Topical Applications of Mathematics

Champions League TEACHER INFORMATION

Key Stage 5 (A-level module, S1)

Target High ability students taking A-level Mathematics

Teaching notes This is an application of probability combined with combinatorics; it is very relevant for a few days but will continue to be of academic interest once the quarter final draw (and semi-final draw) has taken place on Friday 14th March.

Once the draw has been made the analysis is still of interest to see if the outcome with the highest probability actually occurred. Nevertheless, it will be of real relevance during a relatively short time period although similar situations will recur in the future to which the method described here will applicable.

Note that there are a number of possible approaches; we have used the one described here as it requires little previous knowledge, only basic logic and common sense.

Full details, including statistics, are given at the dedicated UEFA website

http://www.uefa.com/competitions/ucl/index.html

and also at the site

http://www.football.co.uk

Solutions and Notes for material in the Pupil Text

Activity 1

See the explanation in the Pupil Text.

Activity 2

2nd match to be drawn : $5+4+3+2+1 = 15$ possible pairings

3rd match to be drawn : $3+2+1 = 6$ possible pairings

Activity 3

$28 \times 15 \times 6 \times 1 = 2520$, possibilities, but where order does not matter.

Activity 4

You could use *Data Sheet 2* to give (i.e. 24 possibilities)

A B C D	B C D A	C D A B	D A B C
A B D C	B C A D	C D B A	D A C B
A C B D	B D C A	C A B D	D B A C
A C D B	B D A C	C A D B	D B C A
A D B C	B A C D	C B A D	D C A B
A D C B	B A D C	C B D A	D C B A

Activity 5

You could use *Data Sheet 2* to give

$$\left.\begin{array}{l} E_1\ E_2 \mid E_3\ E_4 \\ E_1\ E_3 \mid E_2\ E_4 \\ E_1\ E_4 \mid E_2\ E_3 \end{array}\right\} \text{3 possibilities}$$

Activity 6

The only way for this to happen is for two of the matches in the quarter finals to be all ENGLAND, and for the winners to be drawn against one another in the semi final.

This gives $\frac{3}{35} \times \frac{1}{3} = \frac{1}{35}$ as there would be a one third chance of the two winning English teams meeting each other.

Topical Applications of Mathematics

Champions League 2008 SAMPLE LESSON PLAN

Activity		*Notes*
1	**Background information**	T: Teacher P: Pupil
	T: Who can give us some facts about the Champions League? *(Ps give background)*	Many of the Ps will be well-informed about the league: get them to describe the situation to the rest of the class in preference to you giving the information.
	T: How many English teams are expected to be in the draw for the quarter finals? *(4)*	Full details of the Champions League are given on the dedicated UEFA website.
	T: Do you want all the English sides to avoid each other in the quarter finals? Why?	There is an argument for having the English teams play each other: this would guarantee two English sides in the semi-final draw!
	T: How can we find the probability of this happening? *(Ps suggest possible methods)* *5 mins*	Consider any ideas and, if there is time, pursue them to see if the suggested approaches work.
2	**Calculating the number of distinct draws possible in the quarter finals** T: We'll calculate the total number of possibilities, given the there are eight teams, say T_1, T_2, T_3, T_4, T_5, T_6, T_7 and T_8 T: Choosing one team, say T_1, how many possibilities are there for this team's opponent? P: 7 possibilities $(T_2, T_3, \ldots, T_8)$ T: That's right. Now think about T_2: how many possibilities are there? P: 6 possibilities $(T_3, T_4, \ldots, T_8)$ T: Good. Now complete *Data Sheet 1.* You have one minute! T: Answer? P: 28 ways T: Does anyone disagree with 28? So there are 28 ways of choosing the teams in the first match. T: What about the next match? P: $5 + 4 + 3 + 2 + 1 = 15$ possibilities.	Make this as interactive as possible, and try to use Ps' suggestions. You could use Data Sheet 1 here to help with the calculations. It could be shown on the OHP and then Ps write in the teams, etc.

(continued)

Activity		*Notes*
2 *(continued)*	T: And the next match? P: $3 + 2 + 1 = 6$ possibilities. T: The final match? P: 1 (no choice at all!) T: So how many possibilities are there? P: $28 \times 15 \times 6 \times 1 = 2520$ T: Why is this not our final answer? *(Repeats)* T: The order is important in our 2520 possibilities but in practice it makes no difference. So we have repeats, but how many? T: You can determine the number of repeats by considering all combinations of the matches, denoted by A, B, C and D. Try to find out how many combinations there are. You can have 5 minutes to do this: work in pairs. *After 5 minutes:* T: How many repeats? P: 24 T: How did you get 24 ? P: $4 \times 3 \times 2 \times 1 = 24$ T: That's right. So how many distinct possibilities are there for the quarter final? P: $\frac{2520}{24} = 105$ T: Well done! *25 mins*	Use Ps to provide the method and give answers. Make sure that all Ps understand this approach. Ask all Ps for this answer, and get all their responses before clarifying the answer. Give Ps time to reflect on this. You could write about ORDER of games. Again, give Ps time to consider this problem, and how to solve it. Some Ps might get this result very quickly: get them to explain to the others. Or you could use Data Sheet 2 to help confirm the number.
3	**Probability of the 4 English teams avoiding each other** T: We have 105 possible outcomes, but need to calculate how many of these are such that the English teams avoid each other. Any suggestions for a method of approach? T: E_1 could be matched with R_1, R_2, R_3 or R_4 so has 4 possibilities. Then E_2 with 3 possibilities, E_3 with 2, etc. T: So what is the total number of possibilities? P: $4 \times 3 \times 2 \times 1 = 24$ T: What, then, is the probability that NO ENGLISH teams play one another? P: $\frac{24}{105} = \frac{8}{35} \approx 23\%$ *35 mins*	Use the Ps' suggestions as much as possible. One method is to write out all the possibilities: this is worth doing if there is any confusion. Introduce notation E_1, E_2, E_3, E_4 for English teams: R_1, R_2, R_3, R_4 for non-English teams.

4	**Probability of two matches, both with only English teams** T: How can we determine this? P: Write out all possibilities? T: OK. You have 2 minutes to do this. T: In how many ways can the English teams be combined? *(3)* T: How many ways are there for the non-English teams to be combined? *(3)* T: Probability? P: $\frac{3\times 3}{105}\times\frac{9}{105}=\frac{3}{35}\approx 9\%$ *40 mins*	Use Ps' suggestions as much as possible. You will find Data Sheet 3 a useful guide for the calculation. Put on OHP and get Ps to complete.
5	**Outcome most expected** T: There is one more possible outcome. What is it? P: Two English teams play each other and two do not. T: How can we calculate this probability? Be clever? P: $1-\left(\frac{3}{35}+\frac{8}{35}\right)=\frac{24}{35}\approx 68\%$ T: So what is the most likely outcome? P: Just one all-England match. *45 mins*	
6	**Homework** Activity 6	