Text *B3 Number Sequences* Answers

B3.1 Simple Number Patterns

1. (a) 11, 13, 15, 17 (b) 24, 28, 32, 36 (c) 30, 35, 40, 45
   (d) 42, 49, 56, 63 (e) 54, 63, 72, 81 (f) 36, 42, 48, 54
   (g) 60, 70, 80, 90 (h) 66, 77, 88, 99 (i) 48, 56, 64, 72
   (j) 120, 140, 160, 180 (k) 90, 105, 120, 135 (l) 300, 350, 400, 450

2. (a) +3; 20, 23 (b) +8; 42, 50 (c) +5; 32, 37 (d) +11; 61, 72
   (e) +7; 43, 50 (f) $+\frac{1}{2}$; 5, $5\frac{1}{2}$ (g) +9; 49, 58 (h) –3; 11, 8
   (i) -4; 0, -4 (j) -4; –2, –6 (k) –3; -4, –7 (l) –3; –20, –23

3. (a) (i) 22 (ii) 242 (iii) 2442 (iv) 24442 (v) 244442
   (b) (i) 1089 (ii) 10989 (iii) 109989 (iv) 1099989 (v) 10999989
   (c) (i) 968 (ii) 9768 (iii) 97768 (iv) 977768 (v) 97777768
   (d) (i) 63 (ii) 693 (iii) 6993 (iv) 69993 (v) 6999993

4. (a) (i) $11 \times 11 \times 11 = 1331$, $11 \times 11 \times 11 \times 11 = 14641$
   (ii) Each one is a symmetric number (these are found in Pascal's triangle).
   (b) $11 \times 11 \times 11 \times 11 \times 11 = 11^5 = 161051$. This is *not* a symmetric number.

5. (a) (i) 5, 4, 3, 2, 1 (ii) 4, 5, 6, 7, 8
   (iii) In (i), the digits *increase* by 1 each time; in (ii), the numbers *decrease* by 1 each time.
   (b) The numbers sum to 9.

B3.2 Recognising Number Patterns

(d) 0, –3;

30	22	15	9	4	0	–3
–8	–7	–6	–5	–4	–3	
1	1	1	1	1		

(e) 90, 101;

50	56	63	71	80	90	101
6	7	8	9	10	11	
1	1	1	1	1		

(f) 32, 42;

2,	2,	4,	8,	14,	22,	32,	44
0,	2,	4,	6,	8,	10,	12	

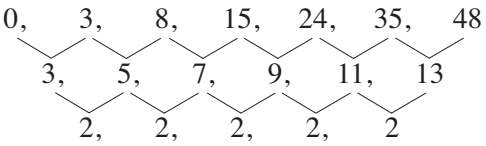
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Text *B3 Number Sequences* Answers

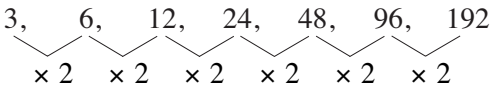
B3.2

(g) 31, 36; +5

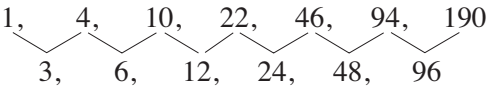
(h) 35, 48;



(i) 96, 192;

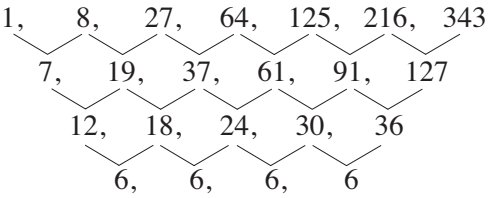


(j) 94, 190;



(k) –71, –101; –25, –30

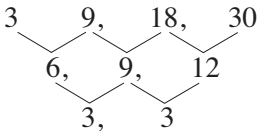
2. 216, 343;

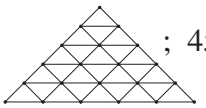


3. (a) (i) 3, 5, 7, 9 (ii) +2 (iii)  ; 11

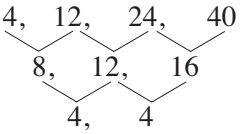
(b) 4, 7, 10, 13 (ii) +3 (iii)  ; 16

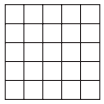
(c)

