) *American* students to *Greek* students 3 : 1

iii) *British* students to *American* students 4 : 3

iv) *British* students to *Greek* students 4 : 1

v) *Greek* students to *American* students 1 : 3

vi) *Greek* students to *British* students? 1 : 4

c) The group is going on a trip in a minibus. They get on the bus in a random order.

How certain are you of these events occurring?

If you think that it is **certain** to happen, write **C**, if you think that it is **possible but not certain**, write **P** and if you think that it is **impossible**, write **I**.

i) The first 4 students to get on the bus are *American*. $\boxed{\text{I}}$

ii) The last student to get on the bus is *American* or *British* or *Greek*. $\boxed{\text{C}}$

iii) The first student to get on the bus is *Greek*. $\boxed{\text{P}}$

iv) The first 4 students to get on the bus are an *American*, a *Greek*, an *American* and a *British* student in that order. $\boxed{P}$

v) Two *Americans*, a *British* and the *Greek* student are the first four to get on the bus. $\boxed{P}$

d) i) Which nationality is the most likely to get on the bus first? British

ii) Is the first student to get on the bus more likely to be *American* or *British*? British

(p131, Q2)

Notes

Parts a) and b) are reinforcing fraction and ratio concepts; note that 1 : 3 is not the same as 3 : 1.

In part c), we are bringing in the probability concepts of

impossible

possible but not certain

certain

This is the first building block in probability.

---

29. *Question and Solution*

How certain are you of these outcomes occurring? Write C for certain, P for possible but not certain or I for impossible.

a) The final of the next *Football World Cup* will be in 2005. $\boxed{I}$

b) The next time I toss a coin I will get a *Head* or a *Tail*. $\boxed{C}$

c) The next time I throw two dice the total will be more than 6. $\boxed{P}$

d) The next time I throw two dice the total will be more than 12. $\boxed{I}$

e) It will rain next week in my home town. $\boxed{P}$

(p135, Q2)

Notes

As in the previous question, you need to work logically.

In part a) the answer is 'Impossible', because the Football World Cup is held every 4 years, in 2010, 2014, 2018, etc, so not only is 2005 in the past, it is also not in this sequence.

Part b) is 'Certain' as the wording asks for a *Head* or a *Tail* and one of these outcomes is certain to happen.

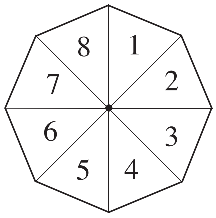
The outcomes for parts c), d) and e) are all logical.

---

30. *Question and Solution*

The diagram shows a spinner used in a board game.

When the spinner is spun, what is the probability that it lands on:



a) 1 $\boxed{\frac{1}{8}}$ b) 8 $\boxed{\frac{1}{8}}$

c) an even number $\boxed{\frac{1}{2}}$ d) a number less than 8 $\boxed{\frac{7}{8}}$

e) a number greater than 8 $\boxed{0}$ f) a number greater than 0? $\boxed{1}$

(p140, Q2)

Notes

Now we are working out probabilities by symmetry. Each number on the wheel is equally likely to occur, so

$$p\,(8) = \frac{1}{8},\quad p(7) = \frac{1}{8},\quad \text{etc.}$$

---

31. *Question and Solution*

Mike is growing two different varieties of tomato plants in his greenhouse.

During one week, he keeps a record of the number of tomatoes he picks from each type of plant and notes the data in a table.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Variety A	5	5	4	1	0	2	5
Variety B	5	3	3	3	7	9	6

a) i) mode $\boxed{5}$ ii) median $\boxed{4}$ iii) mean? $\boxed{\frac{22}{7}}$

b) i) mode $\boxed{3}$ ii) median $\boxed{5}$ iii) mean? $\boxed{\frac{36}{7}}$

c) B - higher median and mean

(p150, Q1)

