STRAND D: Probability

D1 *Probability of One Event*

Text

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D1 Probability of One Event

D1.1 Probabilities

Probabilities are used to describe how likely or unlikely it is that something will happen. Weather forecasters often talk about how likely it is to rain, or perhaps snow, in particular parts of the world.

Note *Assume that any die referred to in this unit is 6-sided and fair. Assume that a coin is fair. A pack of cards comprises 52 cards.*

Worked Example 1

(a) When you roll a die, which number are you most likely to get?

(b) If you rolled a die 600 times how many sixes would you expect to get?

(c) Would you expect to get the same number of ones?

Solution

(a) You are equally likely to get any of the six numbers.

(b) You would expect to get a six in about $\frac{1}{6}$ of the throws, so 100 sixes.

(c) Yes, in fact you would expect to get about 100 of each number.

Worked Example 2

Use one of the following to describe each one of the statements (a) to (d).

Certain	*Very likely*
Likely	*Unlikely*
Very unlikely	*Impossible*

(a) It will snow in Barbados tomorrow.

(b) It will rain in London on Saturday.

(c) You win a car in a competition tomorrow.

(d) You will be late for school tomorrow.

Solution

(a) *Impossible* (It will never snow in the Caribbean because of the islands' close proximity to the Equator.)

(b) *Likely* or *Very likely*.

(c) *Very unlikely* if you have entered the competition. *Impossible* if you have not entered the competition.

(d) *Very unlikely*.

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Exercises

1. If you toss a coin 500 times, how many times would you expect it to land:
   (a) on its side,
   (b) heads up,
   (c) tails up?

2. A tetrahedron is a shape with 4 faces. The faces are numbered 1, 2, 3 and 4. The tetrahedron is rolled 200 times. How many times would you expect it to land on a side numbered
   (a) 4 (b) 2 (c) 5?

3. Describe each of the following events as:

   Impossible,
   Unlikely,
   Likely,
   Certain.

   (a) You roll a normal die and score 7.
   (b) You fall off your bike on the way home from school.
   (c) You complete all your maths homework correctly.
   (d) Your favourite football team wins their next match.
   (e) Your parents decide to double your pocket money next week.
   (f) You have rice with your next school meal.
   (g) The school bus is on time tomorrow.

4. Describe two events that are:
   (a) Certain,
   (b) Impossible,
   (c) Likely to happen,
   (d) Unlikely to happen.

5. How many sixes would you expect to get if you rolled a die:
   (a) 60 times,
   (b) 120 times,
   (c) 6000 times,
   (d) 3600 times?

6. Nathan tossed a coin a large number of times and got 450 heads. How many times do you think he tossed the coin?

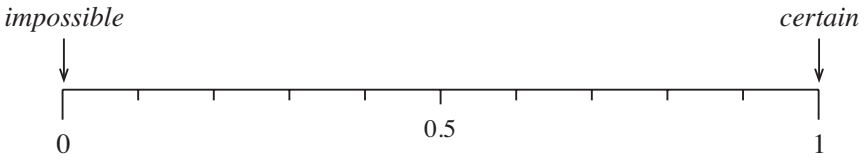
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7. Kea rolled a die and got 250 twos.
   (a) How many times do you think she rolled the die?
   (b) How many sixes do you think she got?

8. Scott chooses a playing card from a full pack 100 times. How many times would you expect him to get:
   (a) a red card,
   (b) a black card,
   (c) a heart,
   (d) a diamond?

D1.2 Straightforward Probability

Probabilities are given values between 0 and 1. A probability of 0 means that the event is *impossible*, while a probability of 1 means that it is *certain*. The closer the probability of an event is to 1, the more likely it is to happen. The closer the probability of an event is to 0, the less likely it is to happen.



Worked Example 1

When you toss a coin, what is the probability that it lands heads up?

Solution

When you toss a coin there are two possibilities, that it lands heads up or tails up. As one of these must be obtained,

$$p(\text{heads}) + p(\text{tails}) = 1$$

But both are equally likely so

$$p(\text{heads}) = p(\text{tails}) = \frac{1}{2}$$

Worked Example 2

The probability that it rains tomorrow is $\frac{2}{3}$.

