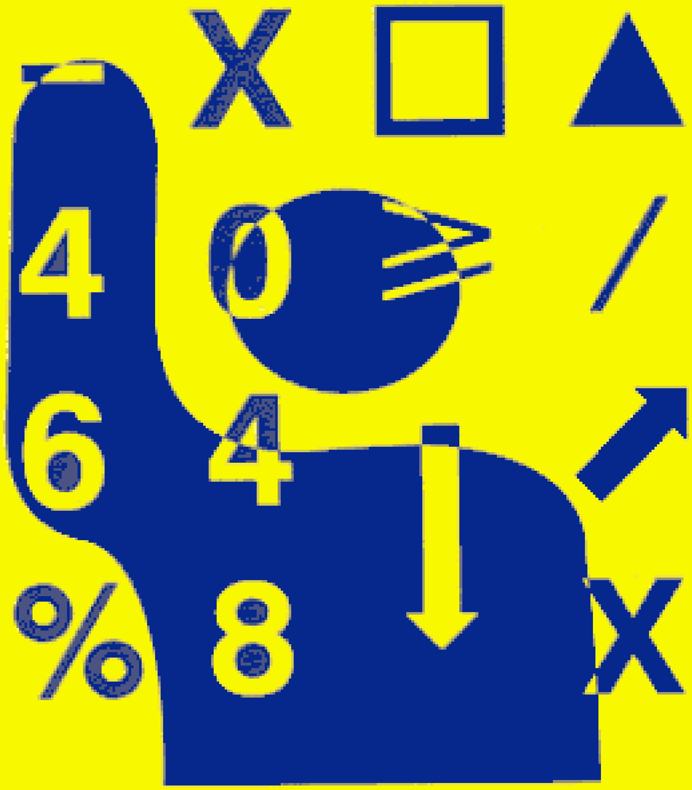
Mathematics Enhancement Programme

Primary Demonstration Project

1A Algebra

Help Booklet



Support for Primary Teachers in Mathematics

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CIMT
School of Education
University of Exeter

Mathematics Enhancement Programme

Help Module 1

ALGEBRA

Part A

Contents of Part A

Contents of Part B

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1.	ALGEBRA	6.	HANDLING DATA
2.	DECIMALS	7.	MENSURATION
3.	EQUATIONS	8.	NUMBERS IN CONTEXT
4.	FRACTIONS	9.	PERCENTAGES
5.	GEOMETRY	10.	PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

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The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

1 *Algebra* Introductory Notes

Historical Background

In one sense, people use 'formulae' (or algebra) all the time – often without realising it: a glazier may use tables to work out the cost of panes of glass, and a decorator will have his own 'rule of thumb' for estimating how much paint is needed for the outside of a house. These routines calculate a *required number* (the price of glass or the number of litres of paint needed); but they are often hard to understand because they are not expressed mathematically.

A mathematical formula should take the form if an *equation* which expresses some required quantity in terms of other, easily measured quantities. For example,

$$\text{area of rectangle} = \text{length} \times \text{breadth}.$$

This is a good beginning, but it is only a beginning. The ancient Babylonians and Egyptians (c. 2000 BC) used many approximate calculational routines and 'formulae' of this kind, but one cannot do mathematics with words.

To go further we have to replace friendly words by abstract symbols and extend the familiar arithmetic numbers to an 'arithmetic of letters': that is, we need to develop *algebra.*

The word 'algebra' comes from the title of a book

Al - jabr w'al muqabala

written in 830 AD by the Arabic astronomer Al - Khurarizuri. The extract translation of the title is disputed: 'al - jabr' means something like "restoring and balancing" (which refers to the idea of shifting things from one side of an equation to the other), and 'w'al muqabala' means something like "cancelling and simplifying". These two ideas reflect the central art of algebra - namely that of 'rearranging and simplifying expressions'.

Full blooded elementary algebra (in which, for example, the general quadratic equation can be written as

$$ax^2 + bx + c = 0$$

and its solution given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}) ,$$

was only developed after 1600 AD. The effect was dramatic. After waiting 3500 years for an effective language, the next century saw an explosive growth - with the rise of coordinate geometry (Descartes 1637) and calculus (Newton, Leibniz 1660 - 1684).

Around 1600, Galileo observed that "The Book of Nature is written in the language of mathematics". It soon became clear that the language of all mathematics is algebra!

Key Issues

Introduction

This is your chance to begin to use algebra, which is central to all subsequent mathematics and to nearly all meaningful applications of mathematics. You need to be equally confident at numerical approaches and algebraic approaches – in fact, the algebraic approach is of much more general applicability. Also, in this module you will be using some of the algebraic rules of notation: these are clear and precise rules, and it is important that you learn to abide by them and understand why they are correct. You will need to sort out any misconceptions that you may already have picked up.

This module deals with the beginnings of algebra, and explains the role of notation in algebra. HELP Module 3 on *Equations* is the key follow-up to this introduction to algebra.

Language / Notation

- Note that $3 \times n = 3n$

  $(3n)^2 = (3n) \times (3n) = 9n^2$

Key Points

- Equations *must always balance, that* is, what is on one side of the equation must *equal* what is on the other side.
- You must always write down clearly the operation which has taken place on each line, e.g. make a the subject of

$$4a + b = c \quad \text{(add } (-b) \text{ to both sides)}$$

$$\Rightarrow \quad 4a = c - b \quad \text{(divide both sides by 4)}$$

$$\Rightarrow \quad a = \frac{c - b}{4}$$

Misconceptions

There are numerous misconceptions with algebra, so here are just a few:

1. that $a \times a \times a = a^3$, not $3a$;
2. that $\dfrac{1}{a + b}$ is not equal to $\dfrac{1}{a} + \dfrac{1}{b}$,

   e.g. $\dfrac{1}{1 + 2}$ is not $\dfrac{1}{3}$ but $\dfrac{1}{1} + \dfrac{1}{2} = \dfrac{2}{2} + \dfrac{1}{2} = \dfrac{2 + 1}{2} = \dfrac{3}{2}$;
3. that $a^2 = a \times a$, not $2a$;
4. that $(a + b)^2 = (a + b) \times (a + b) = a^2 + 2ab + b^2$,

   not $a^2 + b^2$.

WORKED EXAMPLES
and EXERCISES

1 Algebra

1.1 Squares, Cubes, Square Roots and Cube Roots

When a number is multiplied by itself, we say that the number has been *squared*.

For example, 3 squared means $3 \times 3 = 9$. This is written as $3^2 = 9$.
We could also say that 9 is the square of 3.

When a number is *cubed* it is written down 3 times and multiplied.

For example 2 cubed means $2 \times 2 \times 2 = 8$. This is written as $2^3 = 8$.
We could also say that 8 is the cube of 2.

Sometimes the reverse process is needed to answer questions such as:

What number squared gives 25?

The answer would be 5. We say that 5 is the square root of 25, or write $\sqrt{25} = 5$.
Another question might be:

What number cubed gives 8?

The answer would be 2. We would say that the cube root of 8 is 2.

We could also write $\sqrt[3]{8} = 2$.

Worked Example 1

Find

(a) 8^2 (b) 4^2 (c) 5^3.

Use your answers to find

(d) $\sqrt{64}$ (e) $\sqrt{16}$ (f) $\sqrt[3]{125}$

Solution

(a) $8^2 = 8 \times 8 = 64$

(b) $4^2 = 4 \times 4 = 16$

(c) $5^3 = 5 \times 5 \times 5 = 125$

(d) $\sqrt{64} = 8$ because $8^2 = 64$

(e) $\sqrt{16} = 4$ because $4^2 = 16$

(f) $\sqrt[3]{125} = 5$ because $5^3 = 125$

1.1

Exercises

1. Find

(a) 5^2 (b) 6^2 (c) 1^2 (d) 7^2

Use your answers to find

(e) $\sqrt{36}$ (f) $\sqrt{1}$ (g) $\sqrt{49}$ (h) $\sqrt{25}$

2. Find

(a) 3^3 (b) 4^3 (c) 6^3 (d) 10^3

Use your answers to find

(e) $\sqrt[3]{27}$ (f) $\sqrt[3]{1000}$ (g) $\sqrt[3]{216}$ (h) $\sqrt[3]{64}$

3. Find

(a) 10^2 (b) 2^2 (c) 4^2 (d) 7^2

(e) 8^2 (f) 9^2 (g) 1^3 (h) 7^3

(i) 8^3 (j) 0^2 (k) 0^3 (l) 2^3

4. Find

(a) $\sqrt{100}$ (b) $\sqrt{4}$ (c) $\sqrt{81}$ (d) $\sqrt{64}$

(e) $\sqrt{16}$ (f) $\sqrt{9}$

5. Use a calculator to find

(a) 12^2 (b) 11^2 (c) 15^3 (d) 13^3

(e) 13^2 (f) 15^2 (g) 20^2 (h) 11^3

Without a calculator, find

(i) $\sqrt{121}$ (j) $\sqrt{400}$ (k) $\sqrt{169}$ (l) $\sqrt{225}$

(m) $\sqrt[3]{3375}$ (n) $\sqrt[3]{2197}$ (o) $\sqrt{144}$ (p) $\sqrt[3]{1331}$

6. Find

(a) $6^2 + 4^2$ (b) $3^2 - 2^2$ (c) $10^2 + 4^2$ (d) $3^2 + 4^2$

(e) $5^2 - 3^2$ (f) $4^3 + 2^3$ (g) $1^3 + 10^3$ (h) $6^2 + 8^2$

Just for Fun

You open a book. Two pages face you. If the product of the two page numbers is 3 192, what are the two page numbers?

1.2 Index Notation

Index notation is a useful way of writing expressions like

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

in a shorter format. The above could be written with index notation as 2^7.
The small number, 7, is called the *index* or *power*.

Worked Example 1

Find (a) 3^4 (b) 4^5 (c) 7^1

Solution

(a) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

(b) $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

(c) $7^1 = 7$

Worked Example 2

Find the missing number.

(a) $3^4 \times 3^6 = 3^?$ (b) $4^2 \times 4^3 = 4^?$ (c) $\dfrac{5^7}{5^4} = 5^?$

Solution

(a)
$$3^4 \times 3^6 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3) = 3^{10}$$

(b)
$$4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4^5$$

(c)
$$\frac{5^7}{5^4} = \frac{5 \times 5 \times 5 \times \cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1}{\cancel{5}_1 \times \cancel{5}_1 \times \cancel{5}_1 \times \cancel{5}_1} = 5 \times 5 \times 5 = 5^3$$

Note

$$\boxed{a^m \times a^n = a^{m+n}} \quad \text{and} \quad \boxed{\frac{a^n}{a^m} = a^{n-m}}$$

These rules apply whenever index notation is used.

1.2

Using these rules,

$$\frac{a^3}{a^3} = a^{3-3} = a^0 \qquad \text{or} \qquad \frac{a^3}{a^3} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}}} = 1$$

So $\boxed{a^0 = 1}$

Worked Example 3

Find

(a) $\left(2^3\right)^4$ (b) $\left(3^2\right)^3$

Solution

(a)
$$\begin{aligned}\left(2^3\right)^4 &= (2\times 2\times 2)\times(2\times 2\times 2)\times(2\times 2\times 2)\times(2\times 2\times 2)\\ &= 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\\ &= 2^{12}\end{aligned}$$

(b)
$$\begin{aligned}\left(3^2\right)^3 &= (3\times 3)\times(3\times 3)\times(3\times 3)\\ &= 3\times 3\times 3\times 3\times 3\times 3\\ &= 3^6\end{aligned}$$

Note

$$\boxed{\left(a^m\right)^n = a^{m\times n}}$$

Exercises

1. Write each of the following using index notation.

(a) $4\times 4\times 4\times 4\times 4$ (b) $3\times 3\times 3$

(c) $6\times 6\times 6\times 6\times 6\times 6$ (d) $7\times 7\times 7\times 7$

(e) $18\times 18\times 18$ (f) 19×19

(g) $4\times 4\times 4\times 4\times 4\times 4$ (h) $7\times 7\times 7\times 7\times 7$

(i) $10\times 10\times 10\times 10\times 10\times 10$ (j) $100\times 100\times 100\times 100\times 100$

2. Find the value of each of the following.

(a) 3^4 (b) 5^4 (c) 7^4 (d) 10^4

(e) 5^0 (f) 3^6 (g) 2^7 (h) 2^1

(i) 8^4 (j) 4^1 (k) 3^0 (l) 5^2

1.2

3. Fill in the missing numbers.

(a) $2^7 \times 2^4 = 2^?$ (b) $3^4 \times 3^5 = 3^?$ (c) $3^6 \times 3^7 = 3^?$

(d) $4^? \times 4^2 = 4^7$ (e) $5^? \times 5^2 = 5^6$ (f) $5^4 \times 5^? = 5^9$

(g) $?^2 \times 4^4 = 4^6$ (h) $5^7 \div 5^4 = 5^?$ (i) $3^4 \div 3^2 = 3^?$

(j) $7^{14} \div 7^{10} = 7^?$ (k) $17^5 \div 17^? = 17^3$ (l) $9^7 \div 9^? = 9^3$

(m) $4^6 \times 4^? = 4^{11}$ (n) $4^? \div 4^6 = 4^{10}$ (o) $3^? \times 3^2 = 3^8$

(p) $3^6 \div 3^6 = ?$ (q) $3^7 \div 3^6 = ?$ (r) $3^0 \times 3^? = 3^5$

(s) $3^0 \times 3^7 = 3^?$ (t) $4^1 \times 4^? = 4^8$ (u) $5^2 \times 5^? = 5^2$

4. Fill in the missing numbers.

(a) $4 = 2^?$ (b) $8 = 2^?$ (c) $16 = 2^?$

(d) $64 = 2^?$ (e) $27 = 3^?$ (f) $25 = 5^?$

(g) $64 = 4^?$ (h) $81 = 3^?$ (i) $125 = ?^3$

5. Simplify the following expressions, giving your answer in index notation.

(a) $3^7 \times 3^6 =$ (b) $2 \times 2^7 =$ (c) $4^5 \times 4^6 =$

(d) $3^6 \times 3^4 =$ (e) $2^4 \times 2^5 =$ (f) $2^6 \times 2^4 =$

(g) $3^7 \div 3^2 =$ (h) $3 \times 3^6 =$ (i) $3^6 \div 3 =$

(j) $\frac{8^{12}}{8^2} =$ (k) $\frac{7^6}{7^3} =$ (l) $\frac{9^2}{9^0} =$

(m) $4 \times 2^2 =$ (n) $\frac{2^5}{4} =$ (o) $\frac{2^6}{8} =$

6. Fill in the missing powers.

(a) $8 = 2^?$ (b) $1000 = 10^?$ (c) $16 = 2^?$

(d) $27 = 3^?$ (e) $81 = 3^?$ (f) $10\,000 = 10^?$

(g) $625 = 5^?$ (h) $64 = 4^?$ (i) $1296 = 6^?$

(j) $1 = 2^?$ (k) $36 = 6^?$ (l) $1 = 5^?$

7. Simplify the following, giving your answers in index form.

(a) $\left(2^3\right)^2 =$ (b) $\left(3^2\right)^2 =$ (c) $\left(6^2\right)^3 =$

(d) $\left(5^3\right)^2 =$ (e) $\left(2^2\right)^4 =$ (f) $\left(4^2\right)^3 =$

(g) $\left(3^2\right)^4 =$ (h) $\left(5^2\right)^4 =$ (i) $\left(3^3\right)^2 =$

1.2

8. Fill in the missing numbers.

(a) $\left(2^2\right)^4 = 2^?$ (b) $\left(2^?\right)^3 = 2^{12}$ (c) $\left(3^2\right)^5 = ?^{10}$

