A2 *Using Fractions and Percentages* Introduction

Learning objectives

In this unit we revise the important topic of *percentages* and its relationship to *fractions* and *decimals*, which have been covered in Unit 1. After completing Unit A2 you should

- understand the equivalence of percentages, fractions and decimals and be able to convert between them
- be able to calculate percentages of quantities
- understand how to add, subtract, multiply and divide fractions
- be able to use fractions in contexts
- be able to calculate and use percentages in context
- be able to calculate percentage increase or decrease in quantities
- understand the concept of compound interest and be able to calculate the value of a sum of money invested for a given number of years
- be able to solve reverse percentage problems in context.

Introduction

The work covered in this unit will be revision for all students but these are topics that many find difficult.

The use of percentages is thought to originate from Roman times: *per centum* in Latin means 'for hundred', although the common English use would be 'parts out of hundred'. So a more accurate translation would be, for example, that 30% means '30 parts out of a hundred'. In fact, the Roman emperor Augustus levied a tax on all goods sold at auction, with a rate of $\frac{1}{100}$. Other Roman taxes were $\frac{1}{20}$ on every freed slave and $\frac{1}{25}$ on every slave sold. So they used fractions which were easily reduced to hundreds, e.g. $\frac{1}{20}$ would be 5 parts out of a hundred, that is 5 *per centum*.

There is another theory that the word percentage was derived from the Italian *per cento* or the French *pour cent*.

A version of the percentage symbol was introduced in the 15th century when shorthand notation began to be used in business activities such as interest, profit and loss and taxes, but it was not until the 18th century that the % sign as we know it came into general use.

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Key points and principles

- Percentages, fractions and decimals are equivalent.

  For example, $\frac{1}{2} \equiv 0.5 \equiv 50\%$

- You can easily convert from one form to another.

  For example, $25\% = \frac{1}{4} = 0.25, \quad 10\% = 0.1 = \frac{1}{10}$

- Use the lowest common multiple (LCM) of the denominator when adding or subtracting fractions

- Multiply fractions by multiplying the numerators and the denominators.

  For example, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

- Divide fractions using the fact that dividing by a number is the same as multiplying by the reciprocal of that number.

  For example, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

- The value of a sum of money invested in an account paying compound interest at the rate of $r\%$ each year is found by multiplying the initial value by $\left(1 + \frac{r}{100}\right)$.

- Reverse percentage problems can be solved using the equation

$$\text{price} = \text{original cost} \times \left(1 + \frac{r}{100}\right)$$

  where r is value added rate (e.g. $r = 16\frac{1}{2}\%$ VAT), giving

$$\text{original cost} = \frac{\text{price}}{\left(1 + \frac{r}{100}\right)}$$

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Facts to remember

- Equivalence of percentages, fractions and decimals; see examples shown in the table opposite.

Fraction	*Decimal*	*Percentage*
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
$\frac{1}{2}$	0.5	50%
$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}\%$
$\frac{3}{4}$	0.75	75%
1	1.0	100%

- Percentage increase (decrease) of a quantity is

$$\left(\frac{\text{actual increase (or decrease)}}{\text{initial value}} \times 100\right)\%$$

- $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

- Value of investment, A, is given by

  $A_n = A_0\left(1 + \frac{r}{100}\right)^n$ after n years at $r\%$ interest added yearly and A_0 is the original value invested.

Glossary of Terms

Percentage which is the part out of a hundred.

For example, 3% is $\frac{3}{100}$

Percentage increase (or decrease) in a quantity is

$$\left(\frac{\text{actual increase (or decrease)}}{\text{initial value}} \times 100\right)\%$$

Adding, subtracting, multiplying and *dividing* fractions

Compound interest: when the interest each year is added to the value of the account

Depreciation: the rate at which the value of a commodity (e.g. boat, car) decreases over time

Reverse percentage problems in which you find the original value given the percentage increase/ decrease and the current value after the increase/decrease.

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