What is the probability that is does not rain tomorrow?

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Solution

Tomorrow it must either rain or not rain, so,

$$p(\text{rain}) + p(\text{no rain}) = 1$$

The probability it rains is $\frac{2}{3}$, so

$$\frac{2}{3} + p(\text{no rain}) = 1$$

$$p(\text{no rain}) = 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

So the probability that it does not rain is $\frac{1}{3}$.

Exercises

1. What is the probability that it will not rain tomorrow, if the probability that it will rain tomorrow is:

   (a) 0.9 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$?

2. Ben plays pool with his friends. The probability that he beats Grant is 0.8 and the probability that he beats Martin is 0.6.

   (a) What is the probability that Grant beats Ben?

   (b) What is the probability that Martin beats Ben?

3. The probability that a plane is late arriving at Exeter International Airport is 0.02. What is the probability that it is not late?

4. Joe has bought a trick coin in a joke shop. When he tosses it the probability of getting a head is $\frac{1}{5}$. What is the probability of getting a tail with this coin?

5. A weather forecaster states that the probability that it will rain tomorrow is $\frac{3}{7}$.

   (a) Find the probability that it will not rain tomorrow.

   (b) Is it more likely to rain or not to rain tomorrow?

6. The probability that it will snow during the winter in a certain city in France is 0.01. What is the probability that it does not snow?

7. A school basketball team play 20 matches each year. The probability that they win any match is $\frac{3}{5}$.

   (a) What is the probability that they lose a match?

   (b) How many matches can they expect to win each year?

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8. When Chloe plays battle chess on her home computer the probability that she wins depends on the level at which she plays the game.

Level	**Probability Chloe wins**
Easy	0.9
Medium	0.4
Hard	0.1

What is the probability that the computer wins if the level is set to:

(a) Medium (b) Hard (c) Easy?

9. A student is selected at random from a school. The probability that the student is a girl is $\frac{11}{20}$, the probability that the student is left handed is $\frac{1}{11}$ and the probability that the student wears glasses is $\frac{4}{13}$.

Find the probabilities that a student selected at random,

(a) is a boy (b) is right handed (c) does not wear glasses.

10. It has been estimated that the probability that a person has blue eyes is $\frac{1}{9}$.

Is it true that the probability that a person has brown eyes is $\frac{8}{9}$?

11. A machine makes compact discs. The probability that a perfect compact disc will be made by this machine is 0.85.

Work out the probability that a compact disc made by this machine will not be perfect.

12. Here are three possible events

A A coin when tossed will come down heads.

B There will be a heatwave in Wiltshire next January.

C There will be a baby born tomorrow in Liverpool.

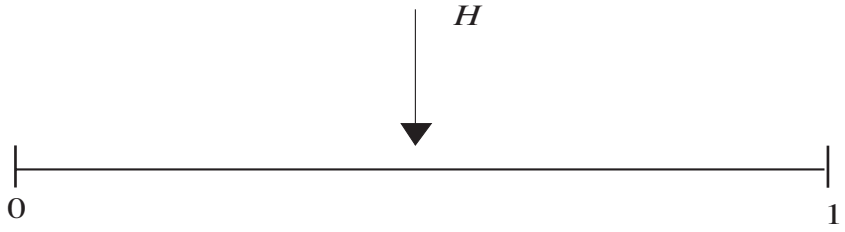
Which of the three events is

(a) most likely to happen?

(b) least likely to happen?

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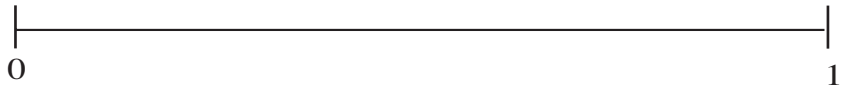
13.



A probability line is shown above.

The arrow *H* on the line shows the probability that, when a coin is tossed, it will come down 'heads'.

