Mathematics Enhancement Programme

Primary Demonstration Project

8B Numbers

Help Booklet

Support for Primary Teachers in Mathematics

Primary Project
funded by
Pricewaterhouse
Coopers

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CIMT
School of Education
University of Exeter

Mathematics Enhancement Programme

Help Module 8

NUMBERS IN CONTEXT

Part B

Contents of Part B

Contents of Part A

CIMT
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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1. ALGEBRA
2. DECIMALS
3. EQUATIONS
4. FRACTIONS
5. GEOMETRY
6. HANDLING DATA
7. MENSURATION
8. NUMBERS IN CONTEXT
9. PERCENTAGES
10. PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

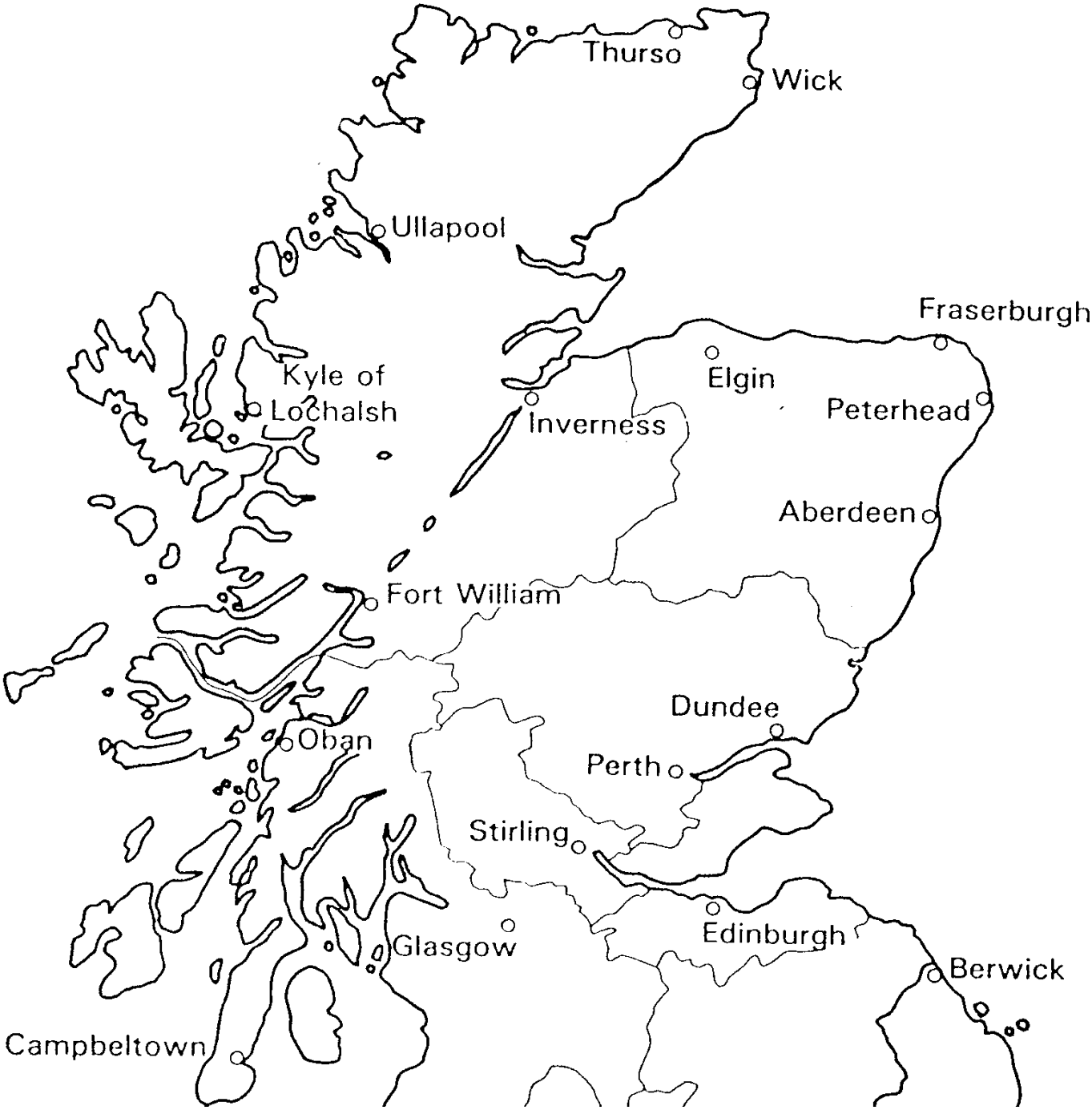
Professor D. N. Burghes
CIMT, School of Education
University of Exeter
EXETER EX1 2LU

The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

ACTIVITIES

ACTIVITY 8.1 *Map Scales*



1. The actual shortest distance between Dundee and Aberdeen is 86 km.

   What is the scale of this map?