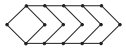


(ii) +15 (iii)  ; 45

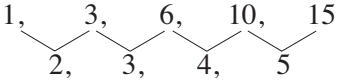
(d)



(ii) +20 (iii)  ; 60

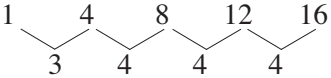
(e) 6, 10, 14, 18 (ii) +4 (iii)  ; 22

4. (a)



+1 to difference to get next number; 21

(b)



If 3 is ignored, the sequence is constant, +4 for next number; 20

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Text *B3 Number Sequences* Answers

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(c) 5, 8, 11, 14, 17 +3 for next number; 20

(d) 4, 6, 8, 10 +2 for next number; 12

(e) 1 5 12 22
4 7 10
3 3
+3 for next difference; 35

(f) 1 6 15 28 45
5 9 13 17
4 4 4
+4 for next difference; 66

5. (a) 1^2, 2^2, 3^2, 4^2, ...

(b) (i)$_n$ $= n^2 + 2$ (ii)$_n$ $= n^2 - 2$ (iii)$_n$ $= 2n^2$ (iv)$_n$ $= 4n^2$

6. (a) (ii)$_n$ $=$ (i)$_n$ $+ 3$ (iii)$_n$ $=$ (i)$_n$ $- 2$

(b) (i) $13 + 21 = 34$ (iii) $16 + 24 - 3 = 37$ (iii) $11 + 19 + 2 = 32$

7. Circle 8

1 2 4 7 11 16 22
1 2 3 4 5 6

8. (a) 35, 42 (b) The next term is obtained by adding 7 to the last term.

9. (a) 41, 122 (b) 125, 216

10. (a) $p = 15$ (b) $q = 52 + 15 = 67$

11. (a) It is increasing by 4. (b) (i) Sequence of square numbers (ii) 49
(iii) 47; minus 2

12. (a) (i) 5, 10, 15, 20, 25, 30, 35, 40 (ii) 5, 0, 5, 0, ..., alternate

(b) Terms are the positive multiples of 3.

(c) $9 \rightarrow 9 + 1 = 10$, $10 \times 2 = 20 \rightarrow 20$
$12 \rightarrow 12 + 1 = 13$, $13 \times 2 = 26 \rightarrow 26$
$15 \rightarrow 15 + 1 = 16$, $16 \times 2 = 32 \rightarrow 32$
$18 \rightarrow 18 + 1 = 19$, $19 \times 2 = 38 \rightarrow 38$

(d) 85, 109

1 4 10 19 31 46 64 85 109
3 6 9 12 15 18 21 24

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Text *B3 Number Sequences* Answers

B3.3 Extending Number Patterns

1. (a) 40; 80 (b) 50; 100 (c) 101; 201 (d) 25; 45
   (e) 41; 81 (f) 11; 1 (g) -4; -64 (h) 49; 79
   (i) -1; -11 (j) 32; 72 (k) 52; 102 (l) -42; -92

2. (a) (i) $10^2 + 2 = 102$ (ii) $4 + 10 \times 1 = 14$ or $5 + 9 \times 1 = 14$
   (b) (i) 116 (ii) 1428 (iii) 88

3. (a) (i) 99 (ii) 41 (b) 4059

4. (a) 90 (b) 990

5. (a) 28 dots (b) 40 dots (c) 60 dots

6. (a) 17 sticks (b) Pattern 58
   (c) (i) 401 sticks (ii) nth pattern needs $(4n + 1)$ sticks

B3.4 Formulae for Number Sequences

1. (a) 5, 9, 13, 17, 21, 25 (b) –2, 3, 8, 13, 18, 23
   (c) 12, 22, 32, 42, 52, 62 (d) 0, 3, 8, 15, 24, 35
   (e) 3, 9, 19, 33, 51, 73 (f) 2, 4, 8, 16, 32, 64