(a)



Copy the probability line and put an arrow *S* on the line to show the probability that it will rain where you live tomorrow.

Explain why you put your arrow in that position.

(b) Put an arrow, *L* on the line to show the probability that the next lorry you see travelling on the road will have a male driver.

Explain why you put your arrow in that position.

D1.3 Finding Probabilities Using Relative Frequency

Sometimes it is possible to calculate values for the probability of an event by symmetry arguments, like tossing a coin and getting a head. For other events probabilities can be estimated by using results of experiments or records of past events.



Worked Example 1

In a town in Yorkshire there was rain on 18 days in February 2009. Use this information to estimate the probability that it rained there on a particular day in February 2009.



Solution

It rained on 18 out of the 28 days, so the relative frequency of rain is

$$\frac{18}{28} = \frac{9}{14}$$

So the probability that it rained on any day in February can be estimated as $\frac{9}{14}$.

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Worked Example 2

Henry carries out an experiment with a piece of buttered toast. He drops it 50 times and finds that 35 times it lands butter side down. Use these results to estimate the probability that a piece of toast lands butter side up when dropped.

Solution

The toast landed butter side down 35 of the 50 times, so the relative frequency is $\frac{35}{50} = \frac{7}{10}$.

So the probability that the toast lands butter side down can be estimated as $\frac{7}{10}$. Hence, the probability of it landing butter side up is estimated as $\frac{3}{10}$.

Exercises

1. (a) Conduct an experiment with a drawing pin, by dropping it in the same way a large number of times. You could drop it 100 times and record whether it lands point up or point down.
   (b) Use your results to estimate the probability that a drawing pin lands point up.

2. (a) Obtain a short stick, such as a cocktail stick. On a sheet of paper draw parallel lines that are 6 cm apart. Drop the stick onto the sheet of paper a large number of times and record whether or not it lands on a line.
   (b) Use your results to estimate the probability that the stick lands on a line.

3. When you toss a coin you would expect to get a head half of the time.
   (a) Toss a coin 20 times and record the results. How well do they compare with your expectation?
   (b) Toss the coin another 30 times, so that you have 50 results. How well do they compare with your expectation now?

4. Mrs Oliver observed that her bus was late on 6 of her 24 work days in March.
   Estimate the probability that the bus was late on any one day.

5. A football team plays on average 40 matches each season and wins 32 of them.
   (a) Estimate the probability that this team wins a match.
   (b) Give a reason why this probability could change.

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6. Six students play regularly in a chess club. The number of games that each student has won is recorded in the table below.

Player	Games Won	Games Lost
Tom	4	10
Andrew	7	3
Danielle	3	9
Rachel	4	16
Connor	6	12
Malene	12	6

(a) Use this data to find the probability that each student wins a match.

(b) Which student is the best player?

(c) Which student is the worst player?

(d) If Connor played Tom, who do you think would be more likely to win?

7. A garage records the number of cars that they sell each week over a 24 week period. The numbers for each week are given below.

3, 4, 8, 6, 5, 7, 4, 3, 6, 5, 2, 4,

5, 7, 6, 9, 2, 4, 5, 6, 7, 4, 3, 5.

Use this data to establish the probability that in any week;

(a) more than 5 cars are sold,

(b) fewer than 5 cars are sold,

(c) exactly 5 cars are sold.

8. A gardener plants 40 seeds and 32 of them produce healthy plants.

(a) Estimate the probability that a seed produces a healthy plant.

(b) If 120 seeds were planted, how many healthy plants can the gardener expect to obtain?



Investigation

A student types 3 different letters and 3 different addresses on 3 envelopes. She puts the letters into the envelopes randomly and sends then to 3 of her friends, A, B and C. What is the probability that

(a) only one of her friends will receive the correct letter,

(b) only two of them will receive the correct letters?

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D1.4 Determining Probabilities

When the outcomes of an event are all equally likely, then probabilities can be found by considering all the possible outcomes.

For example, when you toss a coin there are two possible outcomes, either heads or tails.

