C3 *Areas* Introduction

Learning objectives

The work in this unit is focused on the very important topic of areas. After completing Unit C3 you should be able to

- use formulae to calculate the areas of *squares*, *rectangles* and *triangles*
- understand and calculate the area and circumference of a *circle*
- use formulae to calculate the areas of *parallelograms, trapeziums* and *kites*
- calculate the surface areas of solids including *cuboids, cylinders* and *spheres*
- find the arc length and area of a *sector* of a circle.

Historical introduction

This introduction to Units C3 and C4 provides general background to the topic of GEOMETRY.

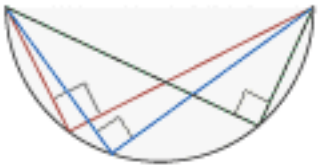
The word 'geometry' is derived from the Greek, geo for 'earth' and metreia for 'to measure'. This is an accurate description of the works of the earliest geometers, who were concerned with problems such as measuring the size of fields and laying out accurate right angles for the corners of buildings.

Much of this introduction is taken from the MSN Encarta Encyclopedia at

http://encarta.msn.com/encyclopedia_761569706/Geometry.html

Geometry in ancient Greece

The first important geometer mentioned in history is **Thales of Miletus**, a Greek who lived about 600 BC. Thales is credited with several simple but important theorems, including the proof that an angle inscribed in a semicircle is a right angle. This is covered in Unit H2.



Thales was the first to demonstrate the truth of a geometric relationship by showing that it followed in a logical, orderly fashion from a set of universally accepted statements, called postulates or axioms.

One of Thales' most famous pupils was **Pythagoras**. Pythagoras and his associates proved many new theorems about triangles, circles, proportions and certain solids. His most famous proof, a theorem that bears his name, states that the square of the longest side of a right-angled triangle is equal to the sum of the squares of the other two sides. This is reviewed in Unit I1.

However, it was **Euclid**, who lived about 300 BC, who brought the various unconnected theorems together into one system in his publication *Elements*. The 13 'books', or parchment rolls, of *Elements* are among the greatest achievements of the human mind. For more than 1000 years mathematicians could add little of importance to them. Euclid's text served as a basic textbook in geometry almost without alteration into the 20th century.

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C3 *Areas* Introduction

He realised that only a small number of postulates (that is, axioms, unproven but universally accepted assumptions; for example, there is one and only one line that passes through two distinct points) underlay the various geometric theorems known at the time. He determined that these theorems could be deduced from just five postulates.

1. *A straight line may be drawn through any two given points.*
2. *A straight line may be drawn infinitely or be limited at any point.*
3. *A circle may be drawn using any given point as the centre, and with any given radius (the distance from the centre to any point on the circle).*
4. *All right angles are congruent. (A right angle is an angle that measures 90°).*
5. *Given a straight line and a point that does not lie on the line, one and only one straight line may be drawn that is parallel to the first line and passes through the point.*

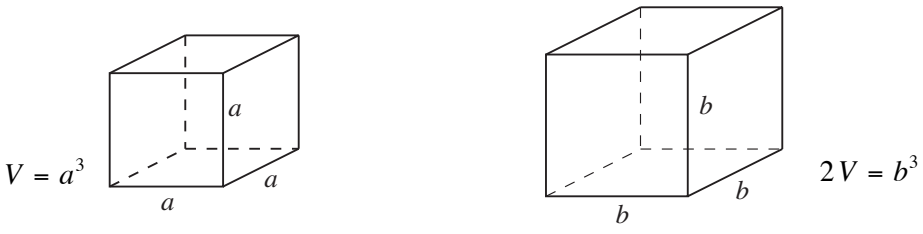
These five postulates can be used in combination with various defined terms to prove the properties of two- and three-dimensional figures, such as areas and circumferences.

The major importance of Euclid's work lay in his method rather than his results. Most of the theorems he proved had been known for many years. However, it had not been known that they were all closely related or that most of them could be derived from a few basic axioms. By proving this, Euclid established the value of the deductive method.

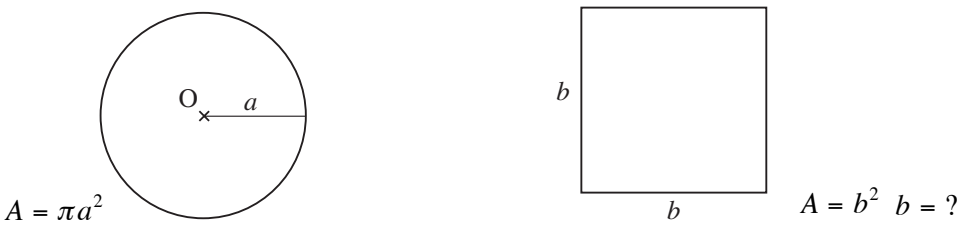
The Greeks introduced construction problems, which require a certain line or figure to be constructed by using straightedge and compass alone. (A compass is an instrument used to draw circles; it consists of two pointed arms joined at an adjustable angle.) Simple examples of these problems include the construction of a line that is twice as long as another line or of a line that divides a given angle into two equal angles.