Notes

Yet more concepts are introduced here, namely

mode the most popular (frequent) outcome; for example, in part b), 3 is the mode as it occurs 3 times (other numbers occur less frequently)

median the middle value when all values are put in numerical order; for example, in part b), 3, 3, 3, 5, 6, 7, 9

$$3,\ 3,\ 3,\ \underset{\substack{\uparrow \\ \text{middle value}}}{5},\ 6,\ 7,\ 9$$

mean the average value; for example, in part b)

$$\frac{3+3+3+5+6+7+9}{7} = \frac{36}{7}$$

32. *Question and Solution*

We have 80 books altogether. They are arranged on 3 shelves.
If we moved 7 books from the top shelf to the middle shelf and took 8 books away from the bottom shelf, there would be an equal number of books on each shelf.

How many books are on each shelf?

Top shelf	31 books
Middle shelf	17 books
Bottom shelf	32 books

(p151, Q5)

Notes

There is an algebraic way of tackling this question (see below) but the most straightforward method is to note that after the two changes, there are

$$80 - 8 = 72$$

books equally distributed between the three shelves; that is

$$72 \div 3 = 24 \text{ books on each shelf}$$

Hence there were

$$24 + 7 = 31 \text{ on the bottom shelf}$$

$$24 - 7 = 17 \text{ on the middle shelf}$$

and $$24 + 8 = 32 \text{ on the top shelf}$$

(*Check*: $31 + 17 + 32 = 80$)

Here is the algebraic method:

If we let T = number of books on top shelf,
M = number of books on middle shelf,
S = number of books on bottom shelf,

then

$$T + M + S = 80$$

and

$$T - 7 = M + 7 = S - 8$$

Algebraically, you can express M and S in terms of T:

$$\begin{aligned} M + 7 &= T - 7 && \text{(take 7 from each side)} \\ M + 7 - 7 &= T - 7 - 7 \\ M + 0 &= T - 14 \\ M &= T - 14 \end{aligned}$$

and similarly

$$\begin{aligned} S - 8 &= T - 7 && \text{(add 8 to both sides)} \\ S - 8 + 8 &= T - 7 + 8 \\ S + 0 &= T + 1 \\ S &= T + 1 \end{aligned}$$

But $\quad T + M + S = 80 \Rightarrow T + (T - 14) + (T + 1) = 80$

$$T + T - 14 + T + 1 = 80$$

$$3T - 13 = 80$$

Add 13 to each side,

$$3T - 13 + 13 = 80 + 13$$

$$3T = 93$$

Divide both sides by 3 to give $\quad T = 31$

Hence $\quad M = T - 14 = 31 - 14 = 17$

$\quad S = T + 1 = 31 + 1 = 32$

Check: $\quad 31 + 17 + 32 = 80$

---

33. *Question and Solution*

What is the smallest possible, 3-digit, positive integer which fulfils these conditions?

- If it is multiplied by 3, the result is also a 3-digit number.
- If it is multiplied by 4, the result is a 4-digit number.

$\boxed{250}$

(p153, Q4)

Notes

Note that $4 \times 250 = 1000$, so clearly this is the smallest 3-digit number that, multiplied by 4, gives a 4-digit number, and $3 \times 250 = 750$, a 3-digit number. So both conditions are satisfied.

---

34. *Question and Solution*

What are the four consecutive **odd** numbers which add up to 80?

$\boxed{17}$ $\boxed{19}$ $\boxed{21}$ $\boxed{23}$

(p155, Q7)

Notes

As $80 \div 4 = 20$, numbers must be close to 80 and equally spread out; hence 17, 19, 21, 23.

---

35. *Question and Solution*

How many positive 3-digit numbers less than 500 are there in which the middle digit is half of the sum of the two outside digits?

$\boxed{20}$

(p158, Q4)

Notes

Work through logically:

Middle digit 1 $\Rightarrow$ 111 and 210 since $\dfrac{1+1}{2} = 1$ and $\dfrac{2+0}{2} = 1$

and both numbers are less than 500

Middle digit 2 $\Rightarrow$ 420, 321, 222, 123 and all are less than 500

Middle digit 3 $\Rightarrow$ 630, 531, 432, 333, 234, 135

not allowed as greater than 500

Working in this way, we get the 20 numbers.