2. Using this scale, estimate the distance in km between Edinburgh and Glasgow.

   Check your answer from an atlas.

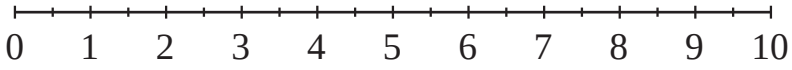
ACTIVITY 8.2 *Proportional Division*

(A)

The line below measures 10 cm.

Divide it in the ratio

(a) 7 : 3 (b) 4 : 1 (c) 3 : 2 (d) 3 : 1

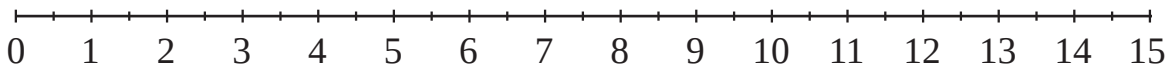


(B)

The line below measures 15 cm.

Divide it in the ratio

(a) 4 : 1 (b) 8 : 7 (c) 3 : 2 (d) 2 : 1



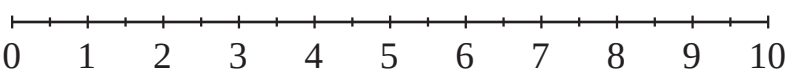
ACTIVITIES 8.1 - 8.2 *Notes for Solutions*

Notes for solutions are given only where appropriate.

8.1 1. 1 cm to 2 km or 1 : 200 000

2. About 68 km

8.2 (A)



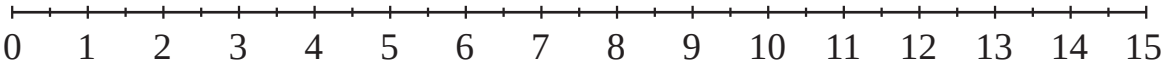
(a) 7 : 3

(b) 4 : 1 (or 8 : 2)

(c) 3 : 2 (or 6 : 4)

(d) 3 : 1 (or 7.5 : 2.5)

(B)



(a) 4 : 1 (or 12 : 3)

(b) 8 : 7

(c) 3 : 2 (or 9 : 6)

(d) 2 : 1 (or 10 : 5)

TESTS

Test 8.1 Mental Practice

Answer these questions as quickly as you can, but without the use of a calculator.

1. Put the following numbers in increasing order:

   $-3,\ -7,\ 2,\ -1,\ -6$

2. What is -3 times -5 ?

3. In a school, the ratio of teachers to pupils is $1 : 15$. If there are 4 teachers, how many pupils are there in the school?

4. In another school, there are 8 teachers and 160 pupils. What is the teacher : pupil ratio in its lowest terms?

5. The ratio of flour to sugar in a recipe is $3 : 2$. How much:
   (a) flour should be mixed with 200 grams of sugar,
   (b) sugar should be mixed with 150 grams of flour?

6. A map has scale $1 : 10\,000$. What is the actual length in km of a 5 cm line?

7. The distance between Exeter and Bristol is 100 km. What is this length, in centimetres, on a map with scale
   (a) $1 : 1\,000\,000$,
   (b) $1 : 2\,000\,000$,
   (c) $1 : 500\,000$?

Test 8.2 Mental Practice

Answer these questions as quickly as you can, but without the use of a calculator.

1. Put the following numbers in increasing order:

   — 10, 4, – 2, – 7, – 1

2. What is – 4 times 5 ?

3. In a company, the ratio of directors to factory workers is 1 : 12. If there are 96 factory workers, how many directors are there?

4. In another company, there are 5 directors and 80 factory workers. What is the director : factory worker ratio in its lowest terms?

5. The ratio of oil to petrol needed for a motorbike is 2 : 5. How much:
   (a) oil should be used with 25 litres of petrol,
   (b) petrol should be used with 6 litres of oil?

6. A map has scale 1 : 20 000. What is the actual length in km of a 5 cm line on this map?

7. The distance between London and Birmingham is 200 km. What length, in centimetres, represents this distance on a map with scale:
   (a) 1 : 2 000 000,
   (b) 1 : 1 000 000,
   (c) 1 : 4 000 000 ?