Three famous construction problems dating from the time of the ancient Greeks resisted the efforts of many generations of mathematicians to solve them:

- *duplicating the cube (constructing a cube with double the volume of a given cube)*



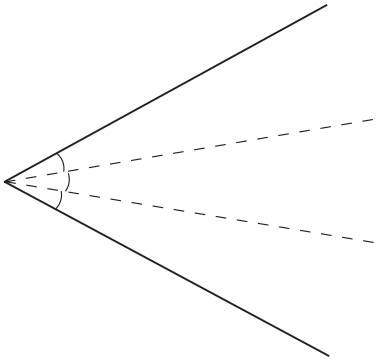
- *squaring the circle (constructing a square equal in area to a given circle)*



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C3 *Areas* Introduction

- *trisecting the angle (dividing a given angle into three equal parts).*



construction using straight line and pair of compasses

None of these constructions is possible with straightedge and compass alone, but the impossibility of squaring the circle was not finally proved until 1882. Constructions are covered in Unit H4.

Archimedes, one of the greatest Greek scientists, made a number of important contributions to geometry during the 3rd century BC. He devised ways to measure the areas of a number of curved figures and the surface areas and volumes of solids bounded by curved surfaces, such as cylinders. He also worked out a method for approximating the value of π (the ratio between the diameter and circumference of a circle) and stated that numerically it lay between 3 10/70 and 3 10/71.

Geometry during the Middle Ages

Geometry, like most other sciences, advanced little from the fall of the Roman Empire in the 5th century AD to the end of the Middle Ages in the 15th century although **Aryabhata**, an Indian mathematician living in the 6th century, discovered, or perhaps rediscovered, the formula for the area of an isosceles triangle. He also determined the value of π with remarkable accuracy to four decimal places, setting it equal to 62832/20000, or 3.1416.

The next great stride in geometry was taken by French philosopher and mathematician **René Descartes**, whose influential treatise *Discourse on Method* was published in 1637. Descartes introduced a method of representing geometric figures within a coordinate system. His work forged a link between geometry and algebra by showing how to apply the methods of one discipline to the other. This link is the basis of analytic geometry, a subject that underlies much modern work in geometry that features in Strand G of this course.

Key points and principles

- There are formulae for the area of many 2D shapes. You should be familiar with these and able to use the formulae in contexts and with compound shapes.
- The formulae for sector area and arc length can be derived from the formula for circles.
- Always be careful to ensure that units used are consistent throughout.

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C3 *Areas* Introduction

Glossary of terms

Area of 2D shape For example, area of circle $= \pi r^2$

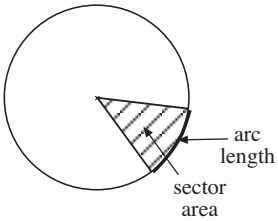
Surface area of 3D shape For example, surface area of cuboid $= 2(ab + bc + ca)$

surface area of a cylinder $= 2\pi r^2 + 2\pi rh$

surface area of a sphere $= 4\pi r^2$

Sector area

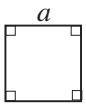
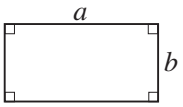
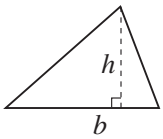
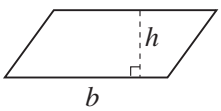
Arc length



Chord (a straight line joining any two points on a curve)

Facts to remember

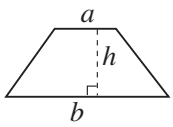
You will find it useful to be fully familiar with the facts listed here.

Shape	***Sketch***	***Area***
Square		a^2
Rectangle		ab
Triangle		$\frac{1}{2}bh$
Parallelogram		bh

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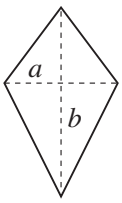
C3 *Areas* Introduction

Trapezium



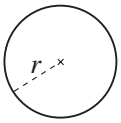
$$\frac{1}{2}(a + b)h$$

Kite



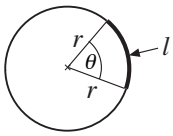
$$\frac{1}{2}ab$$

Circle



πr^2 (and circumference $= 2\pi r$)

Arc length of sector



$$l = \frac{\theta}{360} \times 2\pi r$$

Area of sector

$$A = \frac{\theta}{360} \times \pi r^2$$