(d) $\left(5^?\right)^4 = 5^{12}$ (e) $\left(10^5\right)^? = 10^{15}$ (f) $\left(7^5\right)^? = 7^{20}$

9. Simplify each of the following, giving your answer in index notation.

(a) $3^2 \times 3^0 \times 3^4 =$ (b) $2^6 \times 2^7 \times 2 =$ (c) $5^2 \times 5^7 \times 5^3 =$

(d) $\frac{7^2 \times 7^4}{7^3} =$ (e) $\frac{7^4 \times 7^5}{7^2 \times 7^3} =$ (f) $\frac{2^3 \times 2^8}{2^3 \times 2} =$

(g) $\frac{3^2 \times 3^3}{3^5} =$ (h) $\frac{4^7 \times 4^8}{4^5 \times 4^9} =$ (i) $\frac{2^3 \times 2^0}{2^2} =$

10. Simplify each of the following expressions.

(a) $a^3 \times a^2 =$ (b) $a^4 \times a^6 =$ (c) $x^2 \times x^7 =$

(d) $x^4 \div x^2 =$ (e) $y^3 \times y^0 =$ (f) $p^7 \div p^4 =$

(g) $q^6 \div q^3 =$ (h) $x^7 \times x =$ (i) $b^4 \div b =$

(j) $\frac{b^6}{b^0} =$ (k) $\frac{c^7}{c^4} =$ (l) $\frac{x^8}{x^3} =$

(m) $\frac{y^3}{y} =$ (n) $\frac{x^4}{x^4} =$ (o) $x^2 \times x^3 \times x^3 =$

(p) $\frac{p^2 \times p^7}{p^5} =$ (q) $\frac{x^{10}}{x^2 \times x^5} =$ (r) $\frac{y^3 \times y^7}{y^2 \times y^4} =$

(s) $\frac{x^2 \times x^3}{x^5} =$ (t) $\frac{x^7 \times x}{x^3 \times x^4} =$ (u) $\frac{x^8 \times x^4}{x^0} =$

(v) $\left(x^2\right)^4 =$ (w) $\left(x^3\right)^5 =$ (x) $\left(x^2 \times x^7\right)^6 =$

11. 243 can be written as 3^5.

Find the values of p and q in the following:

(a) $64 = 4^p$ (b) $5^q = 1$

(SEG)

12. Express as simply as possible:

$$\frac{4x^2 \times 6x^5}{12x^3}$$

(MEG)

1.3 Factors

A factor of a number will divide exactly into it.

Worked Example 1

List all the factors of 20.

Solution

The factors of 20 are:

$$1,\ 2,\ 4,\ 5,\ 10,\ 20$$

These are all numbers that divide exactly into 20.

Worked Example 2

Write the number 12 as the product of two factors in as many ways as possible.

Solution

$12 = 1 \times 12$	$12 = 4 \times 3$
$12 = 2 \times 6$	$12 = 6 \times 2$
$12 = 3 \times 4$	$12 = 12 \times 1$

Exercises

1. List the factors of these numbers.

(a)	14	(b)	27	(c)	6	(d)	15
(e)	18	(f)	25	(g)	40	(h)	100
(i)	45	(j)	50	(k)	36	(l)	28

2. Write each number below as the product of two factors in as many ways as possible.

(a)	10	(b)	8	(c)	7	(d)	9
(e)	16	(f)	22	(g)	11	(h)	24

3. Fill in the missing numbers.

(a)	$32 = 4 \times 2 \times ?$	(b)	$45 = ? \times 3 \times 5$
(c)	$27 = 3 \times 3 \times ?$	(d)	$40 = 5 \times ? \times 2$
(e)	$50 = 5 \times 2 \times ?$	(f)	$88 = 11 \times 2 \times ?$
(g)	$66 = 2 \times 3 \times ?$	(h)	$21 = ? \times 3 \times 7$

1.3

4. Here is a Bingo card.

6		10		20		9	
	3		8		17		15
2		24		55		4	

(a) Circle those numbers that 2 will divide into exactly.

(b) Cross out those numbers that 5 will divide into exactly.

(LON)

5. 20 21 22 23 24 25 26 27 29

(a) In the row of numbers above:

(i) *circle* all numbers divisible by 2, e.g. 20

(ii) *cross out* all numbers divisible by 3, e.g. ~~24~~

(iii) *underline* all numbers divisible by 5. e.g. 25

(b) Describe the numbers which are not circled, crossed out or underlined.

(MEG)

6. A pattern of counting numbers is shown.

14, 15, 16, 17, 18, 19, 20, ...

(a) (i) Which of these numbers is a square number?

(ii) Which of these numbers is a multiple of nine?

The pattern is continued.

(b) (i) What is the next square number?

(ii) What is the next number that is a multiple of nine?

(SEG)

1.4 Prime Factors

Any number can be written as the product of a number of prime factors. For example,

$$20 = 2^2 \times 5$$

or

$$180 = 2^2 \times 3^2 \times 5 .$$

Note

A *prime number* is a number that can be divided exactly by only 1 and itself.

The first few prime numbers are 2, 3, 5, 7, 11, ...

1.4

Worked Example 1

Write the number 276 as a product of prime numbers.

Solution

Write 276 as a product of two factors:

$$276 = 2 \times 138$$

But $138 = 2 \times 69$ so $276 = 2 \times 2 \times 69$

But $69 = 3 \times 23$ so $276 = 2 \times 2 \times 3 \times 23$

This expression contains only prime numbers, so

$$276 = 2^2 \times 3 \times 23 .$$

This is called the *product of prime factors*.

Worked Example 2

(a) Write the numbers 660 and 470 as the product of prime factors.

(b) Find the largest common factor that will divide into both 660 and 470.

Solution

(a)

$$\begin{aligned} 660 &= 2 \times 330 \\ &= 2 \times 2 \times 165 \\ &= 2 \times 2 \times 3 \times 55 \\ &= 2 \times 2 \times 3 \times 5 \times 11 \end{aligned}$$

So as a product of prime factors,

$$660 = 2^2 \times 3 \times 5 \times 11.$$

$$\begin{aligned} 470 &= 2 \times 235 \\ &= 2 \times 5 \times 47 \end{aligned}$$

So as a product of prime factors,

$$470 = 2 \times 5 \times 47.$$

(b) To find the largest common factor that will divide into both 660 and 470, look at the factors common to each of the products of primes.

The numbers that appear in both are 2 and 5, so the largest number that will divide into both 660 and 470 is $2 \times 5 = 10$.

This number is called the *highest common factor* or *HCF*.

1.4

Exercises

1. Which of the following are prime numbers?

   1, 2, 3, 5, 7, 9, 13, 15, 18, 19, 21, 23, 25

2. Which numbers between 50 and 60 are prime numbers?

3. Write each number below as a product of prime factors.

(a)	10	(b)	42	(c)	68
(d)	168	(e)	250	(f)	270
(g)	429	(h)	825	(i)	1001

4. (a) Express 32 and 56 as the product of prime factors.

   (b) By comparing the answers to (a) find the HCF of 32 and 56.

5. Find the highest common factors of each pair of numbers below.

(a)	36, 42	(b)	30, 42	(c)	45, 105
(d)	42, 50	(e)	50, 80	(f)	70, 315
(g)	216, 240	(h)	156, 234	(i)	735, 1617

6. (a) Express each of the following numbers as the product of prime factors:

   45, 99, 135.

   (b) By considering the products of the prime factors, find the highest common factor of

   (i) 45 and 99 (ii) 99 and 135 (iii) 45 and 135

   (c) What is the highest common factor of all three numbers?

7. Find the highest common factor (HCF) for each set of three numbers given below.

(a)	20, 35, 105	(b)	90, 225, 405	(c)	16, 24, 56
(d)	200, 210, 220	(e)	72, 168, 312	(f)	330, 450, 630
(g)	216, 324, 432	(h)	660, 572, 528	(i)	1008, 1260, 1764

Investigation

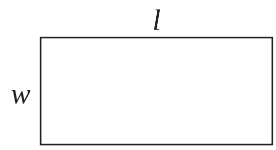
Han Sin, a Chinese general, devised a method to count the number of soldiers that he had. First, he ordered his soldiers to form groups of 3, followed by groups of 5 and then groups of 7. In each case he noted down the remainder. Using the three remainders, he was able to calculate the exact number of soldiers he had without doing the actual counting.
Do you know how he did it?

1.5 Using Formulae

In formulae, letters are used to represent numbers. For example, the formula

$$A = lw$$

can be used to find the area of a rectangle. Here A is the area, l the length and w the width. In this formula, lw means $l \times w$. Formulae are usually written in this way without multiplication signs.



The perimeter of the rectangle would be given by the formula

$$P = 2l + 2w$$

Here again there are no multiplication signs, and $2l$ means $2 \times l$ and $2w$ means $2 \times w$.

Worked Example 1

The perimeter of a rectangle can be found using the formula

$$P = 2l + 2w$$

Find the perimeter if $l = 8$ and $w = 4$.

Solution

The letters l and w should be replaced by the numbers 8 and 4.

This gives

$$\begin{aligned} P &= 2 \times 8 + 2 \times 4 \\ &= 16 + 8 \\ &= 24 \end{aligned}$$

Worked Example 2

The final speed of a car is v and can be calculated using the formula

$$v = u + at$$

where u is the initial speed, a is the acceleration and t is the time taken.

Find v if the acceleration is 2 m s^{-1}, the time taken is 10 seconds and the initial speed is 4 m s^{-1}.

Solution

The acceleration is 2 m s^{-1} so $a = 2$. The initial speed is 4 m s^{-1} so $u = 4$.

The time taken is 10 s so $t = 10$.

Using the formula

$$v = u + at$$

gives

$$\begin{aligned} v &= 4 + 2 \times 10 \\ &= 4 + 20 \\ &= 24 \text{ m s}^{-1} \end{aligned}$$

1.5

Exercises

1. The area of a rectangle is found using the formula $A = lw$ and the perimeter using $P = 2l + 2w$. Find the area and perimeter if:

   (a) $l = 4$ and $w = 2$ (b) $l = 10$ and $w = 3$

   (c) $l = 11$ and $w = 2$ (d) $l = 5$ and $w = 4$

2. The formula $v = u + at$ is used to find the final speed.

   Find v if`:

   (a) $u = 6$, $a = 2$ and $t = 5$ (b) $u = 0$, $a = 4$ and $t = 3$

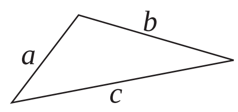
   (c) $u = 3$, $a = 1$ and $t = 12$ (d) $u = 12$, $a = 2$ and $t = 4$

3. Use the formula $F = ma$ to find F if:

   (a) $m = 10$ and $a = 3$ (b) $m = 200$ and $a = 2$

4. The perimeter of a triangle is found using the formula

$$P = a + b + c$$



   Find P if:

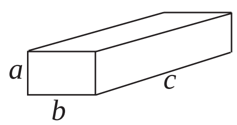
   (a) $a = 10$, $b = 12$ and $c = 8$

   (b) $a = 3$, $b = 4$ and $c = 5$

   (c) $a = 6$, $b = 4$ and $c = 7$

5. The volume of a box is given by the formula

$$V = abc$$



   Find V if:

   (a) $a = 2$, $b = 3$ and $c = 10$

   (b) $a = 7$, $b = 5$ and $c = 3$

   (c) $a = 4$, $b = 4$ and $c = 9$

6. Find the value of Q for each formula using the values given.

   (a) $Q = 3x + 7y$, $x = 4$ and $y = 2$ (b) $Q = x^2 + y$, $x = 3$ and $y = 5$

   (c) $Q = xy + 4$, $x = 3$ and $y = 5$ (d) $Q = 5x - 2y$, $x = 10$ and $y = 26$.

1.5

(e) $Q = xy - 2$

$x = 10$ and $y = 2$

(f) $Q = \dfrac{x}{y}$

$x = 24$ and $y = 2$

(g) $Q = \dfrac{x + 4}{y}$

$x = 8$ and $y = 3$

(h) $Q = \dfrac{4x + 2}{y}$

$x = 5$ and $y = 11$

(i) $Q = 3x + 2y + z$

$x = 4$, $y = 2$ and $z = 10$

(j) $Q = xy + yz$

$x = 2$, $y = 5$ and $z = 8$

(k) $Q = xyz$

$x = 2$, $y = 5$ and $z = 3$

(l) $Q = xy + 4z$

$x = 8$, $y = 3$ and $z = 4$

(m) $Q = \dfrac{x + y}{z}$

$x = 8$, $y = 10$ and $z = 3$

(n) $Q = \dfrac{x}{y + z}$

$x = 50$, $y = 2$ and $z = 3$

7. This formula is used to work out Sharon's pay.

Sharon works for 40 hours.
Her rate of pay is £3 per hour.

Pay = Number of hours worked × Rate of pay + £10.

Work out her pay.

(LON)

8. A rectangle has a length of *a* cm and a width of *b* cm.

The perimeter of a rectangle is given by the formula $p = 2(a + b)$.

Calculate the perimeter of a rectangle when $a = 4.5$ and $b = 4.2$.

(SEG)

1.6 Construct and Use Simple Formulae

A *formula* describes how one quantity relates to one or more other quantities. For example, a formula for the area of a rectangle describes how to find the area, given the length and width of the rectangle.

The perimeter of the rectangle would be given by the formula

$$P = 2l + 2w$$

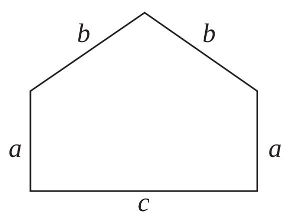
Here again there are no multiplication signs and $2l$ means $2 \times l$ and $2w$ means $2 \times w$.