Test 8.3 Revision

40 minutes are allowed

1. Write down all the integers which lie between -5 and -2. *(2 marks)*

2. Calculate the value of each of the following expressions:

   (a) $\big(3-(-4)\big)\times(-2)$

   (b) $\big(8\times(-2)\big)+\big((-7)\times(-4)\big)$

   (c) $\big((-15)\div 3\big)\times\big(12\div(-4)\big)$ *(3 marks)*

3. Siobhan is asked to make 60 fairy cakes for the stall at her local church fete. The recipe in her cookery book gives the following information:

FAIRY CAKES

Ingredients to make 24 fairy cakes

4 oz margarine
8 oz flour
2 eggs
pinch of salt
4 fluid oz milk
4 oz currants

Cooking time: 25 minutes

   (a) How many eggs will she need to make 60 fairy cakes? *(1 mark)*

   (b) Siobhan makes enough mixture for 60 cakes. She can cook only 24 cakes at a time. She starts cooking the cakes at 8.45 am. At what time will they all be cooked?

(3 marks)
(SEG)

4. A 3-tier wedding cake is made using 6.4 kg of mixture.

Not to scale

The weights of the tiers are in the ratio 2:5:9. What is the weight, in grams,of the smallest tier? *(4 marks)*
(SEG)

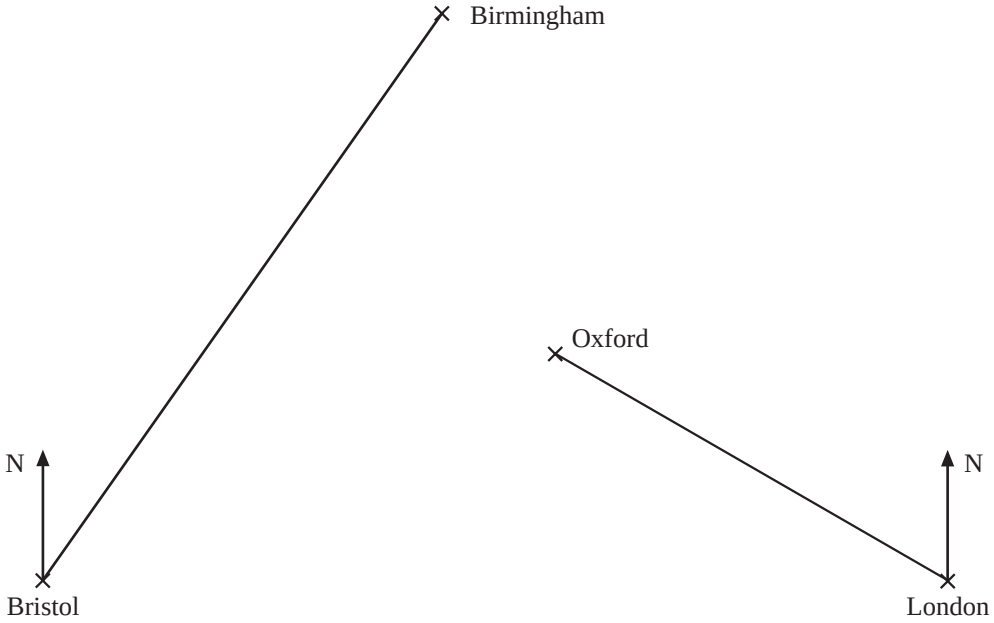
Test 8.3 Revision

5. An Ordnance Survey map is drawn to a scale of 1:50 000.

   (a) Find the actual distance between two railway stations which are 12 cm apart on the map. *(2 marks)*

   (b) Find the area, in square kilometres, of a lake which has an area of 20 cm^2 on the map. *(3 marks)*
   (SEG)

6.



The scale diagram shows the relative positions of Birmingham, Oxford, Bristol and London.

(a) Measure and write down the distance, in cm, from Birmingham to London on the diagram. *(1 mark)*

The actual distance from Birmingham to London is 100 miles.