So
$$p(\text{head}) = \frac{1}{2}$$
$$p(\text{tail}) = \frac{1}{2}$$

The probability of an outcome is given by

$$\frac{\text{number of ways of obtaining outcome}}{\text{number of possible outcomes}}$$

provided all the outcomes are equally likely.



Worked Example 1

A card is taken at random from a full pack of playing cards with no jokers. What is the probability that the card:

(a) is an ace, (b) is black,

(c) is a heart, (d) has an even number on it?



Solution

First note that each card is equally likely to be selected, and that there are 52 possible outcomes.

(a) There are 4 aces, so
$$p(\text{ace}) = \frac{4}{52}$$
$$= \frac{1}{13}$$

(b) There are 26 black cards, so
$$p(\text{black}) = \frac{26}{52}$$
$$= \frac{1}{2}$$

(c) There are 13 hearts in the pack, so;
$$p(\text{heart}) = \frac{13}{52}$$
$$= \frac{1}{4}$$

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(d) There are 20 cards with even numbers on them, so;

$$p(\text{even number}) = \frac{20}{52}$$
$$= \frac{5}{13}$$



Worked Example 2

In a class of 30 students, 16 are girls, 4 wear glasses and 3 are left handed.

A student is chosen at random from the class. What is the probability that this student is:

(a) a girl, (b) right-handed, (c) wearing glasses.



Solution

All the students in the class are equally likely to be selected, when the choice is made at random.

(a) In the class there are 16 girls, so

$$p(\text{girl}) = \frac{16}{30}$$
$$= \frac{8}{15}$$

(b) There are 3 left handed students and so the other 27 must be right handed.

So,

$$p(\text{right handed}) = \frac{27}{30}$$
$$= \frac{9}{10}$$

(c) There are 4 students wearing glasses so,

$$p(\text{wears glasses}) = \frac{4}{30}$$
$$= \frac{2}{15}$$



Exercises

1. Robert takes a card at random from a full pack of 52 playing cards. What is the probability that his card:

   (a) is a diamond, (b) is a spade, (c) is a seven,

   (d) is a king, (e) has a prime number on it?

2. Repeat question 1, this time for a pack of playing cards containing 2 jokers (a total of 54 cards).

3. When you roll a die, what is the probability of obtaining:

   (a) a six, (b) a five,

   (c) an even number, (d) a prime number?

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4. A new game includes an octagonal roller with faces numbered from 1 to 8. When the roller is rolled, what is the probability of obtaining:

   (a) a number 8, (b) a number 1,

   (c) an odd number, (d) a number greater than 3,

   (e) a number less than 3?

5. In a class of 32 students, 20 have school lunches and the rest bring their own food. What is the probability that a student chosen at random from the class brings their own food?

6. In a 'lucky dip' game at a school fair, a tub contains 50 prizes at the start of the fair. There are 20 superballs, 10 pens, 10 toy cars and 10 packets of sweets.

   What is the probability that the first person to visit the lucky dip:

   (a) wins a superball, (b) does not win a pen,

   (c) wins a packet of sweets, (d) does not win a toy car.

   If the first person wins a pen, what is the probability that the second person:

   (e) wins a pen, (f) does not win a toy car,

   (g) wins a packet of sweets?

7. An express bus leaves Edinburgh with 18 passengers. It stops at Perth, where another 12 passengers join the bus. At Dundee it stops again and 20 more passengers get on board. When the bus arrives at its destination, Aberdeen, all the passengers get off and one is chosen at random to be interviewed about the journey.

   Find the probabilities that this passenger:

   (a) was on the bus for the whole journey,

   (b) got on the bus at Perth,

   (c) got on the bus at Perth or Edinburgh,

   (d) got on the bus at Perth or Dundee.

8. Lester has the following 5 coins in his pocket:

   £2, £1, 50 p, 20 p, 10 p

   He selects *one* coin at random to put in a charity collection box.

   What is the probability that he:

   (a) gives more than £1,

   (b) has less than £2 left in his pocket,

   (c) has more than £3 left in his pocket,

   (d) gives away less than half the money in his pocket?