Worked Example 1

(a) Write down a formula for the perimeter of the shape shown.

(b) Find the perimeter if

$a = 2$ cm, $b = 3$ cm and $c = 5$ cm



1.6

Solution

(a) The perimeter is found by adding together the lengths of all the sides, so the formula will be

$$P = a + b + b + a + c$$

but as a and b are both added in twice, this can be simplified to

$$P = 2a + 2b + c$$

(b) If $a = 2$, $b = 3$ and $c = 5$

$$P = 2 \times 2 + 2 \times 3 + 5$$

$$= 4 + 6 + 5$$

$$= 15 \text{ cm}$$

Worked Example 2

An emergency engineer charges a basic fee of £20, plus £8 per hour, when repairing central heating systems.

Find a formula for calculating the engineer's charge.

Solution

Let C = charge and n = number of hours.

The charge is made up of

a fixed £20 and £8 $\times$ the number of hours, or £$8n$.

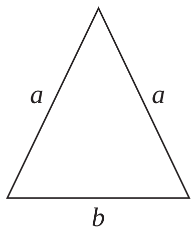
So the total charge is given by

$$C = 20 + 8n$$

Exercises

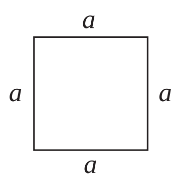
1. Find a formula for the perimeter of each shape, and find the perimeter for the specified values.

(a)



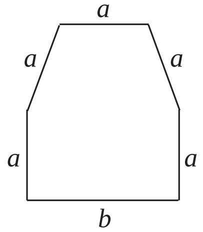
$a = 6$ cm, $b = 4$ cm

(b)



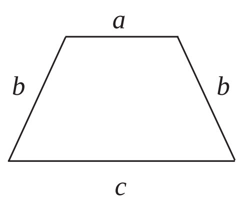
$a = 5$

(c)



$a = 6$ cm, $b = 10$ cm

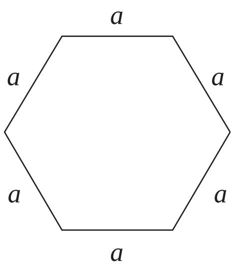
(d)



$a = 5$ cm, $b = 6$ cm, $c = 10$ cm

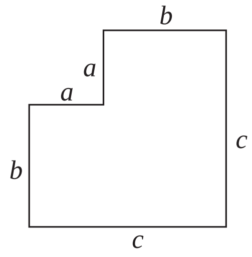
1.6

(e)



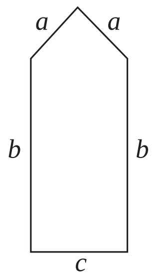
$a = 10$ cm

(f)



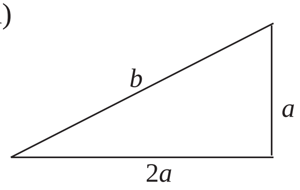
$a = 4$ cm, $b = 5$ cm, $c = 9$ cm

(g)



$a = 60$ cm, $b = 160$ cm, $c = 80$ cm

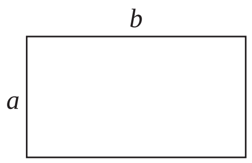
(h)



$a = 4$ cm, $b = 9$ cm

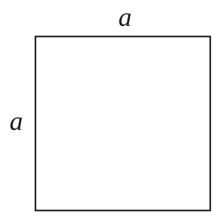
2. Find a formula for the area of each of the shapes below and find the area for the values given.

(a)



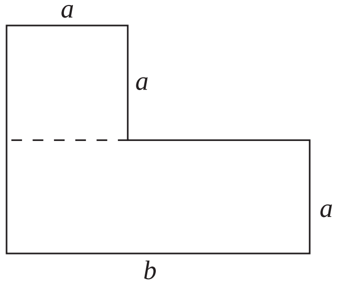
$a = 6$ cm, $b = 10$ cm

(b)



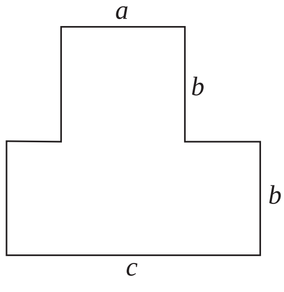
$a = 3$ cm

(c)



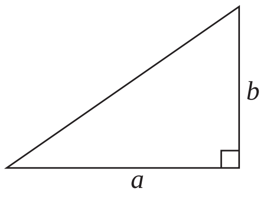
$a = 2$ cm, $b = 8$ cm

(d)



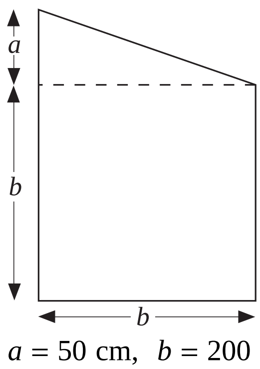
$a = 3$ cm, $b = 4$ cm, $c = 9$ cm

(e)



$a = 4$ cm, $b = 5$ cm

(f)



$a = 50$ cm, $b = 200$ cm

1.6

3. Three consecutive numbers are to be added together.

   (a) If x is the smallest number, what are the other two?

   (b) Write down a formula for the total, T, of the three numbers, using your answer to (a).

4. (a) Write down a formula to find the mean, M, of the two numbers x and y.

   (b) Write down a formula to find the mean, M, of the five numbers $p, q, r, s,$ and t.

5. Tickets for a school concert are sold at £3 for adults and £2 for children.

   (a) If p adults and q children buy tickets, write a formula for the total value, T, of the ticket sales.

   (b) Find the total value of the ticket sales if $p = 50$ and $q = 20$.

6. A rectangle is 3 cm longer than it is wide.

   If x is the width, write down a formula for:

   (a) the perimeter; P;

   (b) the area, A, of the rectangle.

7. Rachel is one year older than Ben. Emma is three years younger than Ben

   If Ben is x years old, write down expressions for:

   (a) Rachel's age;

   (b) Emma's age;

   (c) the sum of all three children's ages.

8. A window cleaner charges a fee of £3 for visiting a house and £2 for every window that he cleans.

   (a) Write down a formula for finding the total cost C when n windows are cleaned.

   (b) Find C if $n = 8$.

9. A taxi driver charges a fee of £1, plus £2 for every mile that the taxi travels.

   (a) Find a formula for the cost C of a journey that covers m miles.

   (b) Find C if $m = 3$.

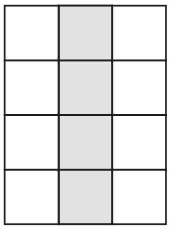
   (a) most likely to happen?

   (b) least likely to happen?

(LON)

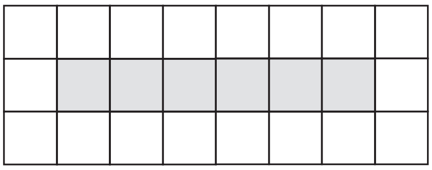
1.6

10. A gardener builds paths using paving slabs laid out in a pattern as shown, with white slabs on each side of a row of red slabs.



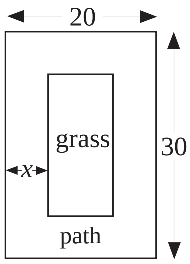
(a) If n red slabs are used, how many white slabs are needed?

(b) Another gardener puts a white slab at each end of the path as shown below.



If n red slabs are used, how many white slabs are needed?

11. A path of width x is laid around a rectangular lawn as shown.



(a) Find an expression for the perimeter of the grass.

(b) Find an expression for the area of the grass.

12. Choc Bars cost 27 pence each.

Write down a formula for the cost, C pence, of n Choc Bars.

(LON)

13. (a) Petrol costs 45 pence per litre.

Write down a formula for the cost, C pence, of l litres of petrol.

(b) Petrol costs x pence per litre.

Write down a formula for the cost, C pence, of l litres of petrol.

(SEG)

14. (a) Vijay earns £P in his first year of work.

The following year his salary is increased by £Q.

Write down an expression for his salary in his second year.

(b) Julie earns £X in her first year of work.

Her salary is increased by £650 every year.

How much will she earn in

(i) the 5th year (ii) the nth year?

1.7 Substitution into Formulae

The process of replacing the letters in a formula is known as *substitution*.

Worked Example 1

The length of a metal rod is l. The length changes with temperature and can be found by the formula

$$l = 40 + 0.02\,T$$

where T is the temperature.

Find the length of the rod when

(a) $T = 50$ °C and (b) $T = -10$ °C

Solution

(a) Using $T = 50$ gives

$$\begin{aligned} l &= 40 + 50 \times 0.02 \\ &= 40 + 1 \\ &= 41 \end{aligned}$$

(b) Using $T = -10$ gives

$$\begin{aligned} l &= 40 + (-10) \times 0.02 \\ &= 40 + (-0.2) \\ &= 40 - 0.2 \\ &= 39.8 \end{aligned}$$

Worked Example 2

The profit made by a salesman when he makes sales on a day is calculated with the formula

$$P = 4n - 50$$

Find the profit if he makes

(a) 30 sales (b) 9 sales

Solution

(a) Here $n = 30$ so the formula gives

$$\begin{aligned} P &= 4 \times 30 - 50 \\ &= 120 - 50 \\ &= 70 \end{aligned}$$

(b) Here $n = 9$ so the formula gives

$$\begin{aligned} P &= 4 \times 9 - 50 \\ &= 36 - 50 \\ &= -14 \end{aligned}$$

So a loss is made if only 9 sales are made.

1.7

Exercises

1. The formula below is used to convert temperatures in degrees Celsius to degrees Fahrenheit, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

$$F = 1.8\,C + 32$$

Find F if:

(a) $C = 10$ (b) $C = 20$ (c) $C = -10$

(d) $C = -5$ (e) $C = -20$ (f) $C = 15$

2. The formula

$$s = \frac{1}{2}(u + v)t$$

is used to calculate the distance, s, that an object travels if it starts with a velocity u and has a velocity v, t seconds later.

Find s if:

(a) $u = 2, \ v = 8, \ t = 2$ (b) $u = 3, \ v = 5, \ t = 10$

(c) $u = 1.2, \ v = 3.8, \ t = 4.5$ (d) $u = -4, \ v = 8, \ t = 2$

(e) $u = 4, \ v = -8, \ t = 5$ (f) $u = 1.6, \ v = 2.8, \ t = 3.2$

3. The length, l, of a spring is given by the formula

$$l = 20 - 0.08F$$

where F is the size of the force applied to the spring to compress it.

Find l if:

(a) $F = 5$ (b) $F = 20$

(c) $F = 24$ (d) $F = 15$

4. The formula

$$P = 120n - 400$$

gives the profit, P, made when n cars are sold in a day at a showroom.

Find P if:

(a) $n = 1$ (b) $n = 3$

(c) $n = 4$ (d) $n = 10$

How many cars must be sold to make a profit?

1.7

5. Work out the value of each function by substituting the values given, *without* using a calculator.

(a) $V = p^2 + q^2$
$p = 8$ and $q = 4$

(b) $p = a^2 - b^2$
$a = 10$ and $b = 7$

(c) $z = \sqrt{x + y}$
$x = 10$ and $y = 6$

(d) $Q = \sqrt{x - y}$
$x = 15$ and $y = 6$

(e) $P = \dfrac{x + y}{2}$
$x = 4$ and $y = -10$

(f) $Q = \sqrt{\dfrac{a}{b}}$
$a = 100$ and $b = 4$

(g) $V = \dfrac{x + 2y + z}{5}$
$x = 2$, $y = -5$ and $z = 8$

(h) $R = \dfrac{1}{a} + \dfrac{1}{b}$
$a = 4$ and $b = 2$

(i) $S = \dfrac{a}{b} + \dfrac{b}{c}$
$a = 3$, $b = 4$ and $c = 16$

(j) $R = 0.2a + 0.4b$
$a = 10$ and $b = 20$

(k) $T = \dfrac{a}{2} + \dfrac{b}{5}$
$a = -20$ and $b = 40$

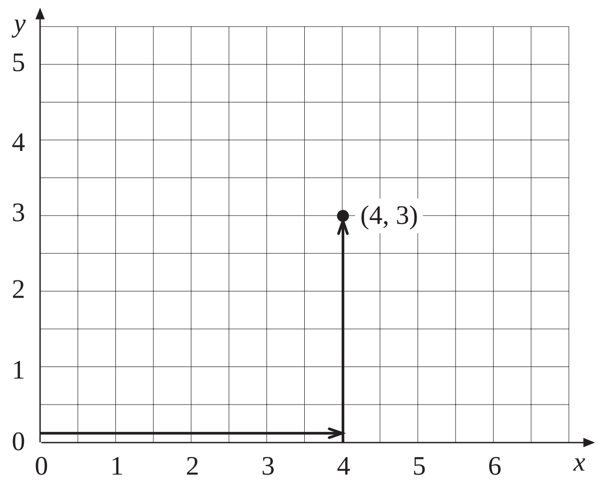
(l) $C = \dfrac{ab}{a + b}$
$a = 10$ and $b = -5$

(m) $P = 2\sqrt{\dfrac{x^2}{y}}$
$x = 10$ and $y = 4$

(n) $A = \dfrac{ab^2}{c}$
$a = 2$, $b = 3$ and $c = 100$

1.8 Positive Coordinates

Coordinates are pairs of numbers that uniquely describe a position on a rectangular grid. The first number refers to the horizontal (x-axis) and the second the vertical (y-axis). The coordinates (4, 3) describe a point that is 4 units across and 3 units up on a grid from the origin (0, 0).