2. (a) $u_n = n^3 - 1$ (b) $u_n = 3n + 5$ (c) $u_n = 8n - 2$ (d) $u_n = n^2 + 1$

3. (a) 80 (b) 10 (c) 87 (d) 0 (e) 396 (f) $\frac{1}{2}$

4. (a) Because the difference sequence is 7, 7, 7, ..., (b) $u_n = 7n + 1$

5. (a) $u_n = 5n - 1$ (b) $u_n = 3n + 8$ (c) $u_n = 6n - 8$
   (d) $u_n = -2n + 102$ (e) $u_n = \frac{1}{2}n + \frac{1}{2}$ (f) $u_n = -7n + 12$ (g) $u_n = -\frac{1}{2}n + 10\frac{1}{2}$

6. (e) $u_n = 0.1n + 0.9$ (g) $u_n = 4n - 1$

7. 1, 4, 9, 16, 25, 36 (a) $u_n = n^2 + 2$ (b) $u_n = n^2 - 5$ (c) $u_n = 2n^2$
   (d) $u_n = 2n^2 - 2$ (e) $u_n = -n^2 + 100$

8. (a) $u_n = n^3 - 1$ (b) $u_n = n^3 + 9$ (c) $u_n = -n^3 + 200$

9. $u_n = 4n + 10$; 14, 18, 22, 26, 30

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B3.4

10. $u_n = 6n + 8$; 68

11. $u_n = 5n + 2$; 7

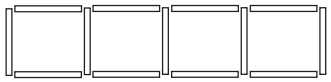
12. (a) $u_8 = 29$, add 4 (b) $n = 100$; 100th term is 397

13. $u_n = 4n + 3$

14. (b)

Number of enclosures	4	5	6	7	8
Number of posts	15	18	21	24	27

(c) 63 posts (d) $u_n = 3n + 3$

15. (a) (i)  (ii) 13 sticks (b) 3 more sticks

(c) number $= 3s + 1$

16. (a) (i) add 5 (ii) $u_n = 5n + 2$

(b) area $= n(n + 1) = n^2 + n$

B3.5 General Laws

1. (a) $u_n = \dfrac{n^2}{n + 9}$ (b) $u_n = \dfrac{2n + 3}{n + 1}$ (c) $u_n = 2 \times 3^{n-1}$ (d) $u_n = 0.9^{n-1}$

(e) $u_n = 1.2^{n-1}$ (f) $u_n = \dfrac{2}{3^{n-1}}$ (g) $u_n = 2^n$ (h) $u_n = 2^n + 1$

(i) $u_n = \dfrac{3n}{n + 3}$

(b), (d), (f), (i) converge; (a), (c), (e), (g), (h) diverge;

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
2 0 0 3

2. (a) $u_1 = 8$; $u_{n+1} = u_n - 3$ (b) $u_1 = 5$; $u_{n+1} = 4u_n$

(c) $u_1 = 2$; $u_{n+1} = 2u_n - 1$ (d) $u_1 = 4000$; $u_{n+1} = \dfrac{u_n}{2}$

(e) $u_1 = 3$, $u_2 = 3$; $u_{n+2} = u_n + u_{n+1}$

(f) $u_1 = 1$, $u_2 = 1$, $u_3 = 1$
$u_{n+3} = u_n + u_{n+1} + u_{n+2}$

3. (a) 1, 3.5, 2.6071, 2.4543, 2.4495 (b) $\sqrt{6} \approx 2.449489743$

4. Yes; converges to 3. If u_1 is a different value it still converges to 3.

5. 0.1, 0.13, 0.1417, 0.14284777, 0.142857142; $\dfrac{1}{7}$

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Text *B3 Number Sequences* Answers

B3.5

6. $\frac{1}{3}$; 0.5, 0.25, 0.3125, 0.33203125

7. (a) (i) $40 = 5 \times 8 = 5 \times (5 + 3)$ (ii) $n(n + 3)$

   (b) (i) $40 = 6 \times 7 - 2$ (ii) $(n + 1)(n + 2) - 2$

   (c) $(n + 1)(n + 2) - 2 = n^2 + 3n + 2 - 2 = n^2 + 3n = n(n + 3)$

8. (a) 64 (b) $u_n = 2^{n-1}$ (c) $u_{11} = 1024$, $u_{10} = \frac{1024}{2} = 512$

9. (a) Row 4: 13 15 17 19 Sum $= 64 = 4^3$

   (b) Row 10 $\rightarrow 10^3 = 1000$ (c) Row 20 $\rightarrow 20^3 = 8000$ (d) $x + 2$

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