D2 *Probability of Two or More Events* Activities

Activities

D2.1 Open and Shut Case

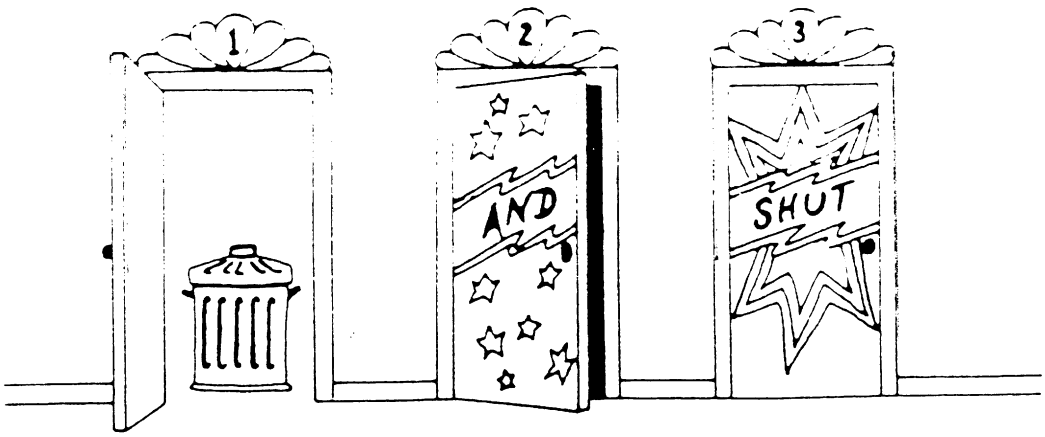
D2.2 Fruit Machines

D2.3 Birthdays

Notes for Solutions (1 page)

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ACTIVITY D2.1 *Open and Shut Case*



In a Game Show in America, the contestant is offered a choice of one of three doors to open. Behind one of these doors is the star prize, a *car,*– but behind the other two doors are *dustbins*!

Once the contestant has chosen say Door 2, the host, who already knows what is behind each door, opens one of the doors, say Door 1, to reveal a dustbin.

He then asks the contestant,

"Do you want to stick with your original choice (Door 2) or switch to the other closed door (Door 3)?"

1. Is it to your advantage to change your choice from Door 2 to Door 3?

It is easy to provide an argument for either policy.

ARGUMENT 1 When one door is opened, there is an equal chance of the car being behind either of the other two doors, so there is no need to change.

ARGUMENT 2 There is a *2* in *3* chance of being *wrong* initially. If you were wrong and changed, you would now be right, so the probability is reversed and you will now be *right* 2 out of 3 times.

2. Simulate this Game Show by playing it with a partner. One of you is the contestant and the other the Game Show host. You will need to play the game at least 20 times in order to gain insight into the solution

If this simulation does not convince you, then try using a *computer program* to simulate the situation 10000 or 20000 times.

Extension

Suppose there are now *four* doors with a star prize behind one door and dustbins behind each of the other doors.

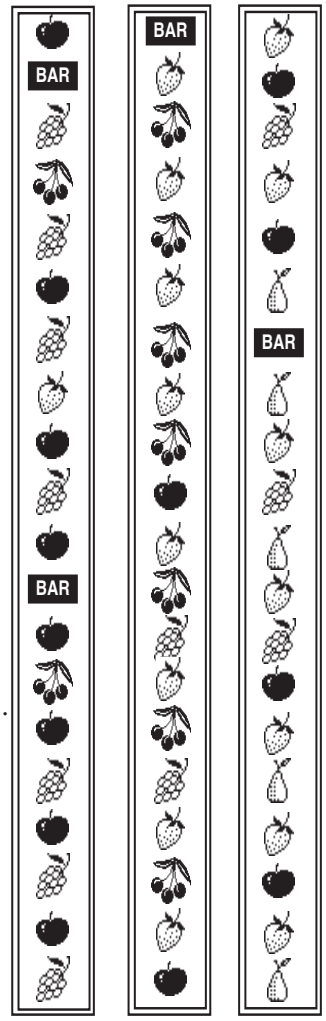
Again the contestants are offered the chance of changing their choices.

Should they change, and if they do, what is now their probability of winning?