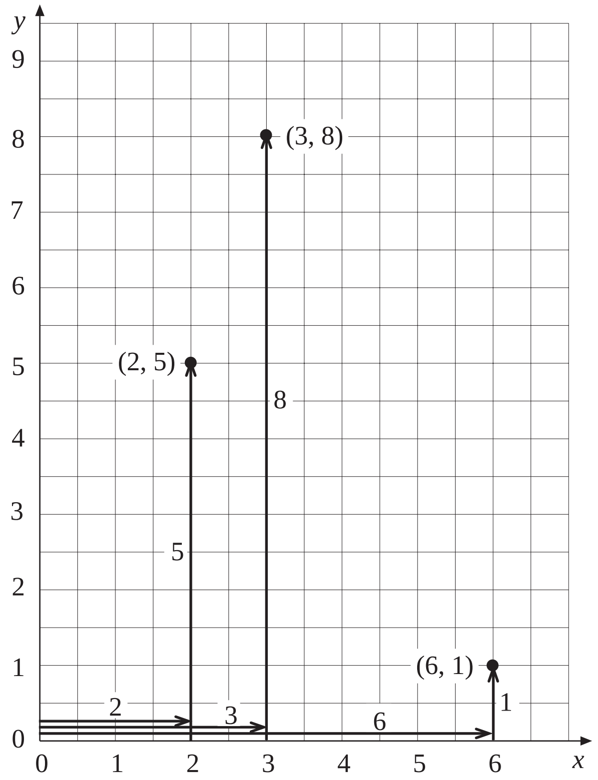
1.8

Worked Example 1

Plot the points with coordinates

(3, 8), (6, 1) and (2, 5)

Solution



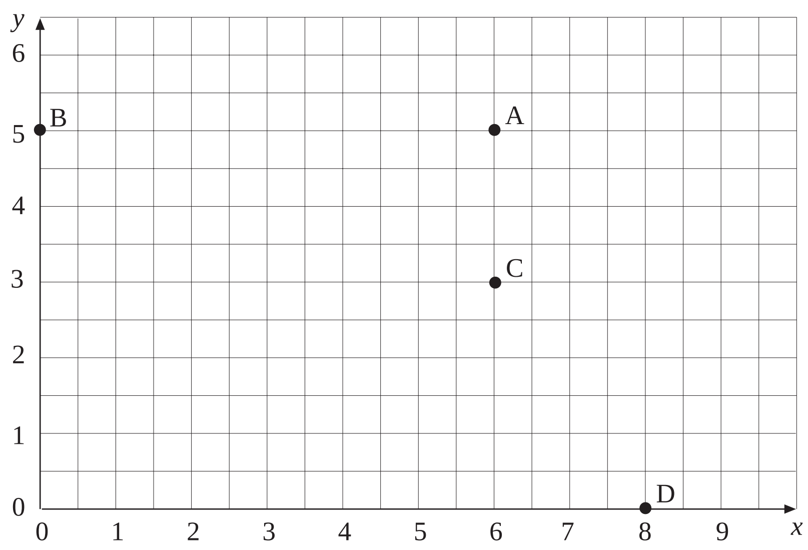
For (3, 8) move 3 across and 8 up.

For (6, 1) move 6 across and 1 up.

For (2, 5) move 2 across and 5 up.

Worked Example 2

Write down the coordinates of each point in the diagram below.



Solution

A is 6 across and 5 up, so the coordinates are (6, 5).

B has no movement across and is straight up 5, so the coordinates are (0, 5).

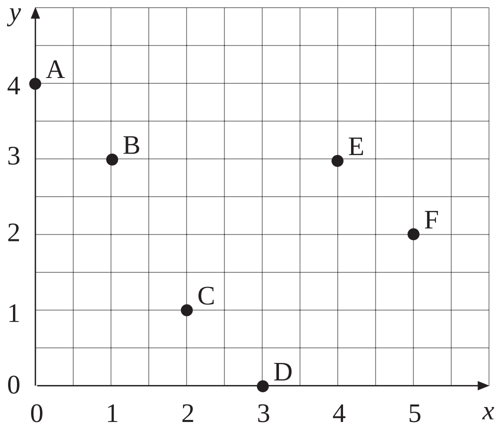
1.8

C is 6 across and 3 up, so the coordinates are (6, 3).

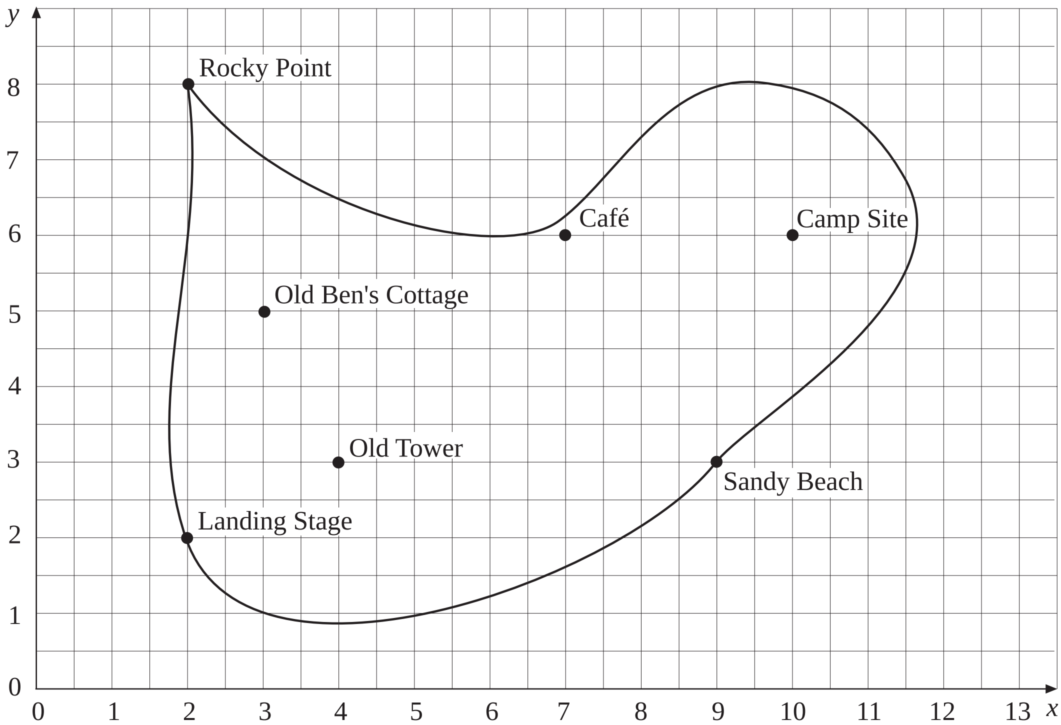
D is 8 across and no movement up, so the coordinates are (8, 0).

Exercises

1. Write down the coordinates of each point on the diagram below.



2. The map of an island has been drawn on a grid.



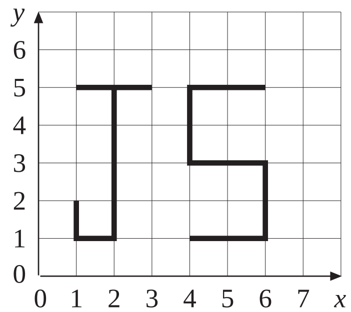
Write down the coordinates of each place marked on the map.

1.8

3. On a grid, join the points with the following coordinates and write down the name of the shape you draw.

(a)	(4, 2)	(8, 2)	(8, 5)	(4, 5)		
(b)	(2, 1)	(6, 1)	(4, 6)			
(c)	(1, 4)	(3, 7)	(5, 4)	(3, 1)		
(d)	(4, 0)	(3, 2)	(5, 4)	(7, 2)	(6, 0)	
(e)	(1, 1)	(0, 3)	(1, 5)	(3, 5)	(4, 3)	(3, 1)

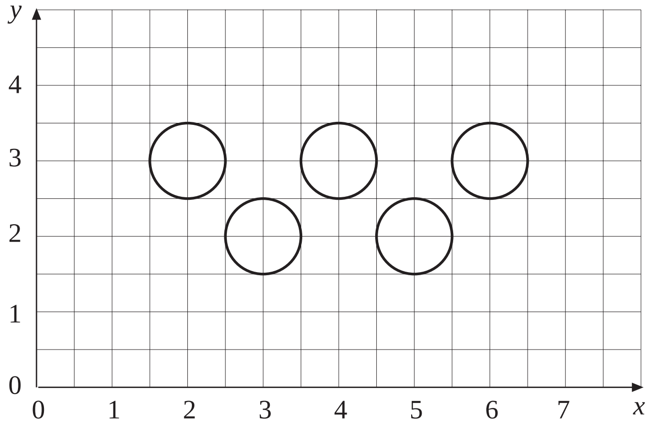
4. Jenny writes her initials on a grid.



(a) Write down the coordinates of the corners of each letter.

(b) Write your initials in the same way and write down the coordinates of your initials.

5. The pattern below is made up of 5 circles. Write down the coordinates of the centre of each circle.



6. (a) On the co-ordinate grid opposite, plot the following points

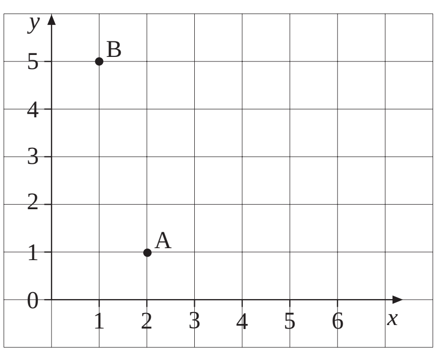
P (3,4), Q (0,2) R (4,0)

(b) Write down the co-ordinates of the points

(i) A,

(ii) B.

(LON)

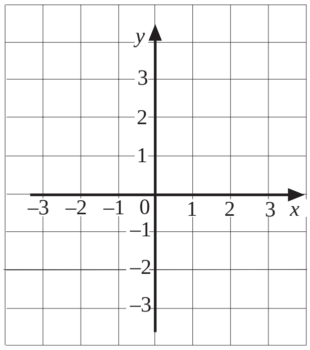


1.9 Coordinates

The coordinates of a point are written as a pair of numbers, (x, y), which describe where the point is on a set of axes.

The x-axis is always horizontal (i.e. *across* the page) and the y-axis always vertical (i.e. *up* the page).

The x-coordinate is always given first and the y-coordinate second.



Worked Example 1

On a grid, plot the point A which has coordinates (–2, 4), the point B with coordinates (3, –2) and the point C with coordinates (– 4, –3).

Solution

For A, begin at (0, 0), where the two axes cross.

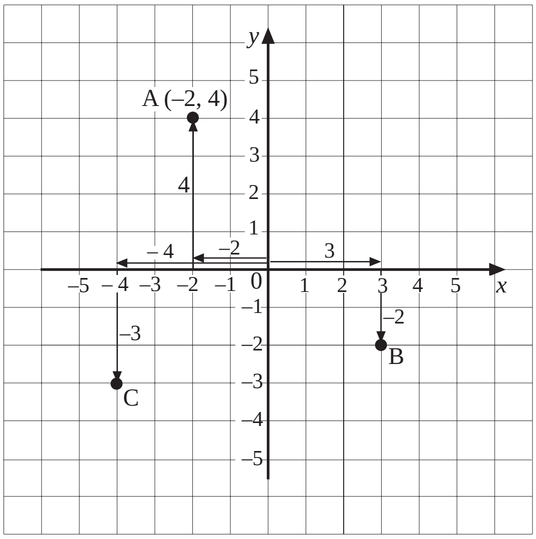
Move –2 in the x direction.

Move 4 in the y direction.

Points B and C are plotted in a similar way.

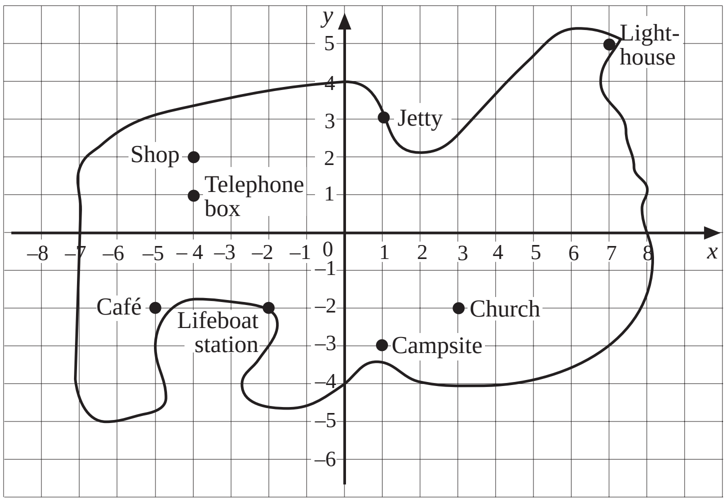
For B, move 3 in the x direction and –2 in the y direction.

For C, move – 4 in the x direction and –3 in the y direction.



Worked Example 2

Write down the coordinates of each place on the map of the island.



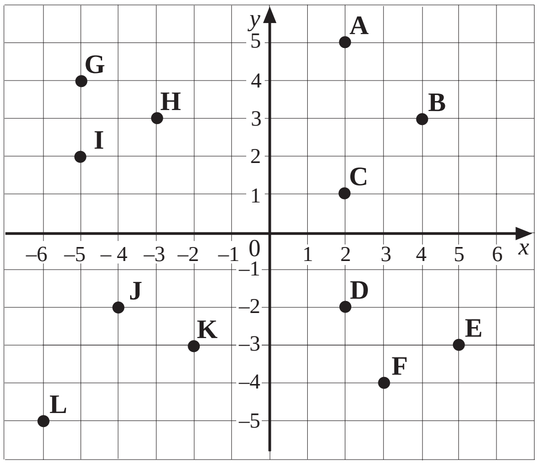
1.9

Solution

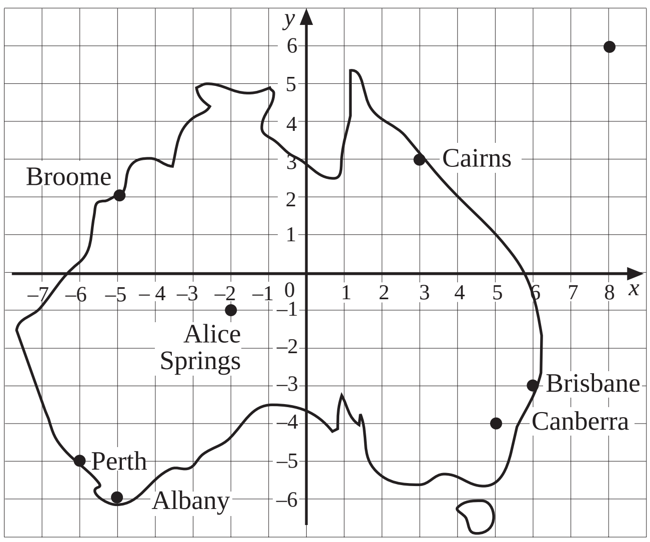
Lighthouse	(7, 5)	}	All coordinates positive
Jetty	(1, 3)		
Church	(3, −2)	}	Negative y-coordinates
Camp Site	(1, −3)		
Shop	(−4, 2)	}	Negative x-coordinates
Telephone Box	(−4, 1)		
Café	(−5, −2)	}	All coordinates negative
Lifeboat Station	(−2, −2)		