111,	123,	135,	147,	159
210,	222,	234,	246,	258
321,	333,	345,	357,	369
420,	432,	444,	456,	468

---

36. *Question and Solution*

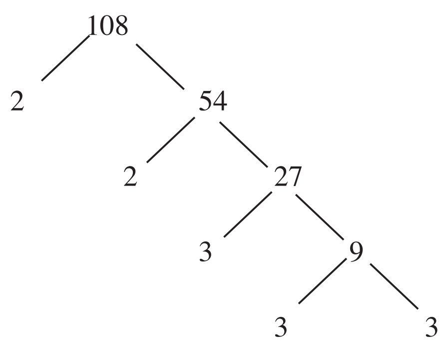
What is the greatest 3-digit natural number in which the **product** of its digits is 108?

$\boxed{962}$

(p159, Q4)

Notes

First find the factors of 108.



So $108 = 2 \times 2 \times 3 \times 3 \times 3$

You now need to determine the 3 digits that give the greatest product, i.e.

$$108 = 2 \times 6 \times 9 \Rightarrow 962$$

---

37. *Question and Solution*

Use each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 **only once** to make **five** whole numbers, so that one number is twice, another number is three times, another number is four times and the last number is five times the smallest number.

18, 36, 54, 72, 90 (09, etc. not allowed)

(p167, Q3)

Notes

Trial and improvement is one method and this should give the possible solution

09, 18, 27, 36, 45

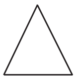
As '09' is not the usual way to write 9, you can say this is not allowed but it is a good clue to the actual solution,

18, 36, 54, 72, 90

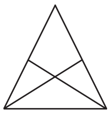
---

38. *Question and Solution*

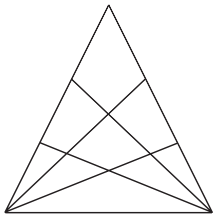
How many triangles can you see in each of these diagrams?

a) 

$\boxed{1}$

b) 

$\boxed{8}$

c) 

$\boxed{27}$

How many triangles do you think will be in the next triangle in the sequence? $\boxed{64}$

(p170, Q3)

Notes

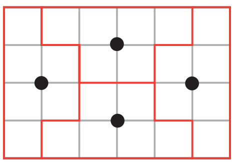
These numbers are cubic numbers, that is

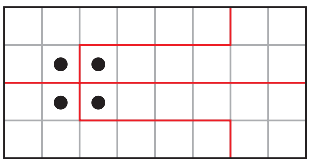
$$1^3\ (=1),\quad 2^3\ (=8),\quad 3^3\ (=27)$$

---

39. *Question and Solution*

Each diagram is the map of a field in which there are 4 wells. Show how the field could be divided into 4 **congruent** parts so that each part has exactly one well.

a) 

b) 

(p172, Q5)

Notes

The question should have included the words " ... using only the grid lines."

Part a) is the easier option for most students. Note that the shape is 8-sided, i.e. an octagon.

Part b) is obvious to some, but is a real challenge to most!

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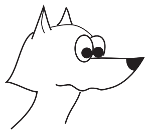
40. *Question and Solution*



Freddy Fox decided that from that day forward he would always tell the truth on Mondays, Wednesdays and Fridays but he would always tell lies on the other days of the week.

One day he said, "*Tomorrow I will tell the truth*."

On which day of the week do you think he said this?

Saturday

(p174, Q2)

Notes

Freddy Fox could not have said this on a Sunday, Tuesday or Thursday because these are the days he told lies.

He could not have said it on a day *before* he told a lie, i.e. on a Monday, Wednesday or Sunday, as he told the truth on these days and he would have said, "Tomorrow I will tell a lie."

He must have said it on a *Saturday,* because he told lies on that day and would also have told a lie the next day, Sunday.