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ACTIVITY D2.2 *Fruit Machines*

A fruit machine with 3 DIALS and 20 SYMBOLS (not all different) on each dial is illustrated opposite. Each dial can stop on any one of its 20 symbols, and each of the 20 symbols on a dial is equally likely to occur.



So, for example, the *Grapes* on DIAL 1 are likely to occur on average 7 times out of 20.

You insert a 10 p coin, press a button and the three dials spin round. You then press more buttons to stop each dial at random.

The three symbols highlighted determine how much, if anything, is won.

For example, Suppose the machine makes the payouts shown opposite.

Combination	Payout (in J$10s)
3 BARS BAR	40
3 STRAWBERRIES	5
3 GRAPES	5
3 APPLES	5
2 BARS	20
2 CHERRIES	5

1. Copy and complete the frequency chart below for each dial.

Symbol	Dial 1	Dial 2	Dial 3
BAR	2	1	1
STRAWBERRY	1	8	..
GRAPE	7	..	..
APPLE	..	..	..
CHERRY	..	..	..
PEAR	..	..	..

We want to find the probability of each of the combinations above to see if it is worth playing. We first consider the 3 BARS combination.

2. (a) In how many ways can you obtain 3 BARS?
   (b) How many possible combinations (including repeats) are there?
   (c) What is the probability of obtaining 3 BARS?

We can find the probability of the other winning combinations in the same way.

3. Find the probabilities of obtaining all the other winning combinations.
   Your expected winnings in 10 pences are

$$40 \times (\text{probability of 3 BARS}) + 5 \times (\text{probability of 3 STRAWBERRIES}) + \ldots..$$

   but you must take off your initial payment of 10 pence.
4. What is the expected gain or loss for each go?

Extension

Design your own fruit machine, work out the probabilities of certain combinations, assign payouts and check whether the player expects to gain or lose money.

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ACTIVITY D2.3 *Birthdays*



First try this experiment. Find out the birthdays of as many of your family as possible. Do any of them have birthdays on the same day of the year?

Now try the same experiment with all the members of your class. You will find out how likely it is that two members of a group have the same birthday.

Consider each member of a group, one by one. The first person will have his/her birthday on a particular day.

1. What is the probability of the second person having a different birthday from the first?

2. What is the probability of the third person having a birthday different from both the first and second person?

3. What is the probability that at least two of the first three people have the same birthday?

This solves the problem of a group of three people. As expected, it is not likely that any 2 out of 3 people will have the same birthday.

4. Repeat the problem above for 4 people. What is the probability that at least 2 of them have the same birthday?

5. Using either a computer programme or a calculator, solve the problem for a group of n people, where $n = 10, 20, 30$, etc.

6. What is the probability that 2 members of your class have the same birthday?

Extension
How many people are needed in the group to be 95% sure that there will be at least two with the same birthday?

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ACTIVITIES D2.1 - D2.3 *Notes for Solutions*

D2.1 1. Yes, you should change choice; or, at least toss a coin to show which of the two doors you now go for - if you stay with your original choice, your original chance of winning, $\left(\frac{1}{3}\right)$, will not change!

(You might need to write a computer simulation, as suggested, to argue this!)

D2.2 2. (a) 2 (b) 800 (c) $\frac{1}{4000}$

3.

Symbols	Number of ways	Probability
3 STRAWBERRIES	56	$\frac{56}{8000}$ $(1 \times 8 \times 7 = 56)$
3 GRAPES	42	$\frac{42}{8000}$
3 APPLES	64	$\frac{64}{8000}$
3 BARS	94	$\frac{94}{8000}$ $(2 \times 1 \times 19 + 2 \times 19 \times 1 + 18 \times 1 \times 1 = 94)$
2 CHERRIES	280	$\frac{280}{8000}$ $(2 \times 7 \times 20 = 280)$

4. On average, you will lose almost 5 p per go.

D2.3 1. $\frac{364}{365}$

2. $\frac{363}{365}$

3. $1 - \frac{364}{365} \cdot \frac{363}{365} \approx 0.008$

4. 0.0164

5. $n = 10 \Rightarrow p = 0.117$; $n = 30 \Rightarrow p = 0.706$; $n = 10 \Rightarrow p = 0.117$

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