Exercises

1. Write down the coordinates of each point marked on the grid below.



2. The map shows some Australian towns and cities.



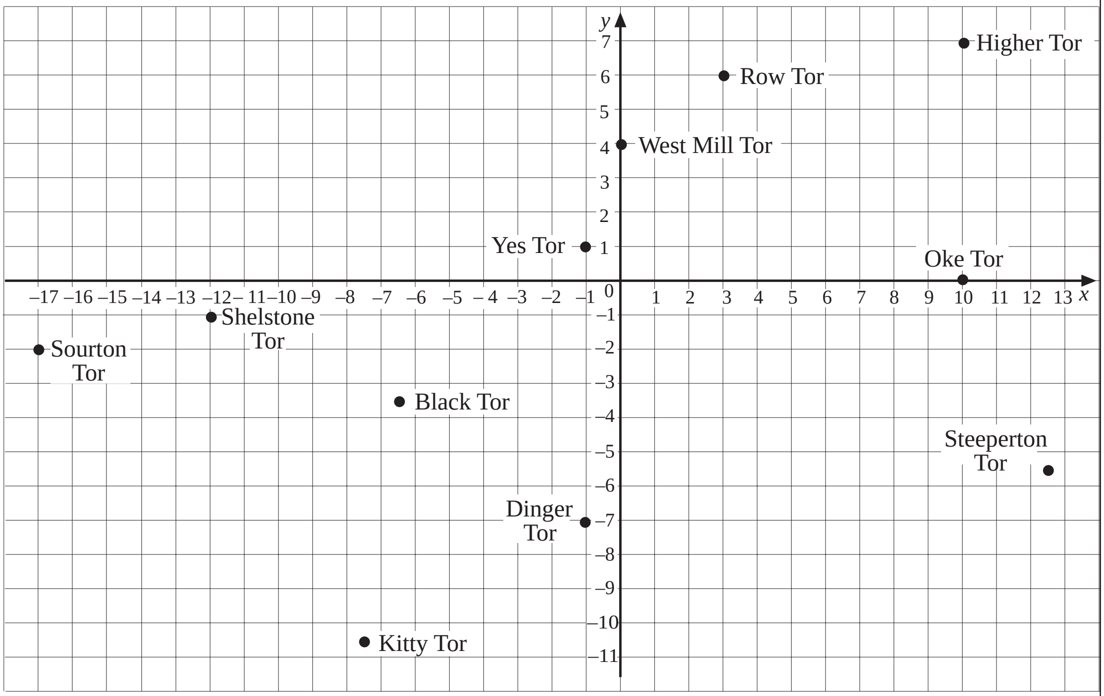
1.9

(a) Write down the coordinates of Canberra, Brisbane and Perth.

(b) A plane flies from the place with coordinates (–5, –6) and lands at the place with coordinates (–2, –1). From where does the plane take off and where does it land?

(c) A ship has coordinates (–5, 2) at the start of a voyage and coordinates (–6, –5) at the end. Where does it start and where does it finish?

3. The map shows some of the tors (rocky outcrops) on Dartmoor in the south west of England.



(a) Write down the coordinates of the following tors.

West Mill Tor

Steeperton Tor

Shelstone Tor

Black Tor

Dinger Tor

(b) The highest tor marked on this map is Yes Tor. Write down the coordinates of this tor.

(c) A boy and his dog walk from Oke Tor to Kitty Tor. Write down the coordinates of the point where they start and the point where they finish.

(d) Sourton Tor is the tor that is the farthest west on this map. What are the coordinates of this tor?

(e) Higher Tor is the tor that is farthest north. What are the coordinates of this tor?

1.9

4. Draw a set of axes with x-values from –5 to 5 and y-values from –3 to 9.

   (a) Join together the points with coordinates (5, 0), (0, 9) and (–5, 0). What shape do you get?

   (b) On the same diagram, join together the points (5, 6), (–5, 6) and (0, –3).

5. Draw a set of axes with x-values from – 4 to 4 and y-values from – 4 to 3.

   Join each set of points below in the order listed.

   (a) (2, 0), (1, 1), (1, 2), (2, 3), (3, 3), (4, 2), (4, 1), (3, 0).

   (b) (0, 1), (1, –1), (–1, –1), (0, 1).

   (c) (–2, 0), (–1, 1), (–1, 2), (–2, 3), (–3, 3), (– 4, 2), (– 4, 1), (–3, 0).

   (d) (3, –1), (2, –3), (0, – 4), (–2, –3), (–3, –1), (–1, –2), (1, –2), (3, –1).

6. (a) Draw a set of axes with x-values from – 4 to 4 and y-values from –5 to 4.

   (b) Plot the following points and join them in the order listed.

   (3, –5), (2, –5), (– 4, –2), (–2, –3), (0, –2), (0, 0), (3, 2), (3, 3), (4, 3), (4, 2), (3, 2), (3, 0), (–2, 2).

7. Three corners of a square have coordinates (4, 2), (–2, 2) and (4, – 4).

   (a) Draw a set of axes with x-values from –2 to 4 and y-values from – 4 to 2. Plot the three points and draw the square.

   (b) Write down the coordinates of the centre of the square.

8. Two corners of a rectangle have coordinates (–3, –1) and (–3, 3). The centre of the rectangle has coordinates (1, 1).

   (a) Plot the three points given and draw the rectangle.

   (b) Write down the coordinates of the other two corners of the rectangle.

9. A dodecagon is a twelve-sided, plane shape.

   (a) Draw a set of axes that have x-values from –7 to 1 and y-values from 0 to 8.

   (b) Plot the points listed below and join them to draw half a dodecagon.

   (–3, 0), (–2, 0), (0, 1), (1, 3), (1, 5), (0, 7), (–2, 8), (–3, 8)

   (c) Draw the other half of the dodecagon. It is symmetric about the line joining (–3, 0) to (–3, 8).

   (d) Write down the coordinates of the six new corners of the dodecagon that you have drawn.

1.9

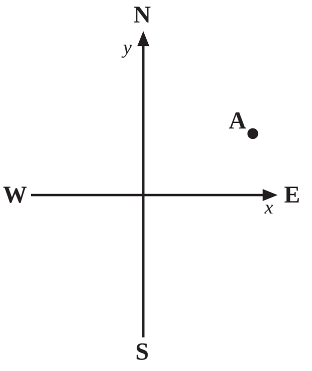
10. A set of axes are arranged so that the x-axis runs from west to east and the y-axis from south to north.

A ship is at the point A which has coordinates (4, 2).

(a) How far south can the ship travel before its y-coordinate becomes negative?

(b) How far west can the ship travel before its x-coordinate becomes negative?

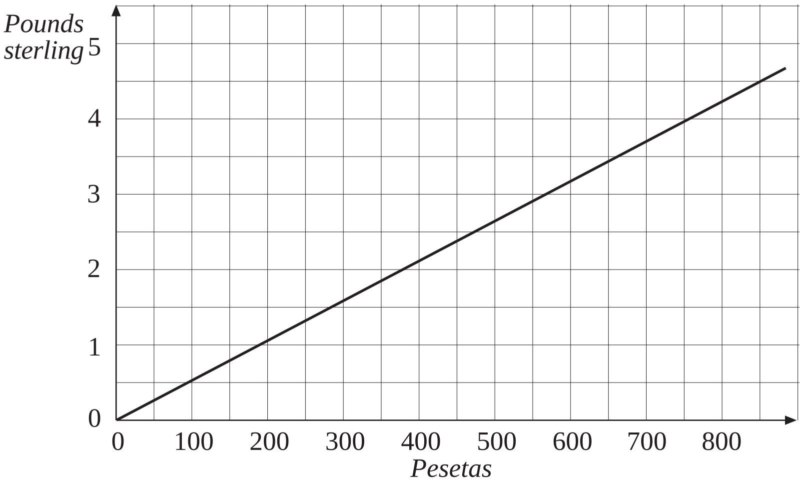
(c) If the ship travels SW, how far does it travel before both coordinates become negative?



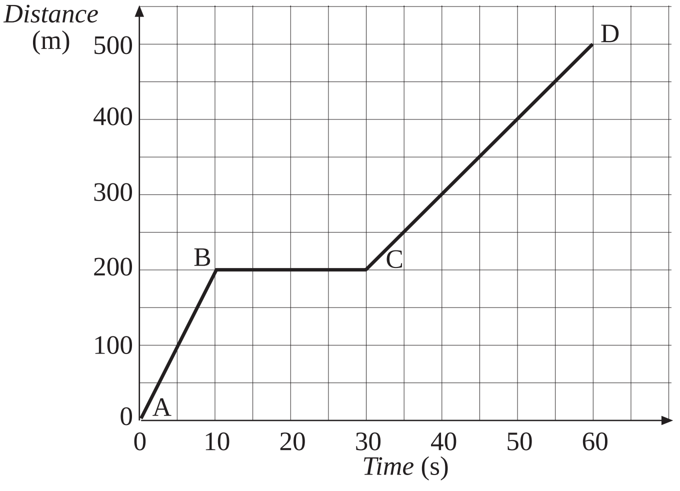
1.10 Applications of Graphs

In this section some applications of graphs are considered, particularly *conversion graphs* and graphs to describe *motion.*

The graph below can be used for converting pounds sterling (British pounds) into and from Spanish pesetas.

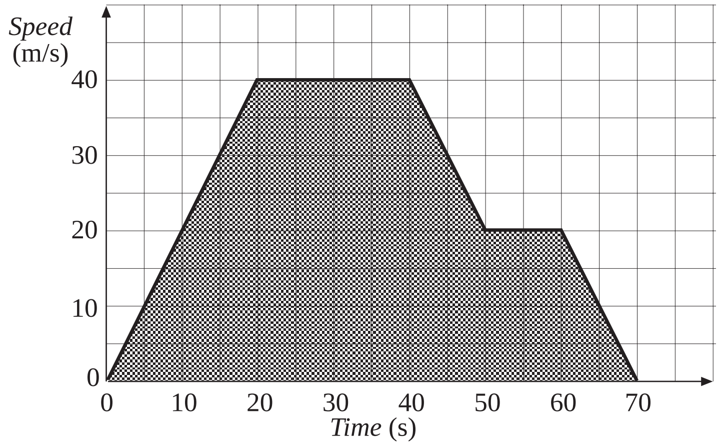


A distance-time graph of a car is shown below. The gradient of this graph gives the speed of the car. The gradient is steepest from A to B, so this is when the care has the greatest speed. The gradient BC is zero, so the car is not moving.



1.10

The area under a speed-time graph gives the distance travelled. Finding the shaded area on the graph below would give the distance travelled.



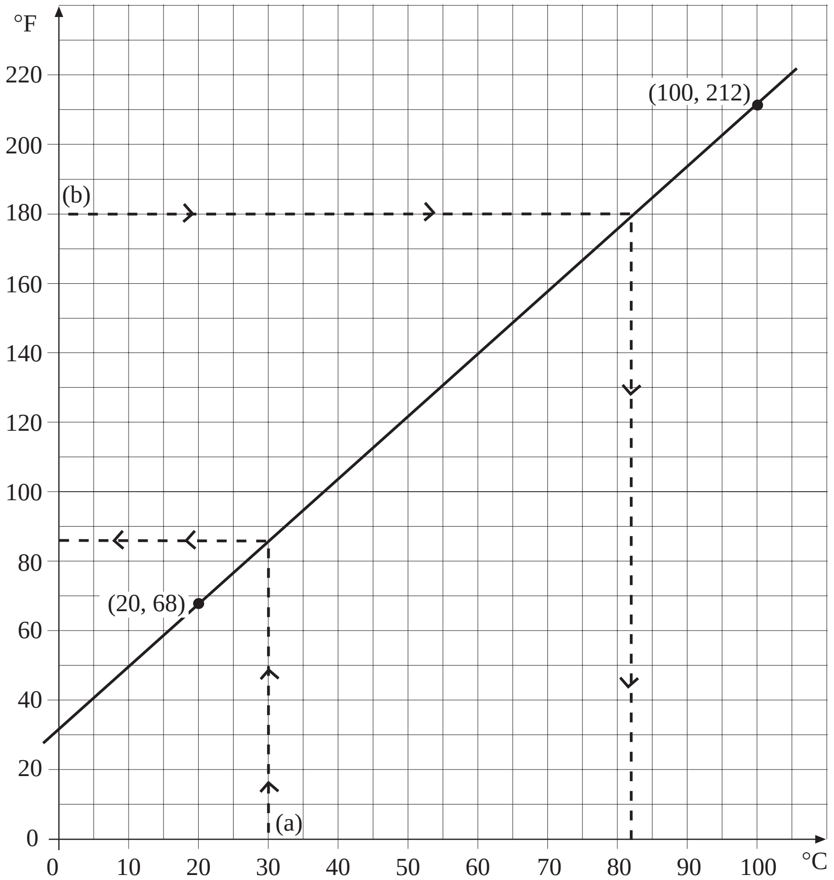
Worked Example 1

A temperature of 20 °C is equivalent to 68 °F and a temperature of 100 °C is equivalent to a temperature of 212 °F. Use this information to draw a conversion graph. Use the graph to convert:

(a) 30 °C to °Fahrenheit, b) 180 °F to °Celsius.

Solution

Taking the horizontal axis as temperature in °C and the vertical axis as temperature in °F gives two pairs of coordinates, (20, 68) and (100, 212). These are plotted on a graph and a straight line drawn through the points.



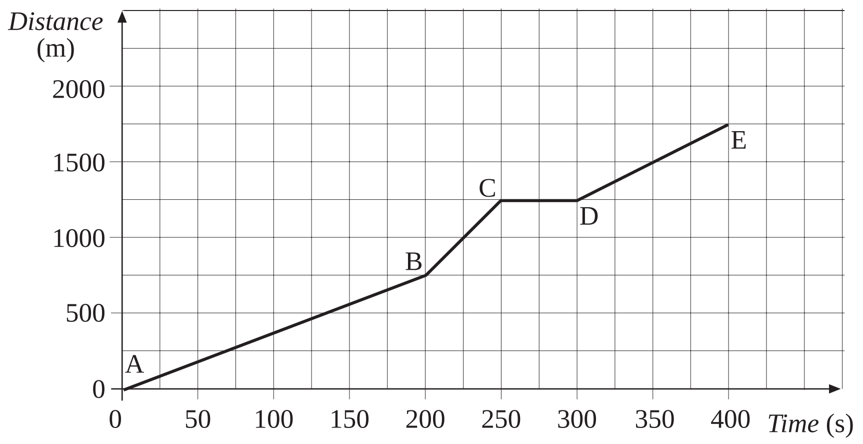
1.10

(a) Start at $30\ °C$, then move up to the line and across to the vertical axis, to give a temperature of about $86\ °F$.

(b) Start at $180\ °F$, then move across to the line and down to the horizontal axis, to give a temperature of about $82\ °C$.

Worked Example 2

The graph shows the distance travelled by a girl on a bike.



Find the speed she is travelling on each stage of the journey.