(b) How many miles are represented by 1 cm on this diagram? (*2 marks*)

(c) Measure and write down the distance, in cm, from Birmingham to Bristol on the diagram. *(1 mark)*

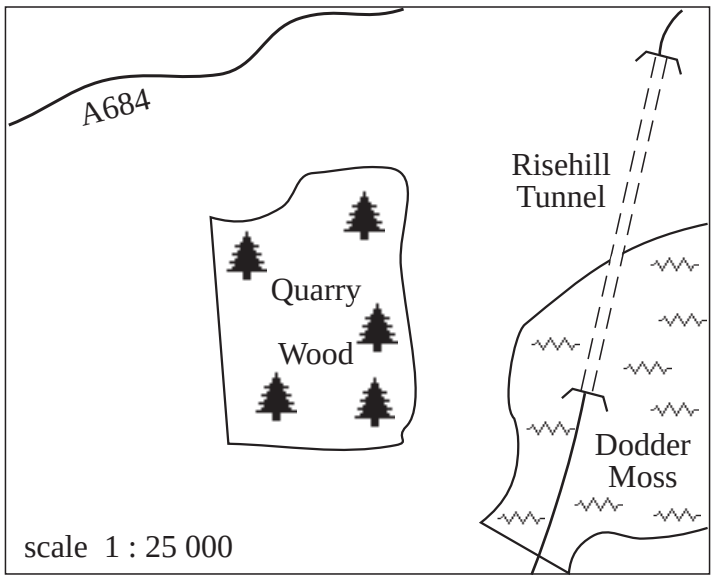
(d) Using your answer to (b) and (c) find the actual distance between Birmingham and Bristol. *(2 marks)*

(e) Measure and write down the bearing from Bristol to Birmingham. *(2 marks)*

(f) Measure and write down the bearing from London to Oxford. *(2 marks)*
(SEG)

Test 8.3 Revision

7. A map of the Yorkshire Dales is drawn to a scale of 1 : 25 000.



(a) On the map the length of Risehill Tunnel is 4.6 cm.
Calculate the actual length of the tunnel in kilometres. *(2 marks)*

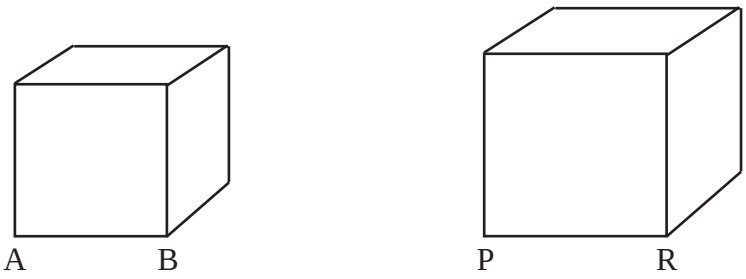
(b) Quarry Wood covers an area of 8 cm^2 on the map.
Calculate the actual area of Quarry Wood in hectares.
(1 hectare = 10 000 m^2) *(4 marks)*
(MEG)

8. Taking 8 kilometres to be 5 miles per hour, find:

(a) the speed in kilometres per hour equivalent to the British speed limit of 30 miles per hour. *(2 marks)*

(b) the speed in miles per hour equivalent to the French speed limit of 60 kilometres per hour. *(2 marks)*
(MEG)

9.



The diagrams show two similar cubes, the smaller cube with edge AB and the larger cube with edge PR. The ratio of the surface areas of the cubes is 9 : 16. Calculate the ratio of:

(a) length AB to length PR, *(1 mark)*

(b) the volume of the smaller cube to the volume of the larger cube. *(2 marks)*
(SEG)

Test 8.3 Revision

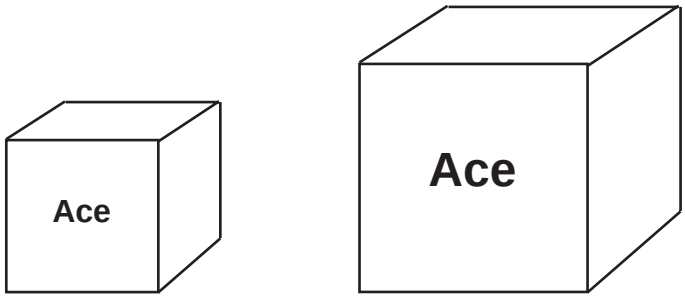
10. A photocopier can print copies smaller or larger than the original. The ratio of lengths in the original to lengths in the copies must be one of the following:

    1 : 0.5 1 : 0.7 1 : 1 1 : 1.4 1 : 2

    A firm had always used the ratio 1 : 1, but now wishes to reduce the area of paper used for each copy by about one half. Which ratio should the firm use? Explain your answer.

