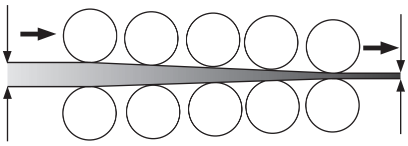
The diagram opposite represents a cold reduction mill.

It consists of five pairs of workrolls. The incoming steel is of thickness 2.1 mm, whilst the mill is producing strip of thickness 0.2 mm.



If the percentage reduction is constant on each pair of workrolls, find that reduction. Also, if the incoming speed is 1 m s^{-1}, what is the outgoing speed?

This example uses mathematics similar to that of compound interest, except that it is equivalent to negative interest. At the first pair of workrolls, the actual reduction is given by

$$2.1 \times \left(1 - \frac{r}{100}\right)$$

if r per cent is the reduction produced by each pair of workrolls. This new thickness is further reduced by the second pair of workrolls, and the outgoing thickness will be

$$\left[2.1\left(1 - \frac{r}{100}\right)\right]\left(1 - \frac{r}{100}\right)$$

$$= 2.1\left(1 - \frac{r}{100}\right)^2$$

Activity 1

What will be the thickness after the third and fourth pairs of workrolls?

Continuing in this way, the final thickness will be given by $2.1\left(1 - \frac{r}{100}\right)^5$.

But this must equal 0.2, so

$$2.1\left(1 - \frac{r}{100}\right)^5 = 0.2$$

$$\Rightarrow \quad \left(1 - \frac{r}{100}\right)^5 = \frac{0.2}{2.1} = 0.095238$$

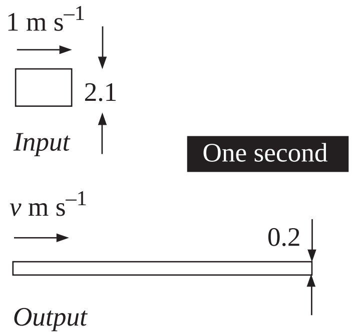
$$\Rightarrow \quad \left(1 - \frac{r}{100}\right) = (0.095238)^{\frac{1}{5}}$$

$$= 0.6248$$

$$\Rightarrow \quad \frac{r}{100} = 1 - 0.6248$$

$$\Rightarrow \quad r = 37.52\%$$

Problem 1



If each pair of workrolls can, at most, provide 25 per cent reduction in thickness, how many pairs of workrolls are needed to reduce the thickness from 2.1 mm to 0.2 mm?

Solve the problem in general when each pair can reduce by p per cent.

Solution

The exit speed can be determined from volume conservation – no steel is lost. So, in one second, the volume of steel being introduced into the mill is

$$1 \times 2.1 \times \text{width}.$$

Assuming an output speed of $v\ \text{m s}^{-1}$ (and the same width), then this must equal

$$v \times 0.2 \times \text{width}.$$

Hence

$$2.1 = v \times 0.2$$

$$\Rightarrow \quad v = \frac{2.1}{0.2}$$

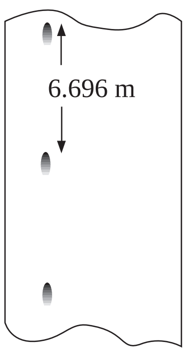
$$v = 10.5\ \text{m s}^{-1} \quad \text{(about 25 mph)} \qquad \text{(a considerable increase).}$$

Activity 2

The output speed is crucial for the next stage in the cycle of production, so it must be carefully controlled. If each pair of workrolls reduces the width by 30 per cent, find the width and output speed after 1, 2, ..., 10 pairs of workrolls for an input speed of 1 m s^{-1}.

Activity 3

Given the input speed and the input thickness and output thickness, does the number of pairs of workrolls and their reduction factor change the output speed?



Returning to the first problem in which there are five pairs of workrolls with an input thickness of 2.1 mm and output thickness of 0.2 mm, a mark is noticed on the finished strip.

If the mark repeats itself every 6.696 metres and is attributable to a defect on one of the pairs of rolls, which pair is producing it? (The diameters of all the rolls are 520 mm.)

Suppose the mark is on the *n*th pair of workrolls; the thickness of the strip coming out of this will be

$$2.1 \times \left(1 - \frac{r}{100}\right)^n$$

$$= 2.1 \times (0.6248)^n.$$

Then the distance apart of marks would be 520π mm. But the volume of steel between marks remains constant, hence

$$520\pi \times \left[2.1 \times (0.6248)^n\right] \times \text{width}$$

$$= L \times 0.2 \times \text{width}$$

where $L = 6.696$ m $= 6696$ mm, the distance between marks on the final output strip.

Problem 2

Solve the equation above for *n*.

Solution

The equation can be written as

$$(0.6248)^n = \frac{6696 \times 0.2}{520\pi \times 2.1}$$

i.e. $$(0.6248)^n = 0.3904$$

You can solve this for *n* by either 'trial and improvement' or by taking natural logarithms on both sides, to give

$$n \ln(0.6248) = \ln(0.3904)$$

$$\Rightarrow \quad n = \frac{\ln(0.3904)}{\ln(0.6248)}$$

$$= 2$$

So the fault is with the second pair of workrolls.

Exercises

1. Find a general formula to give the output thickness and speed in terms of:

   n, the number of pairs of workrolls;

   r, the percentage reduction given by each pair of workrolls;

   l, the input thickness (in mm);

   v, the input speed (in $\text{m s}^{-1\,1}$).

2. If, in the previous example, there is a blemish every 4.184 m, determine which pair of workrolls is at fault.

Answers to Exercises

1. $\text{thickness} = l\left(1 - \frac{r}{100}\right)^n$

   $\text{speed} = \dfrac{v}{\left(1 - \frac{r}{100}\right)^n}$

2. $n = 3$