Solution

For AB the gradient $= \dfrac{750}{200}$

$= 3.75$

So the speed is 3.75 m/s.

Note

The units are m/s (metres per second), as m are the units for distance and s the units for time.

For BC the gradient $= \dfrac{500}{50}$

$= 10$

So the speed is 10 m/s.

For CD the gradient is zero and so the speed is zero

For DE the gradient is $= \dfrac{500}{200}$

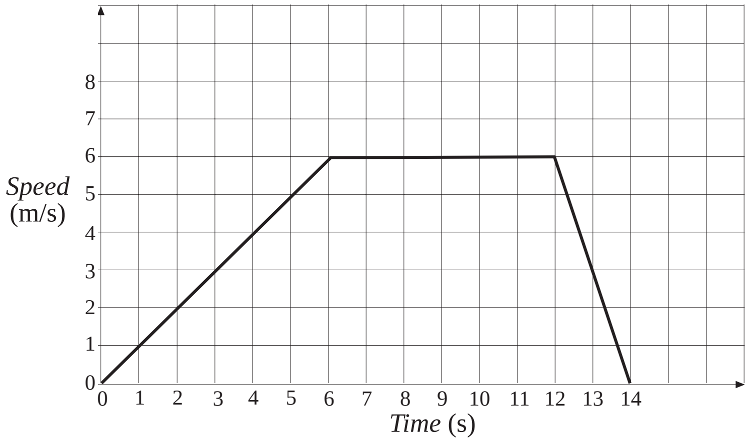
$= 2.5$

So the speed is 2.5 m/s.

1.10

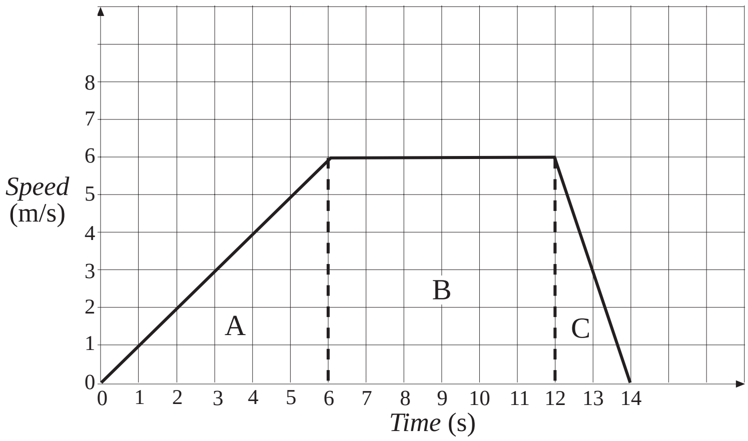
Worked Example 3

The graph shows how the speed of a bird varies as it flies between two trees. How far apart are the two trees?



Solution

The distance is given by the area under the graph. In order to find this area it has been split into three sections, A, B and C.



$$\text{Area of A} = \frac{1}{2} \times 6 \times 6 = 18$$

$$\text{Area of B} = 6 \times 6 = 36$$

$$\text{Area of C} = \frac{1}{2} \times 2 \times 6 = 6$$

$$\text{Total Area} = 18 + 36 + 6 = 60$$

1.10

So the trees are 60 m apart. Note that the units are m because the units of speed are m/s and the units of time are s.

Exercises

1. Use the approximation that 10 kg is about the same as 22 lbs to draw a graph for converting between pounds and kilograms. Use the graph to convert the following:

   (a) 6 lbs to kilograms,

   (b) 8 lbs to kilograms,

   (c) 5 kg to pounds,

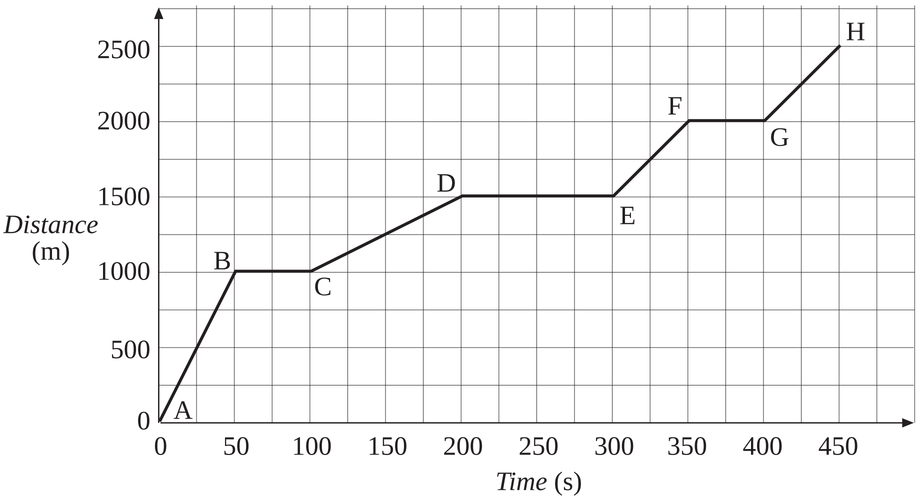
   (d) 3 kg to pounds.

2. Use the approximation that 10 gallons is about the same as 45 litres to draw a conversion graph. Use the graph to convert:

   (a) 5 gallons to litres,

   (b) 30 litres to gallons.

3. The graph shows how the distance travelled by a bus increased.



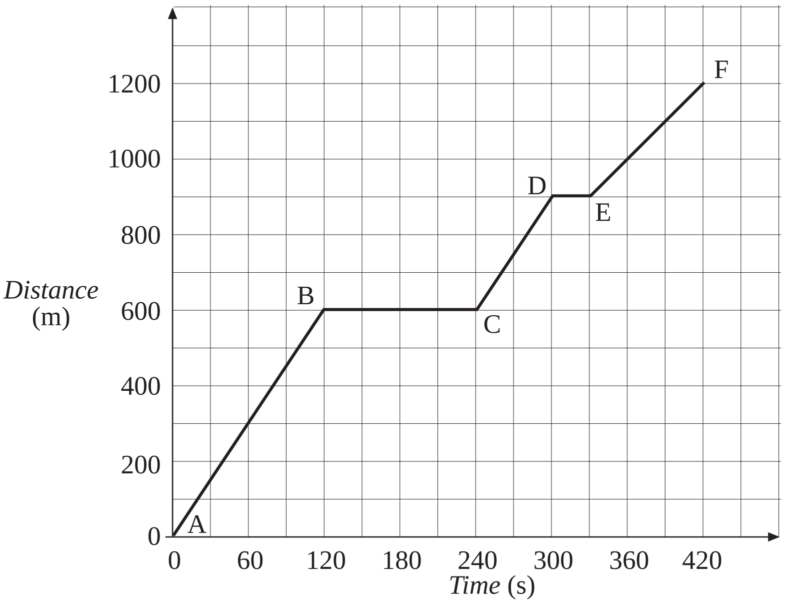
   (a) How many times did the bus stop?

   (b) Find the speed of the bus on each section of the journey.

   (c) On which part of the journey did the bus travel fastest?

1.10

4. The distance-time graph shows the distance travelled by a car on a journey to the shops.

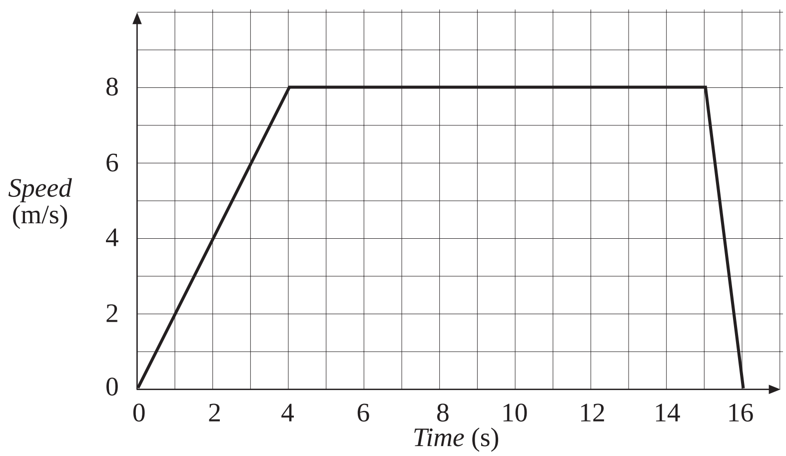


(a) The car stopped at two sets of traffic lights. How long did the car spend waiting at the traffic lights?

(b) On which part of the journey did the car travel fastest? Find its speed on this part.

(c) On which part of the journey did the car travel at its lowest speed? What was this speed?

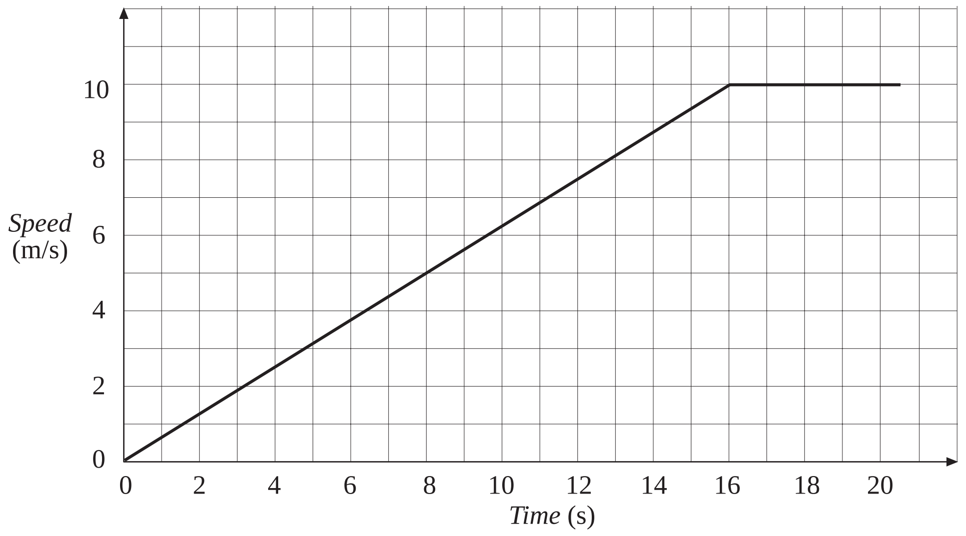
5. The graph below shows how the speed of an athlete varies during a race.



What was the distance of the race?

1.10

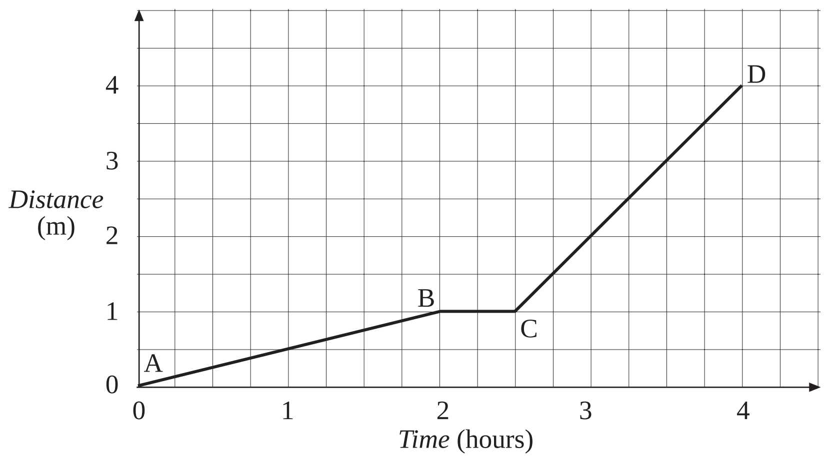
6. The graph below shows how the speed of a lorry varies as it sets off from a set of traffic lights.



Find the distance travelled by the lorry after

(a) 8 seconds, (b) 16 seconds, (c) 20 seconds.

7. The graph shows how the distance travelled by a snail increases.



Find the speed of the snail on each section in m/hour.

8. Hannah runs at 2 m s^{-1} for 5 seconds and then her speed decreases to zero at a steady rate over the next 4 seconds.

Find the distance that Hannah runs.

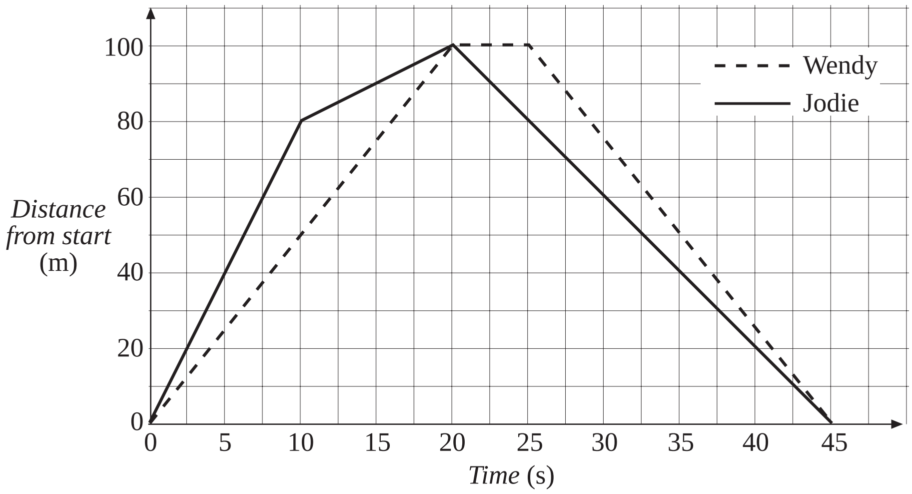
9. Ian runs at a constant speed for 10 seconds. He has then travelled 70 m. He then walks at a constant speed for 8 seconds until he is 86 m from his starting point.

(a) Find the speed at which he runs and the speed at which he walks.

(b) If he had covered the complete distance in the same time, with a constant speed, what would that speed have been?

1.10

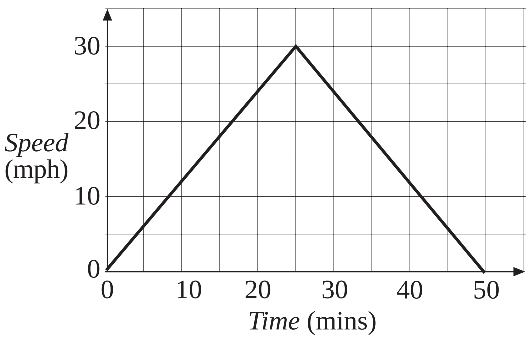
10. The graph shows how the distance travelled by Wendy and Jodie changes during a race from one end of the school field to the other end, and back.