    (3 marks)

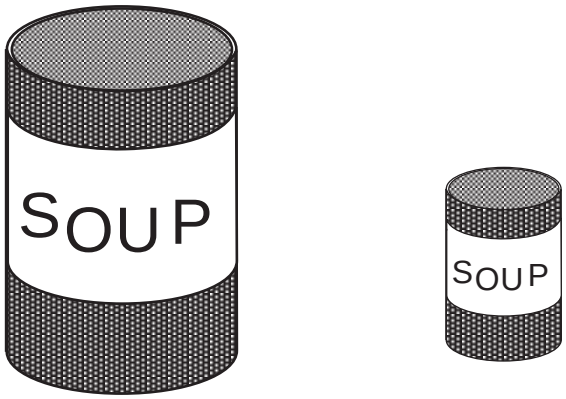
11.



    Ace soap powder is sold in cubical boxes. The small size has a side of 10 cm and contains 600 grams. The large size has a side of 15 cm. Calculate the weight of powder contained in the large size. *(3 marks)*

    (SEG)

12. The diagram shows two soup tins which are drawn to scale.



    The volume of the soup in the large tin is 1 litre.

    Estimate the volume of the soup in the small tin. *(3 marks)*

    (SEG)

Tests 8.1 and 8.2 Answers

Test 8.1

1. $-7,\ -6,\ -3,\ -1,\ 2$
2. 15
3. 60
4. 1 : 20
5. (a) 300 gm (b) 100 gm
6. $\frac{1}{2}$ km
7. (a) 10 cm (b) 5 cm (c) 20 cm

Test 8.2

1. $-10,\ -7,\ -2,\ -1,\ 4$
2. -20
3. 8
4. 1 : 16
5. (a) 10 litres (b) 15 litres
6. 1 km
7. (a) 10 cm (b) 20 cm (c) 5 cm

Test 8.3 Answers

1. (a) $-4,\ -3$ B1 B1 *(2 marks)*

2. (a) -14 (b) 12 (c) 15 B1 B1 B1 *(3 marks)*

3. (a) 5 B1
   (b) 3×25 mins = 75 mins M1 A1
   i.e. 10.00 am A1 *(4 marks)*

4. ratio $\frac{2}{16}\ \left(=\frac{1}{8}\right)$ B2
   $6.4 \times \frac{1}{8} = 0.8$ kg M1 A1 *(4 marks)*

5. (a) $\frac{12 \times 50\,000}{100\,000} = 6$ km (or equivalent) M1 A1
   (b) Since $4\ \text{cm}^2 \equiv 1\ \text{km}^2$, B1
   $20\ \text{cm}^2 \equiv 5\ \text{km}^2$ (or equivalent) M1 A1 *(5 marks)*

6. (a) 10 cm B1
   (b) 10 miles B2
   (c) 9.1 cm (Allow 9.1 to 9.2) B1
   (d) $9.1 \times 10 = 91$ miles M1 A1
   (e) $035°$ (B1 for $125°$) B2
   (f) $300°$ (B1 for $060°$ or $120°$) B2 *(10 marks)*

7. (a) $4.6 \times \frac{25\,000}{100\,000} = 1.15$ km (or equivalent) M1 A1
   (b) Since $1\ \text{cm} \equiv 250$ m, $1\ \text{cm}^2 \equiv (250)^2\ \text{m}^2$ B1
   So $8\ \text{cm}^2 \equiv 8 \times 250 \times 250\ \text{m}^2 = 500\,000\ \text{m}^2$ M1 A1
   $= 50$ hectares B1 *(6 marks)*

8. (a) 30 miles per hour $= 30 \times \frac{8}{5}$ km per hour M1
   $= 48$ km per hour A1
   (b) 60 km per hour $= 60 \times \frac{5}{8}$ miles per hour M1
   $= 37.5$ km per hour A1 *(4 marks)*

9. (a) 3 : 4 B1
   (b) $3^3 : 4^3 \equiv 27 : 64$ M1 A1 *(3 marks)*

Test 8.3 Answers

10. Ratio of areas will be

1 : 0.25 1 : 0.49 1 : 1 1 : 1.96 1 : 4 M1 A1

Nearest to 1 : 0.5 is 1 : 0.49 B1 *(3 marks)*

11. ratio $\left(\frac{15}{10}\right)^3$ B1

$600 \times \left(\frac{15}{10}\right)^3 = 2025$. Weight of powder = 2025 grams M1 A1 *(3 marks)*

12. Decrease in ratio $\left(\frac{1}{2}\right)^3$ B1

$1 \times \left(\frac{1}{2}\right)^3 = 0.125$ M1 A1 *(3 marks)*

(TOTAL MARKS 50)