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9. Five different types of model dinosaurs are being given away in cereal packets. A model dinosaur is put into each packet at random and five dinosaurs are needed for a complete set.

   (a) Bradley already has 3 of the 5 models. What is the probability he gets a different one in the next packet he opens?

   (b) Adam only needs one more dinosaur to complete his set. What is the probability that he gets this dinosaur in the next packet he opens?

   (c) Leo has only one dinosaur in his collection. What is the probability that he gets the same one in his next packet?

10. A bag contains 5 red counters, 3 green counters and 2 blue counters. Counters are taken out of the bag at random, but are not put back into the bag.

   (a) What is the probability that the first counter taken out is green?

   (b) If the first counter is green, what is the probability that the second counter is green?

   (c) If the first two counters are green, what is the probability that the third counter is green?

   (d) If a red counter is followed by a blue counter, what is the probability that the third counter is green?

11. Grant has a bag of 30 marbles. There are 7 red marbles in the bag. He chooses a marble at random from the bag.

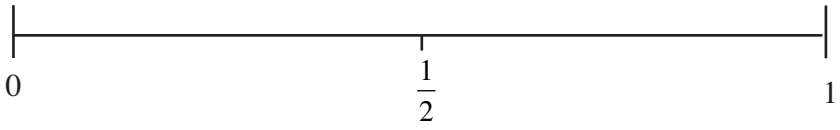
   What is the probability that

   (a) he gets a red marble?

   (b) he gets a marble which is not red?

12. (a) Make a copy of the line below and mark with an *H* the probability of getting a head when one coin is tossed.

   (b) On the same line, mark with an *S* the probability of getting a 5 when a six - sided die is thrown.



13. In a raffle, 200 tickets are sold.

   (a) Holly buys one ticket. What is the probability that she wins first prize?

   There are lots of prizes.

   (b) The probability that Holly wins a prize is $\frac{1}{10}$. How many prizes are there?

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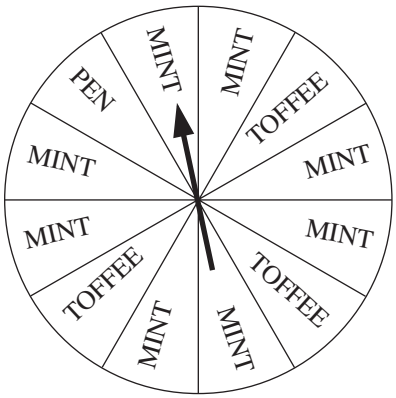
14. Sophie is shown a selection of ski-holiday brochures. There are three for Italy, two for Austria and five for Switzerland.

Sophie takes one of these brochures at random.

(a) What is the probability that it is for Italy?

(b) What is the probability that it is **not** for Austria?

15. To play a game you spin the pointer.
You win the prize on which the pointer stops.



Ryan has one spin.

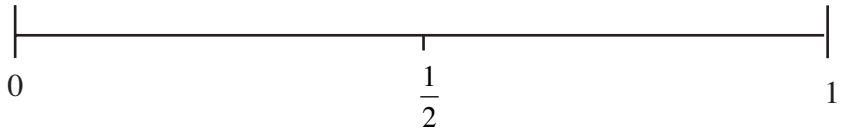
(a) Which prize is Ryan most likely to win?

(b) Explain your answer to part (a)

Donna has one spin.

(c) Make a copy of the line shown below and mark with a P the probability that Donna will win a pen.

(d) On the line mark with a W the probability that Donna will win a watch.



Challenge!

Anthony, Patrick and Millie repeatedly take turns tossing a die. Anthony begins; Patrick always follows Anthony; Millie always follows Patrick; Anthony always follows Millie. What is the probability that Millie will be the first one to toss a six?

Investigations

1. How many students must you gather together so as to ensure that at least two of the students have birthdays falling in the same month?

2. A secondary school has an enrolment of 1100. Is it possible that there will be four students in the school whose birthdays fall on the same day of the year? Explain your answers.

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