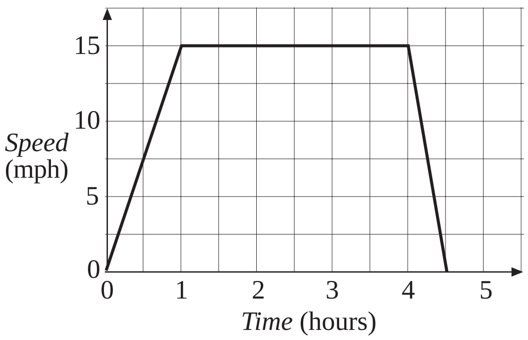
Describe what happens during the race.

11. Find the area under each graph below and state the distance that it represents.

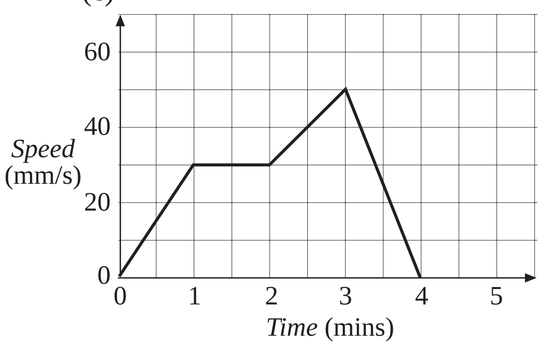
(a)



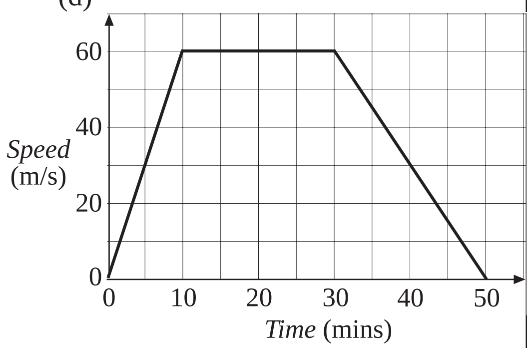
(b)



(c)



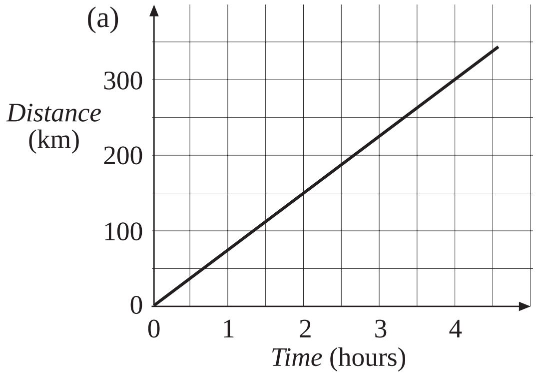
(d)



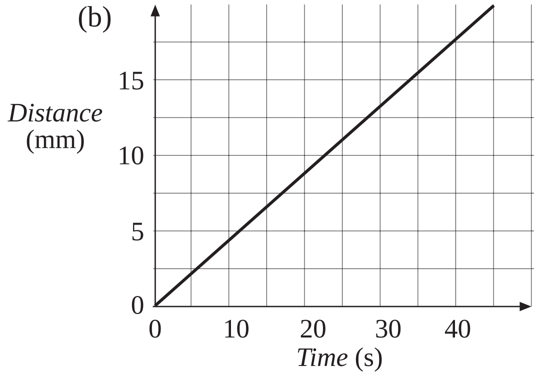
1.10

12. For each distance-time graph, find the speed in the units used on the graph and in m/s.

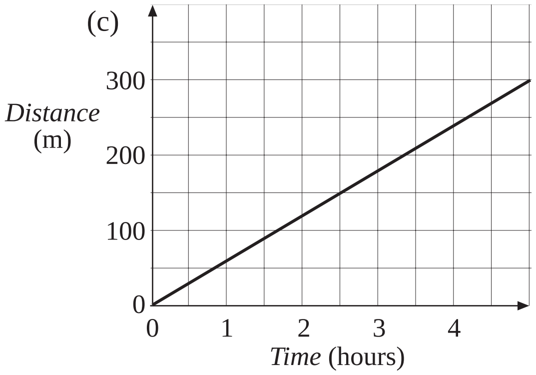
(a)



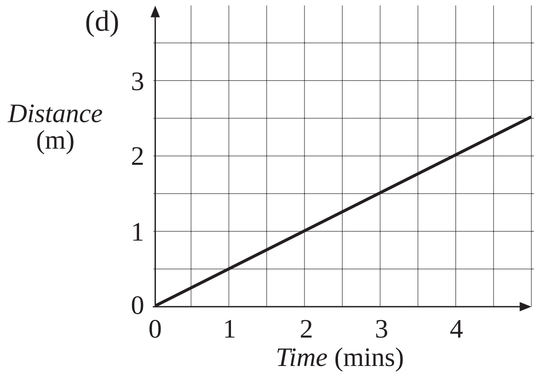
(b)



(c)

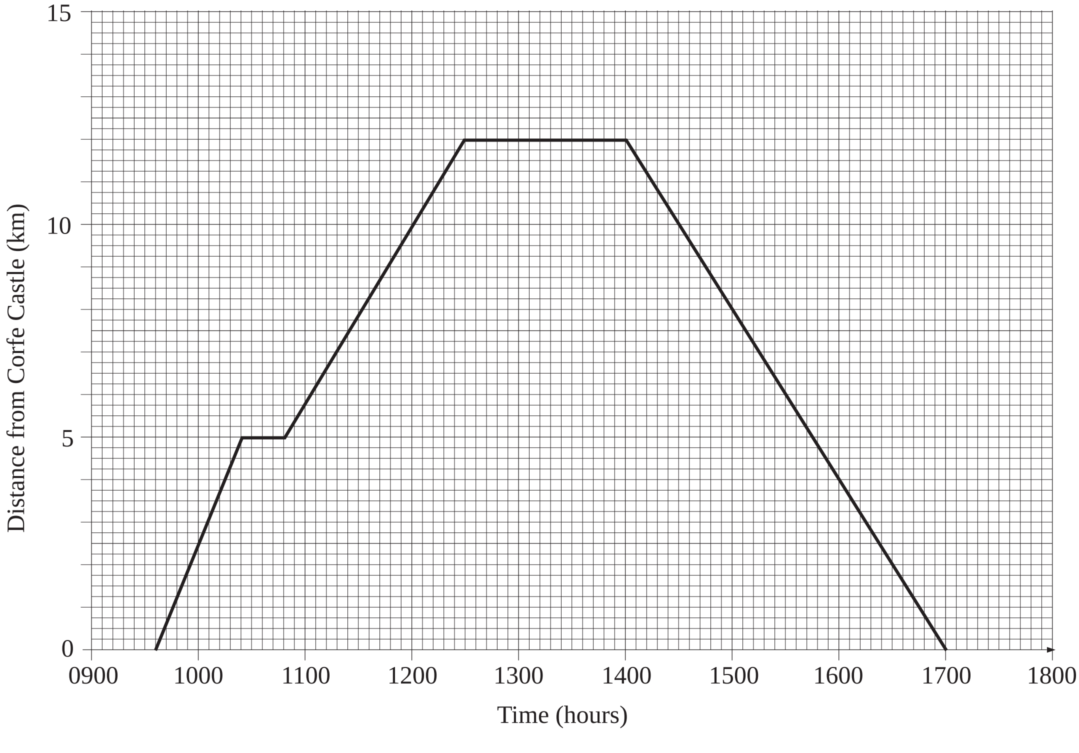


(d)



13. Jennifer walks from Corfe Castle to Wareham Forest and then returns to Corfe Castle.

The following travel graph shows her journey.



(a) At what time did Jennifer leave Corfe Castle?

(b) How far from Wareham Forest did Jennifer make her first stop?

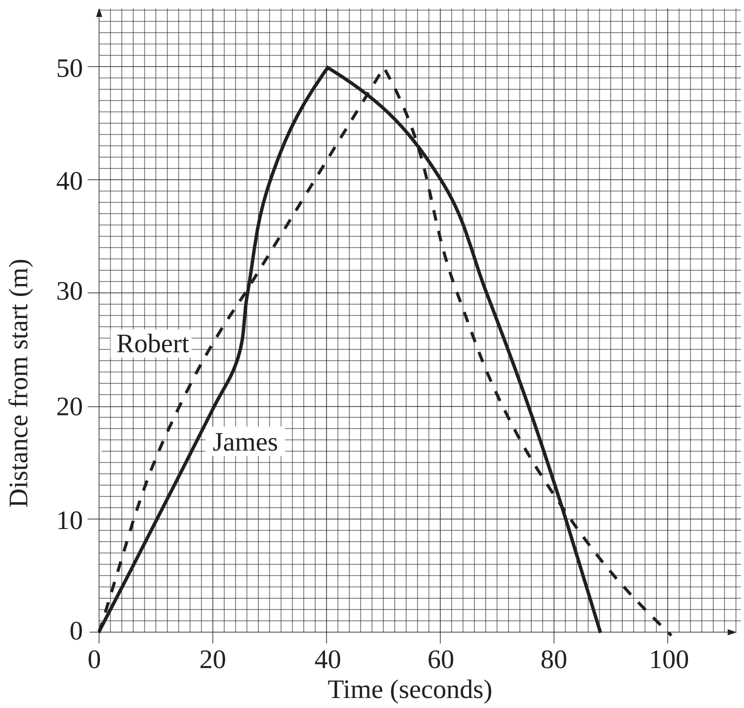
(c) Jennifer had lunch at Wareham Forest.

For how many minutes did she stop for lunch?

(d) At what average speed did Jennifer walk back from Wareham Forest to Corfe Castle?

(SEG)

14. The graph represents a swimming race between Robert and James.



(a) At what time did James overtake Robert for the second time?

(b) What was the **maximum** distance between the swimmers during the race?

(c) Who was swimming faster at 56 seconds? How can you tell?

(SEG)

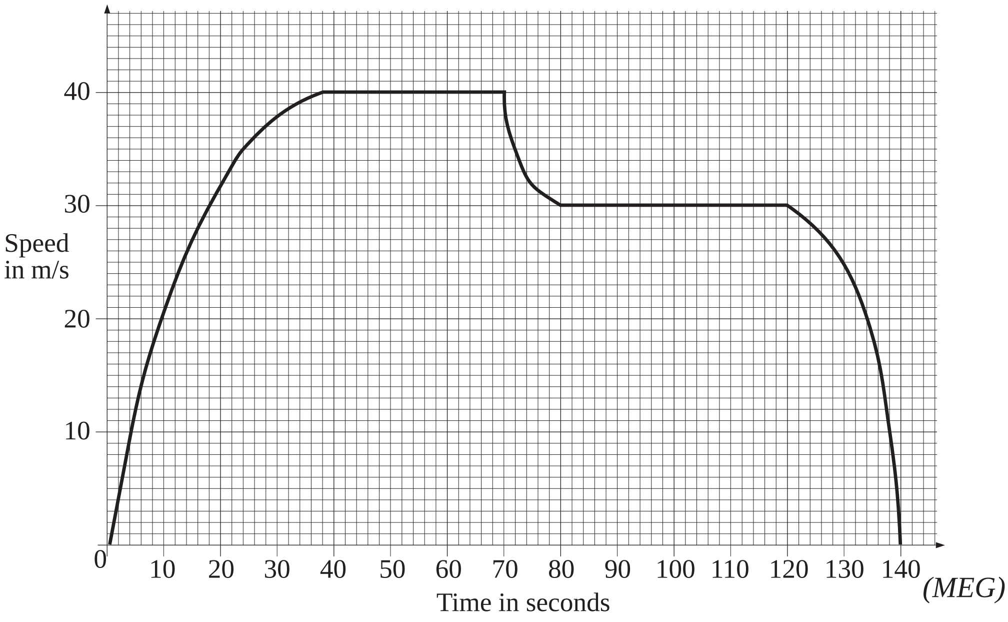
15. The graph at the top of the next page illustrates the journey of a car.

(a) Estimate the area under the graph taking into account the scales of the graph.

(b) State the units of the quantity represented by the area under the graph.

(c) Another car did the same journey in the same time at constant speed. On a copy of the grid, draw the graph which illustrates the second car's journey.

1.10


Speed
in m/s
40
30
20
10
0
10 20 30 40 50 60 70 80 90 100 110 120 130 140
Time in seconds
(MEG)

Answers to Exercises

1.1 Squares, Cubes, Square Roots and Cube Roots

1. (a) 25 (b) 36 (c) 1 (d) 49 (e) 6 (f) 1 (g) 7 (h) 5

2. (a) 27 (b) 64 (c) 216 (d) 1000 (e) 3 (f) 10 (g) 6 (h) 4

3. (a) 100 (b) 4 (c) 16 (d) 49 (e) 64 (f) 81 (g) 1 (h) 343
   (i) 512 (j) 0 (k) 0 (l) 8

4. (a) 10 (b) 2 (c) 9 (d) 8 (e) 4 (f) 3

5. (a) 144 (b) 121 (c) 3375 (d) 2197 (e) 169 (f) 225
   (g) 400 (h) 1331 (i) 11 (j) 20 (k) 13 (l) 15
   (m) 15 (n) 13 (o) 12 (p) 11

6. (a) 52 (b) 5 (c) 116 (d) 25 (e) 16
   (f) 72 (g) 1001 (h) 100

1.2 Index Notation

1. (a) 4^5 (b) 3^3 (c) 6^7 (d) 7^4 (e) 18^3 (f) 19^2
   (g) 4^6 (h) 7^5 (i) 10^6 (j) 100^5

2. (a) 81 (b) 625 (c) 2401 (d) 10 000 (e) 1 (f) 729
   (g) 128 (h) 2 (i) 4096 (j) 4 (k) 1 (l) 25

3. (a) 2^{11} (b) 3^9 (c) 3^{13} (d) 4^5 (e) 5^4 (f) 5^5
   (g) 4^2 (h) 5^3 (i) 3^2 (j) 7^4 (k) 17^2 (l) 9^4
   (m) 4^5 (n) 4^{16} (o) 3^6 (p) $3^0 = 1$ (q) $3^1 = 3$
   (r) 3^5 (s) 3^7 (t) 4^7 (u) 5^0

4. (a) 2^2 (b) 2^3 (c) 2^4 (d) 2^6 (e) 3^3 (f) 5^2
   (g) 4^3 (h) 3^4 (i) 5^3

5. (a) 3^{13} (b) 2^8 (c) 4^{11} (d) 3^{10} (e) 2^9 (f) 2^{10}
   (g) 3^5 (h) 3^7 (i) 3^5 (j) 8^{10} (k) 7^3 (l) 9^2
   (m) 2^4 or 4^2 (n) 2^3 (o) 2^3

Answers

1.2

6. (a) 2^3 (b) 10^3 (c) 2^4 (d) 3^3 (e) 3^4 (f) 10^4
   (g) 5^4 (h) 4^3 (i) 6^4 (j) 2^0 (k) 6^2 (l) 5^0

7. (a) 2^6 (b) 3^4 (c) 6^6 (d) 5^6 (e) 2^8 (f) 4^6
   (g) 3^8 (h) 5^8 (i) 3^6

8. (a) 2^8 (b) 2^4 (c) 3^{10} (d) 5^3 (e) $\left(10^5\right)^3$ (f) $\left(7^5\right)^4$

9. (a) 3^6 (b) 2^{14} (c) 5^{12} (d) 7^3 (e) 7^4 (f) 2^7
   (g) $3^0 = 1$ (h) $4^1 = 4$ (i) $2^1 = 2$

10. (a) a^5 (b) a^{10} (c) x^9 (d) x^2 (e) y^3 (f) p^3
    (g) q^3 (h) x^8 (i) b^3 (j) b^6 (k) c^3 (l) x^5
    (m) y^2 (n) $x^0 = 1$ (o) x^8 (p) p^4 (q) x^3 (r) y^4
    (s) $x^0 = 1$ (t) $x^1 = x$ (u) x^{12} (v) x^8 (w) x^{15} (x) x^{54}

11. (a) $p = 3$ (b) $q = 0$

12. $2x^4$

1.3 Factors

1. (a) 1, 2, 7, 14 (b) 1, 3, 9, 27 (c) 1, 2, 3, 6 (d) 1, 3, 5, 15
   (e) 1, 2, 3, 6, 9, 18 (f) 1, 5, 25 (g) 1, 2, 4, 5, 8, 10, 20, 40
   (h) 1, 2, 4, 5, 10, 20, 25, 50, 100 (i) 1, 3, 5, 9, 15, 45
   (j) 1, 2, 5, 10, 25, 50 (k) 1, 2, 3, 4, 6, 9, 12, 18, 36 (l) 1, 2, 4, 7, 14, 28

2. (a) $1\times10,\ 2\times5,\ 5\times2,\ 10\times1$ (b) $1\times8,\ 2\times4,\ 4\times2,\ 8\times1$
   (c) $1\times7,\ 7\times1$ (d) $1\times9,\ 3\times3,\ 9\times1$
   (e) $1\times16,\ 2\times8,\ 4\times4,\ 8\times2,\ 16\times1$ (f) $1\times22,\ 2\times11,\ 11\times2,\ 22\times1$
   (g) $1\times11,\ 11\times1$
   (h) $1\times24,\ 2\times12,\ 3\times8,\ 4\times6,\ 6\times4,\ 8\times3,\ 12\times2,\ 24\times1$

3. (a) 4 (b) 3 (c) 3 (d) 4 (e) 5 (f) 4 (g) 11 (h) 1

4. (a) 6, 10, 20, 8, 2, 24, 4 (b) 10, 20, 15, 55

5. (a) (i) 20, 22, 24, 26 (ii) 21, 24, 27 (iii) 20, 25
   (b) prime numbers

6. (a) (i) 16 (ii) 18 (b) (i) 25 (ii) 27

Answers

1.4 Prime Factors

1. 2, 3, 5, 7, 13, 19, 23

2. 53, 59

3. (a) 2×5 (b) $2 \times 3 \times 7$ (c) $2^2 \times 17$ (d) $2^3 \times 3 \times 7$

   (e) 2×5^3 (f) $2 \times 3^3 \times 5$ (g) $3 \times 11 \times 13$ (h) $3 \times 5^2 \times 11$

   (i) $7 \times 11 \times 13$

4. (a) $32 = 2^5$ and $56 = 2^3 \times 7$ (b) $2^3\ (= 8)$

5. (a) $2 \times 3 = 6$ (b) $2 \times 3 = 6$ (c) $3 \times 5 = 15$ (d) 2

   (e) $2 \times 5 = 10$ (f) $5 \times 7 = 35$ (g) $2^3 \times 3 = 24$

   (h) $2 \times 3 \times 13 = 78$ (i) $3 \times 7^2 = 147$

6. (a) $45 = 3^2 \times 5$, $99 = 3^2 \times 11$, $135 = 3^3 \times 5$

   (b) (i) $3^2 = 9$ (ii) $3^2 = 9$ (iii) $3^2 \times 5 = 45$

   (c) $3^2 = 9$

7. (a) 5 (b) $3^2 \times 5 = 45$ (c) $2^3 = 8$ (d) $2 \times 5 = 10$

   (e) $2^3 \times 3 = 24$ (f) $2 \times 3 \times 5 = 30$ (g) $2^2 \times 3^3 = 108$

   (h) $2^2 \times 11 = 44$ (i) $2^2 \times 3^2 \times 7 = 252$

1.5 Using Formulae

1. (a) $A = 8$, $P = 12$ (b) $A = 30$, $P = 26$ (c) $A = 22$, $P = 26$

   (d) $A = 20$, $P = 18$

2. (a) 16 (b) 12 (c) 15 (d) 20

3. (a) 30 (b) 400

4. (a) 30 (b) 12 (c) 17

5. (a) 60 (b) 105 (c) 144

6. (a) 26 (b) 14 (c) 19 (d) 46 (e) 18 (f) 12 (g) 4
   (h) 2 (i) 26 (j) 50 (k) 30 (l) 40 (m) 6 (n) 10

7. £130

8. 17.4 cm

Answers

1.6 Construct and Use Simple Formulae

1. (a) $P = 2a + b,\ P = 16$ (b) $P = 4a,\ P = 20$
   (c) $P = 5a + b,\ P = 40$ (d) $P = a + 2b + c,\ P = 27$
   (e) $P = 6a,\ P = 60$ (f) $P = 2a + 2b + 2c,\ P = 36$
   (g) $P = 2a + 2b + c,\ P = 520$ (h) $P = 3a + b,\ P = 21$

2. (a) $A = ab,\ A = 60\ \text{cm}^2$ (b) $A = a^2,\ A = 9\ \text{cm}^2$
   (c) $A = a^2 + ab,\ A = 20\ \text{cm}^2$ (d) $A = ab + bc,\ A = 48\ \text{cm}^2$
   (e) $A = \frac{1}{2}ab,\ A = 10\ \text{cm}^2$ (f) $A = \frac{1}{2}ab + b^2,\ A = 45\,000\ \text{cm}^2$

3. (a) $(x + 1)$ and $(x + 2)$ (b) $T = 3x + 3$

4. (a) $M = \dfrac{x + y}{2}$ (b) $M = \dfrac{p + q + r + s + t}{5}$

5. (a) $T = 3p + 2q$ (b) £190

6. (a) $P = 2x + 2(x + 3) = 4x + 6$ (b) $A = x(x + 3)$

7. (a) $x + 1$ (b) $x - 3$ (c) $S = 3x - 2$

8. (a) $C = 3 + 2n$ (b) £19

9. (a) $C = 1 + 2m$ (b) £7

10. (a) $2n$ (b) $2n + 6$

11. (a) $100 - 8x$ (b) $(20 - 2x)(30 - 2x)$

12. $C = 27n$

13. (a) $C = 45l$ (b) $C = xl$

14. (a) $S = P + Q$ (b) (i) $S = X + 3250$ (ii) $S = X + 650n$

1.7 Substitution into Formulae

1. (a) 50 (b) 68 (c) 14 (d) 23 (e) – 4 (f) 59
2. (a) 10 (b) 40 (c) 11.25 (d) 4 (e) –10 (f) 7.04
3. (a) 19.6 (b) 18.4 (c) 18.08 (d) 18.8

Answers

1.7

4. (a) – 280 (b) – 40 (c) 80 (d) 800 ; 4

5. (a) 80 (b) 51 (c) ± 4 (d) ± 3 (e) – 3 (f) ± 5
   (g) 0 (h) $\frac{3}{4}$ (i) 1 (j) 10 (k) – 2 (l) – 10
   (m) ±10 (n) 0.18 (o) 0.38 (p) ± 5 (q) ±8 (r) ±15

6. (a) 3.8 (b) 0.225 (c) 2.6 (d) 7.5 (e) 9.7 (f) 2.4
   (g) 0.5 (h) 7.12 (i) 3.7

7. – 21.67 (2 d.p.)

8. – 13

9. (a) $-\frac{13}{8}$ (b) $-\frac{5}{8}$

1.8 Positive Coordinates

1. A (0, 4), B (1, 3), C (2, 1), D (3, 0), E (4, 3), F (5, 2)

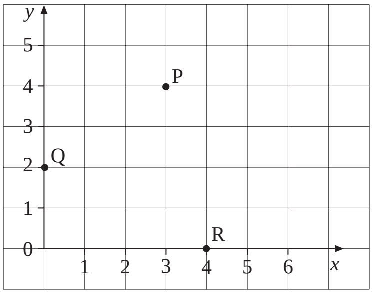
2. Rocky Point (2, 8) , Landing Stage (2, 2) , Old Ben's Cottage (3, 5)
   Old Tower (4, 3) , Café (7, 6) , Sandy Beach (9, 3) , Camp Site (10, 6)

3. (a) rectangle (b) triangle (c) rhombus (d) pentagon (e) hexagon

4. (a) J : (1, 2), (1, 1), (2, 1), (2, 5), (1, 5) and (3, 5)
   S : (4, 1), (6, 1), (6, 3), (4, 3), (4, 5) and (6, 5)

5. (a) (2, 3), (3 2), (4, 3), (5, 2), (6, 3)

6. (a)



(b) (i) A (2, 1) (ii) (B) (1, 5)

1.9 Coordinates

1. A (2, 5), B (4, 3), C (2, 1), D (2, –2), E (5, –3), F (3, – 4), G (–5, 4),
   H (–3, 3), I (–5, 2), J (– 4, –2), K (–2, –3), L (–6, –5)

2. (a) (5, – 4), (6, –3) and (–6, –5)
   (b) Albany to Alice Springs
   (c) Broome to Perth

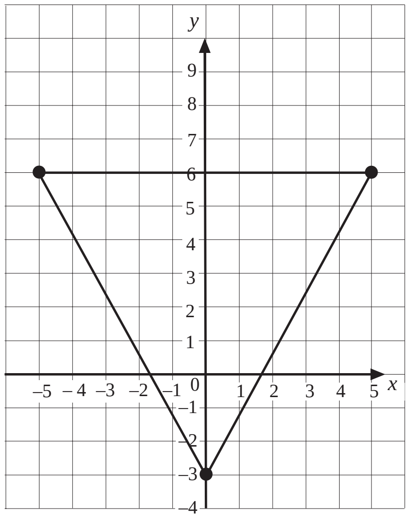
Answers

1.9

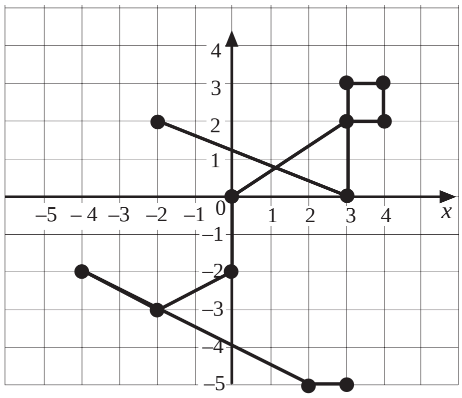
3. (a) (0, 4), (12.5, –5.5), (–12, 1), (–6.5, –3.5), (–1, –7) (b) (–1, 1)

   (c) (10, 0) to (–7.5, –10.5) (d) (–17, –2) (e) (10, 7)

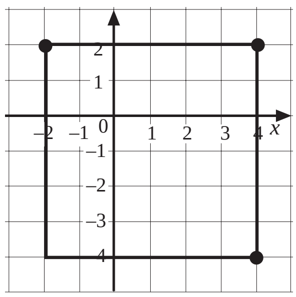
4. (a) triangle (b)



6. (b)

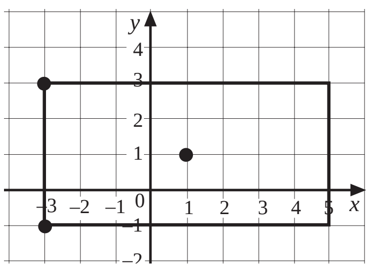


7. (a)



   (b) (1, –1)

8. (a)

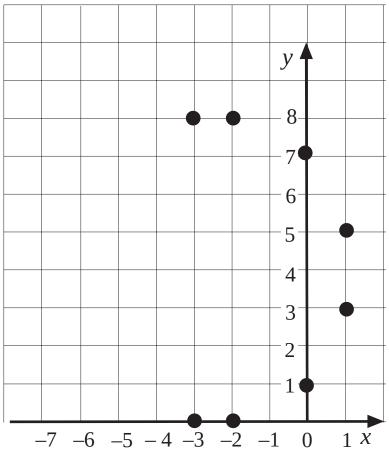


   (b) (5, –1), (5, 3)

Answers

1.9

9. (b)



(c) (– 4, 0), (–6, 1), (–7, 3), (–7, 5), (–6 , 7), (– 4, 8)

10. (a) 2 units (b) 4 units (c) $\sqrt{32} \approx 5.66$

1.10 Applications of Graphs

1. (a) 2.7 kg (b) 3.6 kg (c) 11 lbs (d) 6.6 lbs

2. (a) 22.5 litres (b) 6.7 gallons

3. (a) 3 (b) AB: 20 m/s ; CD: 5 m/s ; EF: 10 m/s ; GH:10 m/s (c) AB

4. (a) 150 s (b) AB and CD; 5 m/s (c) EF; $\frac{10}{3}$ m/s

5. 16 + 88 + 4 = 108 metres

6. (a) 20 m (b) 80 m (c) 120 m

7. AB: 0.5 m/hour ; BC: not moving ; CD: 2 m/hour

8. 14 m

9. (a) 7 m/s ; 2 m/s (b) 4.78 m/s

10. Jodie ran faster for the first 10 s but then slowed down until Wendy caught up at the end of the school field. While Wendy rested, Jodie returned at a constant speed until reaching the starting point, whilst Wendy (after her rest) ran faster, reaching the starting point at the same time.

11. (a) 12.5 miles (b) 56.25 miles (c) 6600 mm (d) 35 m

12. (a) 75 km/hour; 20.83 m/s (b) 0.4375 mm/s; 0.04375 m/s

    (c) 60 m/hour ; $\frac{1}{60}$ m/s (d) 0.5 m/min ; $\frac{1}{120}$ m/s

13. (a) 09.35 (b) 5 km (c) 90 mins (d) 4 km/hour

14. (a) 82 seconds from the start (b) 8.5 m (c) Robert – steeper slope

15. (a) about 4300 (b) metres